Determination of the Pion-Deuteron Scattering Amplitude

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New data on the polarization observables iT_{11}, T_{20}, T_{21}, and T_{22}, supplemented by $d\sigma/d\Omega$ and information from reasonable theoretical models, are used to extract the pion-deuteron scattering amplitude at 256 and 294 MeV. Using these amplitudes, partial-wave components are calculated and the spin-transfer parameter $i(11\,|\,20)$ is predicted. Some observations on helicity phase patterns are also made.

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The pion-deuteron system is one of the basic problems in medium-energy physics, providing the main testing ground for relativistic three-body theories, particularly those effects originating from the coupling to the pion-absorption channel.[1-4] This system can perhaps also provide information about quark degrees of freedom, if the predicted dibaryon resonances[5] are eventually confirmed by partial-wave analyses. Finally, the possible existence of phase patterns in the helicity amplitudes of strongly interacting systems,[6,7] is in itself an interesting question.

Very recently,[8,9] an experiment was performed by the Karlsruhe-SIN (Swiss Institute for Nuclear Research) collaboration in which they measured the vector and tensor analyzing powers iT_{11}, T_{20}, T_{21}, and T_{22} at $\theta_{\mathrm{c.m.}} > 90°$ and pion kinetic energies $T_\pi = 256$ and 294 MeV. Since very accurate data exist also for the differential cross section $d\sigma/d\Omega$, one has then a total of five observ-

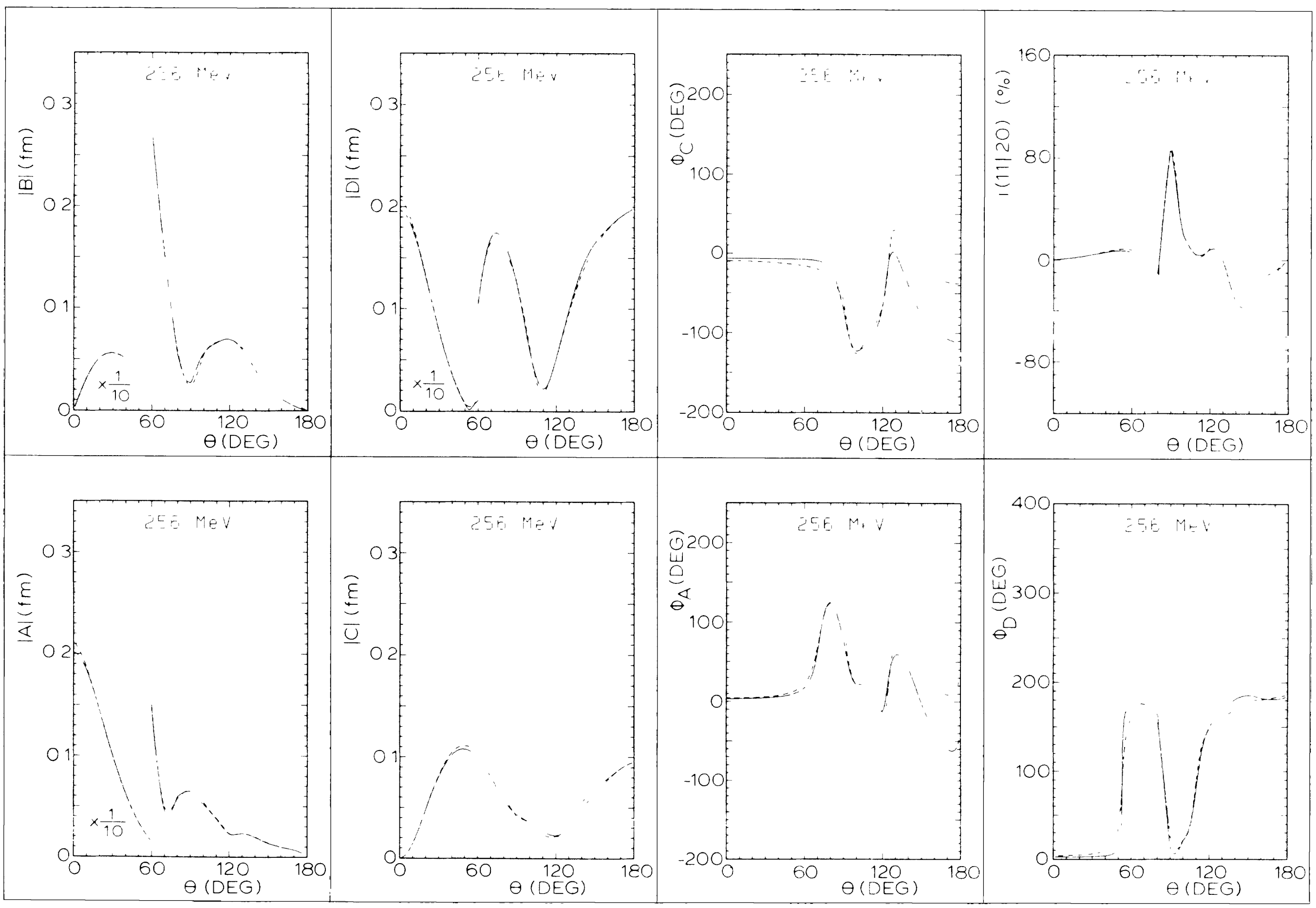

FIG. 1. Helicity amplitudes for $T_\pi = 256$ MeV obtained from the data and models 4 (dashed lines) and 5 (solid lines) and their corresponding predictions for the spin-transfer coefficient $i(11\,|\,20)_{\mathrm{lab}}$.

ables as a function of angle. The pion-deuteron system has four independent helicity amplitudes, which are $A=F_{11}$, $B=F_{10}$, $C=F_{1-1}$, and $D=F_{00}$, where the subscripts of F denote the initial and final helicities of the deuteron. The following bilinear combinations of amplitudes represent the five known observables[10]:

$$4|B|^2+|D|^2+2|A|^2+2|C|^2 \equiv U = 3d\sigma/d\Omega, \quad (1)$$

$$\mathrm{Im}[B^*(A-C+D)] \equiv V = iT_{11}U/\sqrt{6}, \quad (2)$$

$$|A|^2+|C|^2-|B|^2-|D|^2 \equiv W = T_{20}U/\sqrt{2}, \quad (3)$$

$$\mathrm{Re}[B^*(A-C-D)] \equiv X = -T_{21}U/\sqrt{6}, \quad (4)$$

$$2\mathrm{Re}(A^*C)-|B|^2 \equiv Y = T_{22}U/\sqrt{3}, \quad (5)$$

where all the variables are functions of the scattering angle θ. Since the expressions (1)-(5) do not depend on the overall phase of the helicity amplitudes, we can take one of them as real so that we will choose $B=|B|$. Thus, we are left with the seven unknowns $|A|$, $|B|$, $|C|$, $|D|$, ϕ_A, ϕ_C, and ϕ_D. Since we have only five relations between the seven unknowns, we must borrow two additional relations from a theoretical model. We have considered five different theoretical models, namely, (1) the Flinders full model,[2] (2) the Flinders model without P_{11} channel, (3) the Lyon full model,[3] (4) the Lyon model without P_{11} channel, and (5) the author's model.[4] Of these five models, we have found that only models 2, 4, and 5 are already reasonably close to the data so that they can be taken as basis for the analysis (a possible explanation of why the models of Refs. 1-3 without P_{11} channel work better than their full models has been given in Ref. 4). However, since model 2 has complete partial waves only up to $J=4$, we decided to keep it only for comparison, while for the full analysis we used models 4 and 5 which contain partial waves up to $J=9$ and $J=14$, respectively.

Using Eqs. (1)-(5), one can show that $|B|_- \leq |B| \leq |B|_+$, where $|B|_\pm$ are determined in terms of the observables only as

$$|B|^2_\pm = (X^2+V^2)\frac{2U-W-3Y\pm 2[(U+W-3Y)(U-2W)-18(X^2+V^2)]^{1/2}}{3(W-Y)^2+24(X^2+V^2)}. \quad (6)$$

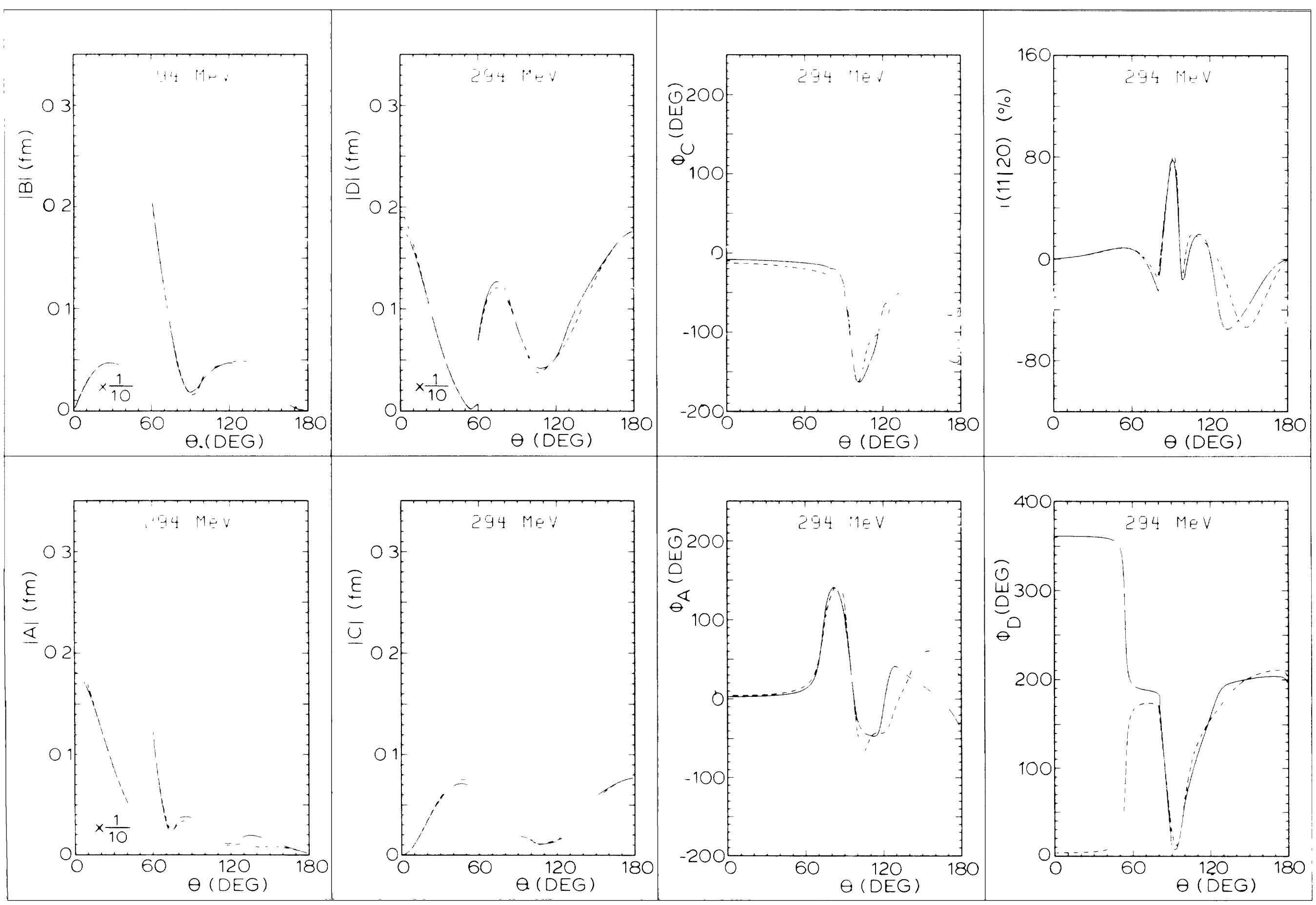

FIG. 2. Same as Fig. 1 for $T_\pi=294$ MeV.

Thus, if we write

$$|B| = |B|_- + (|B|_+ - |B|_-)\beta, \tag{7}$$

then $0 \le \beta \le 1$. For a given theoretical model, Eqs. (6) and (7) are also valid with U, V, W, X, and Y constructed from the theoretical amplitudes through Eqs. (1)-(5). Thus, if we take the parameter β from the theoretical model, that determines $|B|$. If we know $|B|$, it is easy to see from Eqs. (1)-(5) that $|D|$, $(|A|^2 + |C|^2)^{1/2}$, and ϕ_D are also determined. However, in order to determine separately $|A|$ and $|C|$ and the phases ϕ_A and ϕ_C, we need a second relation from the theoretical model. Again, we can show that $R_- \le 2|A||C| \le R_+$, with

$$R_- = |Y + |B|^2|, \tag{8}$$

$$R_+ = \tfrac{1}{3}(U+W) - |B|^2, \tag{9}$$

so that we can write

$$2|A||C| = R_- + (R_+ - R_-)\alpha, \tag{10}$$

with $0 \le \alpha \le 1$. Thus, if we use for α the corresponding theoretical quantity, that would determine the remaining amplitudes. Since $|C| = 0$ at 0° and $|A| = 0$ at 180°, it follows that $|A| = |C|$ at some intermediate angle. In the theoretical models that we used, this angle occurs very near the angle where R_+ has a minimum. Thus, we will also require that $|A| = |C|$ near the angle where the experimental function R_+ has a minimum. This will be achieved by our considering α not as a function of θ but as a function of $\theta' = a\theta + b$, where a and b are determined from the conditions $\theta' = \theta = 180°$ and $\theta' = \theta_{\text{exp}}$ when $\theta = \theta_{\text{theor}}$, where θ_{theor} (θ_{exp}) is the angle where the theoretical (experimental) function R_+ has a minimum.

In order to apply Eqs. (6)-(10), we need to know the five observables as a function of angle. For the vector and tensor analyzing powers iT_{11}, T_{20}, T_{21}, and T_{22}, we used the expressions in terms of Legendre polynomials constructed by the Karlsruhe-SIN group[9,10] to which we stripped off the error bars. For the differential cross section $d\sigma/d\Omega$, we simply interpolated the very accurate data.[11,12] For $\theta > 90°$ the five observables are known, while for $\theta < 90°$ we have only the differential cross section and the total cross section.[13] In the forward direction, however, the amplitudes should be well described by the theoretical model, since in this region the scattering amplitude is dominated by the impulse approximation which is essentially determined by the deuteron wave function and the pion-nucleon on-shell data. Thus, in the region $\theta < 90°$ we used simply the theoretical amplitudes multiplied by a common factor which ranges from 0.84 to 1.13 depending on angle and model. This factor produces agreement with the differential and total cross sections but has no effect on the polarization observables. The requirement that the experimental amplitudes should go into the theoretical ones in the forward direction, also suffices to determine a unique solution.

We show in Fig. 1 the results for $T_\pi = 256$ MeV obtained from the data and the two theoretical models, where we also show the spin-transfer coefficient $i(11|20)$ predicted by the two solutions. As we see, the four magnitudes $|A|$, $|B|$, $|C|$, and $|D|$ and the phase ϕ_D are rather insensitive to the choice of model (this result is also confirmed by use of model 2). The biggest model dependence is observed in the phases ϕ_A and ϕ_C. Since we have found that the combination $\phi_A - \phi_C$ is also largely model independent, that means that only the combination $\phi_A + \phi_C$ is strongly model

TABLE I. Partial-wave amplitudes $T^J_{LL'}$ normalized as in the Argand diagram.

			256 MeV		294 MeV	
J	L	L'	Model 4	Model 5	Model 4	Model 5
0	1	1	$-0.099+i0.138$	$-0.071+i0.151$	$-0.135+i0.131$	$-0.087+i0.160$
1	0	0	$-0.215+i0.247$	$-0.196+i0.267$	$-0.240+i0.209$	$-0.217+i0.242$
1	1	1	$-0.146+i0.201$	$-0.114+i0.210$	$-0.169+i0.171$	$-0.142+i0.179$
1	2	2	$-0.059+i0.042$	$-0.046+i0.053$	$-0.067+i0.041$	$-0.053+i0.054$
1	2	0	$-0.009+i0.023$	$-0.021+i0.028$	$-0.025+i0.018$	$-0.027+i0.025$
2	1	1	$-0.189+i0.343$	$-0.166+i0.347$	$-0.220+i0.266$	$-0.191+i0.279$
2	2	2	$-0.104+i0.128$	$-0.095+i0.146$	$-0.127+i0.106$	$-0.116+i0.132$
2	3	3	$-0.024+i0.026$	$-0.019+i0.028$	$-0.032+i0.017$	$-0.028+i0.023$
2	3	1	$0.000-i0.015$	$0.003-i0.019$	$0.015-i0.013$	$0.013-i0.021$
3	2	2	$-0.102+i0.184$	$-0.094+i0.194$	$-0.127+i0.148$	$-0.117+i0.162$
3	3	3	$-0.045+i0.069$	$-0.034+i0.067$	$-0.061+i0.058$	$-0.051+i0.057$
3	4	4	$-0.009+i0.009$	$-0.010+i0.008$	$-0.014+i0.014$	$-0.011+i0.011$
3	4	2	$0.007-i0.007$	$0.004-i0.006$	$0.002-i0.007$	$0.004-i0.005$
4	3	3	$-0.043+i0.072$	$-0.035+i0.070$	$-0.059+i0.063$	$-0.052+i0.064$
4	4	4	$-0.021+i0.023$	$-0.019+i0.028$	$-0.029+i0.023$	$-0.023+i0.028$
4	5	5	$-0.007+i0.009$	$-0.005+i0.007$	$-0.011+i0.007$	$-0.008+i0.007$
4	5	3	$0.009-i0.002$	$0.008-i0.003$	$0.008-i0.001$	$0.005-i0.002$

dependent. This combination, however, could be determined with use of the spin-transfer coefficient $i(11|20)$ that will be measured at SIN in the near future.[14] We show in Fig. 2 the corresponding results for $T_\pi = 294$ MeV. As we see, the behavior of the phases ϕ_A and ϕ_C as given by the two models, is somewhat different than in the previous case. The spin-transfer coefficient shows also a somewhat different behavior between the two solutions which makes this energy also interesting to measure. We estimate that the error bars in the observables generate errors of approximately ± 0.006 fm in $|A|$, $|B|$, $|C|$, and $|D|$, of $\pm 5°$ in ϕ_D, of $\pm 10°$ in ϕ_C, and of $\pm 15°$ in ϕ_A.

In order to perform a partial-wave decomposition of the scattering amplitude, we need to know the phase ϕ_B which, however, is not measurable. Thus, for each experimental solution we simply multiplied the helicity amplitudes A, B, C, and D by $\exp(i\phi_B)$ where ϕ_B is the corresponding theoretical phase. We projected out the partial-wave components $T^J_{LL'}$, where J is the total and L and L' the initial and final orbital angular momenta of the system. We present in Table I these partial-wave amplitudes for $J < 5$. As we see, both solutions are quite similar (and also similar to the original theoretical models). If we compare the amplitudes of Table I with those obtained in the past from phase-shift analyses,[15-17] we find that our amplitudes T^1_{11} are consistent with those obtained by Arvieux and Rinat,[15] while the amplitudes T^1_{00} and T^2_{11} differ by about 40%. The amplitudes of Hiroshige *et al.*,[16] are about a factor of 2 larger than ours, while those of Stevenson and Shin[17] are smaller than ours by about a factor 0.6. We suspect that the amplitudes of Refs. 16 and 17 are wrongly normalized. Regarding the question of which method is better, we like to point out that given a set of data and a theoretical model, the solution of our helicity reconstruction method is unique, while in general that of a phase-shift analysis is not. Also, since in the backward direction the helicity amplitudes A, B, and D are smaller by more than an order of magnitude, they will be very hard to determine accurately by a phase-shift analysis.

An interesting final point is the observation by Moravcsik and co-workers[6,7] that the helicity amplitudes of hadronic systems (usually taken in the planar transversity frame) tend to follow phase patterns, such that they are predominantly multiples of π with respect to each other. In the present case, where we have determined reliably only the phase ϕ_D (as measured with respect to B) we observe such type of pattern. As we see in Figs. 1 and 2, at low angles we have $\phi_D \simeq 0°$ (or 360°) which then jumps to $\phi_D \simeq 180°$, then back to $\phi_D \simeq 0°$ and finally ending up as $\phi_D \simeq 180°$. There is presently no explanation for the origin of these patterns.

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