Broken Symmetries and Chaotic Behavior in ^{26}Al

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Fluctuation properties of positive-parity states in ^{26}Al from the ground state to the resonance region (0–8 MeV excitation energy) are found to be consistent with the Gaussian-orthogonal-ensemble version of random-matrix theory. Although isospin is an approximately good quantum number in this system, the fluctuation properties of the combined sequence ($T=0$ and $T=1$ states) are consistent with those of a single Gaussian-orthogonal-ensemble sequence. This agrees with earlier predictions by Dyson and Pandey and with recent analysis by Guhr and Weidenmüller.

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The topic of quantum chaos and searches for quantum systems which display chaotic behavior have been the subject of much discussion recently.[1,2] One approach is to study the quantum analog of a system whose classical behavior is known to be chaotic; the canonical example of this approach is the quantization of Sinai's billard by Berry.[3] Bohigas, Giannoni, and Schmit[4] studied the level fluctuations of the quantum Sinai's billiard and concluded that they were "fully consistent" with the predictions of the Gaussian orthogonal ensemble (GOE) of random-matrix theory.[5] They offered the conjecture that this relationship (between time-reversal–invariant systems whose classical analogs are chaotic and the GOE) is universal. Additional results for different systems have tended to support this conjecture,[6] although there is evidence that the fluctuation properties may not be fully governed by random-matrix theory (see, e.g., Casati, Chirikov, and Guarneri[7]). Berry[8] showed that certain systematic deviations from random-matrix theory [the $\Delta_3(L)$ parameter—a measure of spectral rigidity—falls below GOE predictions] can be explained semiclassically. Wintgen[9] demonstrated a connection between classical periodic orbits and the long-range correlations in quantum spectra which Δ_3 measures.

In this circumstance, numerical "experiments" are extremely valuable. For example, Seligman, Verbaarschot, and Zirnbauer[10,11] considered the transition from regular to irregular spectra for a class of Hamiltonians; Wintgen and Marxer[12] examined the level statistics for the anisotropic Kepler problem; Meredith, Koonin, and Zirnbauer[13,14] considered the spectural fluctuations and the overlap distribution for the three-orbital Lipkin-Meshov-Glick model. In each case, a classically chaotic system corresponds to GOE statistics, while a classically regular system corresponds to Poisson statistics.

Nuclear energy levels provide the best experimental data thus far for tests of random-matrix theory. The most extensive set of experimental data used for comparison with GOE predictions has been a collection of high-quality neutron and proton resonance data. The fluctuation properties of this ensemble of energy levels have been studied with a variety of measures[15,16]; the data show both the short- and long-range order required by GOE. It would be extremely interesting to apply these techniques to low-lying nuclear states. The requirements that the data sets be pure (correct spin assignments) and complete (no missing levels) impose severe limitations; von Egidy and co-workers[17,18] have compiled such level schemes for low-lying states in many nuclei. Preliminary analysis by von Egidy and co-workers[17,18] and by Abul-Magd and Weidenmüller[19] suggest that regularity is observed for rotationallike states, while other states are completely or partially chaotic. This effect has been explored in the context of the interacting-boson model by Paar and Vorkapic.[20]

For each nucleus the experimental sample sizes for the low-lying states are extremely small. What is needed is a nucleus for which all of the levels are known from the ground state to the resonance region. With sufficient levels, one could study fluctuation properties of low-lying states *and* resonance states and (if regularity was observed at low energies) examine empirically the transition from regularity to chaoticity. Of course, there is no such ideal nucleus. For medium and heavy nuclei, there is a several-MeV gap between low-lying states and resonance states. For light nuclei, the level density is far below the ideal, but at least it is possible in principle to obtain all of the levels. We believe that at present ^{26}Al most closely approximates the ideal nucleus. For ^{26}Al, the first 100 positive-parity states have been identified.[21-23] The spectroscopic assignments have been confirmed by detailed comparison with the nuclear shell model.[23] (The negative-parity states are on almost as firm ground experimentally, but there is no detailed comparison with theory available.) These data provide the first opportunity to examine the fluctuation properties in the same nucleus from the ground state to the resonance region. In addition, the isospin of the positive-parity states is known; the $T=0$ and $T=1$ states coexist throughout the entire energy region. Thus ^{26}Al also provides the first opportunity to study empirically the fluctuation properties of states with a (known) broken symmetry.

First we describe the corrections required to obtain a set of data suitable for analysis of the fluctuation properties. This analysis yields fluctuations consistent with GOE for all energies. The data are also consistent with GOE independent of whether the isospin quantum number is considered. This latter result is consistent with earlier theoretical predictions by Dyson[24] and Pandey[25] and with recent analysis by Guhr and Weidenmüller.[26]

To evaluate the fluctuation properties, we consider the nearest-neighbor spacing distribution, the linear correlation coefficient between adjacent spacings, and the Dyson-Mehta Δ_3 statistic. The prescription is to take all of the states of the same J^π, and to determine a set of dimensionless spacing parameters $\{x_i\}$, with $x_i \equiv s_i/D$, where s_i is the ith nearest-neighbor spacing and D is the average spacing. However, D is a function of excitation energy, since the average level density $\rho(E)$ changes radically over the 8-MeV range of excitation energy. To remove the energy dependence, the data were fitted with polynomials in energy, thus avoiding the use of semiempirical level-density models. Data are available for five different positive-parity sequences ($J^\pi = 1^+$-5^+); the number of levels in each sequence ranges from 12 to 25. Each J^π sequence was fitted separately; satisfactory fits were always obtained. To avoid possible introduction of bias in the data (e.g., long-range correlations created by overfitting of the data and reduction of the fluctuations), careful attention was paid to the values of χ^2 and to the physical reasonableness of the smooth curve for $\rho(E)$.

Initially we ignored isospin. After correction for the energy dependence of the average level density, a set of x's was obtained for each value of J^π. A nearest-neighbor spacing distribution was generated for each sequence; errors were determined with the bootstrap method.[27] The overall spacing distribution was obtained by averaging of the results from the different values of J^π. In Fig. 1 the nearest-neighbor spacing distribution $P(x)$ and the cumulative probability are plotted versus x. The Poisson and Wigner distributions are shown for comparison. The data clearly prefer the Wigner (GOE) distribution; comparison of the cumulative probability distribution with the Wigner and Poisson distributions yields a χ^2 per point of 1.0 for the Wigner and 8.1 for the Poisson. The apparent "bump" is small x is due to spacings which appear to be randomly distributed in both energy and J^π. The agreement with GOE is good at all energies—no energy dependence was observed in the fluctuation properties. The linear correlation coefficient between adjacent spacings is -0.24 ± 0.08, in good agreement with the GOE prediction of -0.27.

Next we consider Δ_3, as a measure of the long-range correlations. The function

$$\Delta_3(\alpha,L) = (LD)^{-1} \min_{A,B} \int_\alpha^{\alpha+LD} dE\,|N(E) - AE - B|^2$$

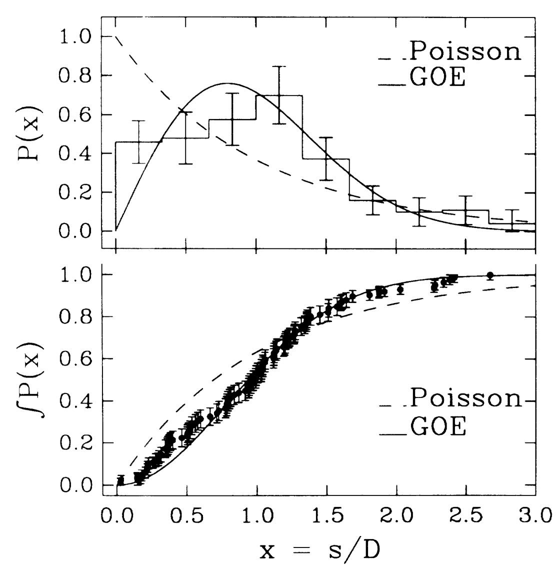

FIG. 1. (Top) The nearest-neighbor spacing distribution $P(x)$ plotted vs the dimensionless spacing parameter $x_i \equiv s_i/D$ for the positive-parity states in ^{26}Al. The energy dependence of the average level density is first removed and a set of x's obtained for each J^π sequence. The resulting sets of x's are then combined. The Poisson and Wigner distributions are shown for comparison. (Bottom) The cumulative probability ($\equiv i/N$ for the ith spacing, where N is the total number of spacings, which are ordered according to the size of x). The integrals of the normalized area under the Poisson and Wigner distributions are shown for comparison.

measures the least-squares deviation of the function $N(E)$ from a straight line for the interval $[\alpha,\alpha+LD]$ on the energy axis. Following Bohigas, Giannoni, and Schmit,[4] we chose successive intervals which overlap by $LD/2$. (One wishes to improve statistics, but avoid the introduction of correlations.) For a given set of $\{x_i\}$ for a given J^π, a value of $\Delta_3(L)$ is calculated by averaging over the different intervals for that L. The results for the different J^π sequences are then combined. The results are shown in Fig. 2. For comparison, the GOE prediction ($\propto \ln L$ for large L) and the classical limit $[\Delta_3(L)=L/15]$ are also shown. The experimental results again agree very well with the GOE prediction (χ^2 per point of 2.5) and not at all with the classical value (χ^2 per point of 176). Qualitatively similar results were obtained for the negative-parity states.

We conclude that all of the states in ^{26}Al, from the ground state to the resonance region, display GOE fluctuation properties. These results confirm (with a much larger sample size in one nucleus) the suggestions by von Egidy and co-workers[17,18] and by Abul-Magd and Weidenmüller[19]: Levels in nuclei may be chaotic even in the ground-state domain. Much more extensive data and analysis are clearly needed to consider more detailed questions about different classes of states and types of nuclei.

At first glance, what seems surprising about these results is that isospin has been neglected. Isospin is thought to be a rather good but approximate symmetry[28] in ^{26}Al and a very useful concept even in the resonance region. Experimentally, the positive-parity states seem (except for a few states of the same J^π which happen to lie very close in energy) to have a well defined isospin.[23] Suppose that isospin was a perfect quantum number. Then (for example) levels with the same J^π, but different T, should not repel each other, and the fluctuation properties would not be those for a single GOE sequence. The absence of level repulsion has been verified experimentally for states of different J and different parity, but not empirically for a broken symmetry such as isospin. These data provide the first opportunity to perform such a test. (Tests with other quantum numbers may be possible in the future—for example, examination of the fluctuation properties of rotational states in heavy nuclei might provide similar tests with the K quantum number.)

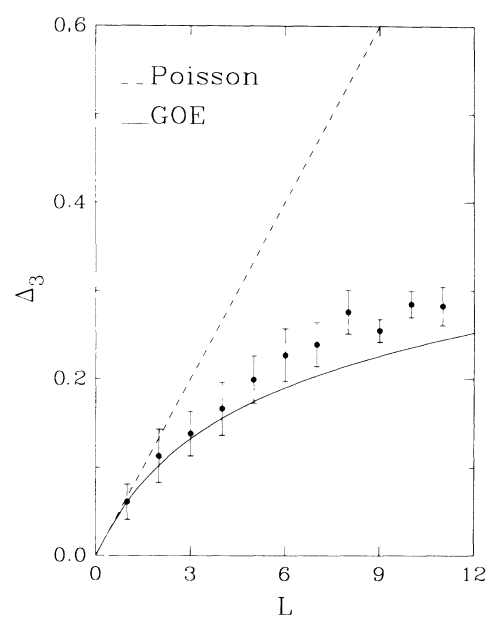

FIG. 2. The Dyson-Mehta Δ_3 statistic plotted as a function of L for the positive-parity states in ^{26}Al. The classical limit $[\Delta_3(L)=L/15]$ and the GOE prediction $[\Delta_3(L)\propto \ln L]$ are shown for comparison.

The combined data set ($T=0$ and $T=1$) agrees extremely well with GOE. When the entire analysis is repeated for the $T=0$ states, one obtains essentially identical results. For example, the χ^2 per point for the cumulative probability distribution for $T=0$ is 0.9, compared with the value of 1.0 obtained for the combined ($T=0$ and $T=1$) sequences. In spite of the limitations of sample size (107 positive-parity levels, with 75 $T=0$ states), the difference between the GOE and Poisson distributions is so great than an unambiguous choice can be made between these alternatives. The difference between one pure GOE sequence and a mixture of two GOE sequences in the appropriate combination is not nearly as large. However, all of our tests of the ^{26}Al spectrum are *consistent* with the fluctuation properties being independent of isospin, in spite of the fact that isospin appears to be a rather good quantum number in ^{26}Al for most properties. These results are consistent with a prediction due to Dyson[24] which was generalized by Pandey.[25] Pandey says, "when the symmetry is exact, the fluctuations are those of a random superposition of independent spectra .... As the (symmetry breaking) grows, we recovery very quickly the asymptotic fluctuations." The present results are the first experimental data to test these predictions. Our results are consistent with expectations—the breaking of a symmetry introduces a very rapid change in the fluctuation properties. Recent analysis by Guhr and Weidenmüller[26] of the fluctuation properties as a function of the isospin-symmetry breaking predict the observed behavior. Thus it may not be so surprising that GOE fluctuation properties are observed even in the ground-state domain of nuclear physics.

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