Renormalization of the Mean-Field Superconducting Penetration Depth in Epitaxial $YBa_2Cu_3O_7$ Films

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(Received 13 June 1988)

The *a-b*-plane penetration depth $\lambda(T)$ in $YBa_2Cu_3O_7$ films, determined from kinetic inductance measurements, follows the temperature dependence of weak-coupling mean-field BCS theory near T_c to within ~0.5 K of T_c, resolution limited either by homogeneity or for the thinnest films by phase fluctuations. The Kosterlitz-Thouless transition in a thin film is the first observation in a clean type-II superconductor, noted by the universal superfluid jump and the algebraic-decay law with a square-root cusp exponent.

PACS numbers: 74.70.Vy, 74.30.Ci, 74.75.+t

A kinetic inductance method in which supercurrent flow is induced in the *a-b* basal plane of *c*-axis-oriented epitaxial $YBa_2Cu_3O_7$ films grown on $SrTiO_3(100)$ was used to measure the weak-field in-plane component of the penetration depth λ.[1] Properties of the high-T_c cuprates such as the short coherence length ξ_0, comparatively long mean free path l, large *a-b* vs *c* anisotropy, superconducting energy gap $\Delta(T)$, together with thermodynamic fluctuations near T_c will influence the detailed temperature dependence of $\lambda(T)$, which is a measure of the superelectron density and of the observed dissipation, which is associated with vortex motion. For temperatures not too close to T_c, the weak-coupling mean-field BCS theory with $\Delta/kT_c = 1.76$ and $\lambda(0) = 0.15$ μm provides a good fit to $\lambda(T)$ in films with the sharpest transitions. Broadening by film inhomogeneity correlates with a systematic increase in the magnitude of $\lambda(T)$. Phase fluctuations driven by the presence of quantized vortices are clearly observed in 500-Å films, where the entire film fluctuates as a two-dimensional (2D) superconductor. The data presented here are the first to show the Kosterlitz-Thouless (KT) transition[2] in an intrinsic ($l > \xi_0$) type-II superconductor, as distinct from the earlier restriction to high-resistivity dirty-limit oxides.[3] The intriguing possibility of renormalization by 2D fluctuations within the CuO_2 planes is consistent with the data.

Various films from 500 to 2000 Å thick were prepared by coevaporation from Cu, Y, and BaF_2 sources onto polished $SrTiO_3(100)$ and by reaction at 800 °C in O_2 with water vapor used to catalyze oxidation.[4] The method yields high-quality *c*-axis perpendicular films with 10-μm grains, critical current density $\gtrsim 2\times10^6$ Å cm^{-2} at 77 K, and *a-b*-plane resistivity of 400 $\mu\Omega$cm at 295 K. We found the penetration-depth measurement itself to be a sensitive and critical test of film quality.

The penetration depth λ was calculated from a measurement of the complex ac sheet impedance of the film, $Z = R + i\omega L$.[1] Under conditions of low dissipation, the sheet kinetic inductance of the superconducting state dominates, and is given by $L = 4\pi c^{-2} d^{-1} \lambda^2$, where d is the film thickness. The London equation may be inadequate very close to T_c, because of fluctuations, the finite transition width, and the appearance of the dissipative component R. Applied perpendicular magnetic fields enhance L (and λ) and contribute to R.

The apparatus contained astatically wound 1-mm-diam drive and receive coils positioned with a 1.5-mm gap along a common axis. Films ~1 cm wide were inserted and the complex mutual inductance signal was measured with phase-sensitive detection. Ambient magnetic fields were nulled to ±5 mG, which nevertheless exceeds the demagnetized H_{c1} in thin films and hence vortices were invariably nucleated below T_c. A weak ac excitation field of ~0.1 mG, chosen to assure linearity, was localized by coil geometry to a 1.5-mm region at the center of the film, giving rise to ambient vortex displacements of ~50 μm. Aided by numerical calculations of the screening current distribution as a function of sheet impedance, the mutual inductance data were transformed to give $Z(\omega)$.[1] Below T_c the mutual inductance drops by a factor up to 10^{-3}, because of screening supercurrents. The small baseline mutual inductance M_s at $Z \to 0$ is not known with sufficient absolute accuracy, and thus it was treated as a fitting parameter in the data analysis. The data presented in the figures were taken with a measuring frequency of 13 kHz. Results for λ were frequency independent, over the range 41 Hz to 130 kHz, except near T_c where dissipation dominates. The results also appear unaffected by vortex pinning, since the same mutual inductance was measured at 4.2 K with excitation levels up to 10 mG, or in ambient fields up to 100 G.

Figures 1 and 2 show the results for two film thicknesses, 500 and 2000 Å, plotted as $\lambda(T)$ or $\lambda^{-2}(T)$ to emphasize low- and critical-temperature regions, respectively. The parameters $\lambda(0)$ and T_c and the experimental constant M_s were evaluated by a least-squares fit of the kinetic inductance component of the data with use of the mean-field BCS-theory expression for $\lambda^{-2}(T)$, with the weak-coupling ratio $\Delta(0)/kT_c = 1.76$,

$$\lambda^2(T) = \lambda^2(0)\left[1 - \frac{\partial \ln \Delta(T)}{\partial \ln T}\right], \tag{1}$$

as given in tabular form by Mühlschlegel.[5] The quantity $\lambda^{-2}(T)$ was fitted so as to minimize sensitivity to the data in the unknown critical region near T_c. Conversely, the results for $\lambda(T)$ near T_c are insensitive to M_s. The results are $\lambda(0) = 0.15$ and 0.21 μm and $T_c = 89.5$ and 91 K for the 500- and 2000-Å films, respectively. The 0.15-μm value for the 500-Å film is close to the microscopic value $\lambda(0) = (m^* c^2/4\pi n e^2)^{1/2}$, which was determined from muon-spin-resonance (μSR) data by Harshman *et al.* to be about 0.14-μm for the *a-b* component.[6] [Although presently applicable only to bulk samples, μSR appears to be useful for measurement of $\lambda(0)$ because of the microscopic nature of the μ^+ probe.] An enhanced λ, such as shown for the 2000-Å film, is more typical of our films, and its origin is discussed below in the context of Josephson coupling between the grains in the film.

Figure 1 shows that the BCS function of Eq. (1) gives a good fit to the 500-Å film $\lambda(T)$ data over most of the temperature range. The fitting comparison at high temperature continues in Fig. 2 to reveal the significant depression in λ^{-2} below the BCS curve near T_c. We attribute this to renormalization by phase fluctuations, as discussed further below. Changes in the effective mass m^* with temperature arising from phonon dressing are unknown and have been neglected in the analysis. Apart from this and the experimental uncertainties, the results at low temperatures are consistent with *s*-wave pairing, as was found previously by Harshman *et al.*[6]

The enhanced $\lambda(0)$ values we find in thicker films can be explained by Josephson coupling between grains in the film, which adds another contribution to the kinetic inductance of the film. This is consistent with structural studies, which find best epitaxial quality close to the interface. A 2D square array of Josephson junctions has an effective penetration depth given by $\lambda^2 = c\phi_0 d/8\pi^2 I_c$, where I_c is the junction or link critical current, d the film thickness, and ϕ_0 the flux quantum.[7] Taking the Ambegaokar-Baratoff expression for the critical current of a resistively shunted Josephson junction,[8] one obtains a result analogous to Eq. (1)[9]

$$\lambda^2(T) = \frac{\hbar c^2 r_n d}{4\pi^2 \Delta(T)}\left[\tanh\frac{\Delta(T)}{2kT}\right]^{-1}, \tag{2}$$

where r_n is the junction shunt resistance. The asymptotic behavior close to T_c for Eqs. (1) and (2) have the same functional form, $\lambda^2(T) \propto [1 - T/T_c]^{-2\beta}$. The exponent $\beta = \frac{1}{2}$ in each case is identical to the critical exponent $\Delta(T)$ in mean-field theory. Comparing magnitudes, if we take $r_n d = 160$ μΩ cm, the normal-state resistivity of the 2000-Å film near T_c, then λ^2 computed from Eq. (2) is 30% of the microscopic result of Eq. (1). The linear temperature dependence of λ^{-2} near T_c in 1000–2000-Å films with sharp superconducting transitions, an example of which is given in Fig. 2, confirms that the $\beta = \frac{1}{2}$ exponent of the order parameter is obeyed to within about 0.5 K of the critical temperature. There is no evidence that mean-field theory fails to describe high-T_c superconductivity in $YBa_2Cu_3O_7$; e.g., we do not find $\beta = \frac{1}{3}$ as for the superfluid transition in ^4He.

The region close to T_c is marked by dissipation, which varies in detailed form among the samples studied. Figure 3 shows an example, where the plotted quantity is $G = \mathrm{Re}(Z^{-1})$. The width of the dissipation peak is denoted as δT_c and gives quantitatively the sharpness of the superconducting transition. Films for which $\delta T_c \lesssim 0.5$ K are the cases where it turns out that $2\beta = 1$ and

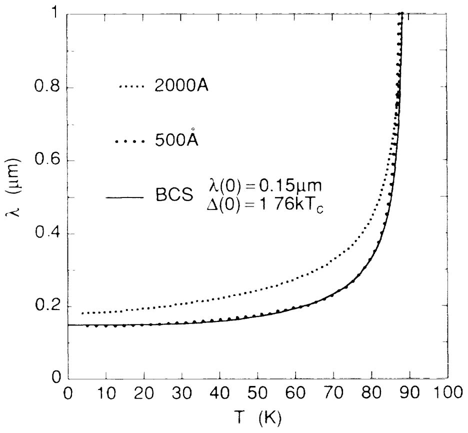

FIG. 1. Temperature dependence of the *a-b*-plane penetration depth λ in two $YBa_2Cu_3O_7$ epitaxial films. A fit with the BCS weak-coupling theory is shown for the 500-Å film.

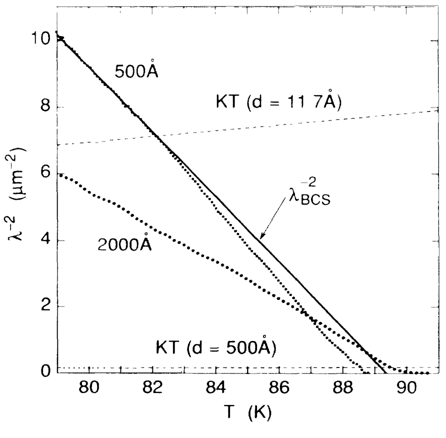

FIG. 2. Temperature dependence of λ^{-2}, which theoretically drops linearly to zero at the mean-field T_c. Dashed curves mark by intersection predicted 2D phase transitions at two values for the distance d in Eq. (3).

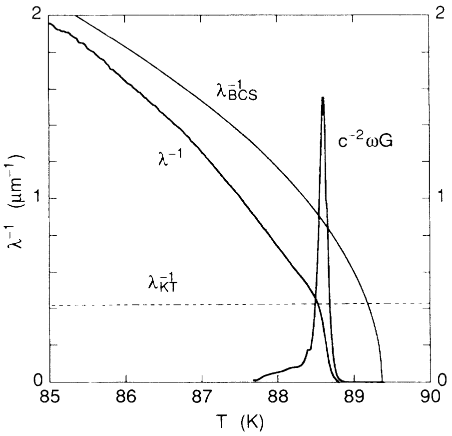

FIG. 3. Inverse penetration depth and dissipation peak ($\omega = 8.4 \times 10^4$ s^{-1}) for a 500-Å film. Also shown are the mean-field BCS curve, the theoretical Kosterlitz-Thouless function $\lambda_{KT}^{-1}(T_{KT})$, which passes 3% below the data at T_{KT}, and dissipation peak G.

$\lambda(0) \lesssim 0.24$ μm. Because of dissipation in the δT_c region, it is uncertain as to whether the mean-field T_c is correctly given by the linear extrapolation $\lambda^{-2}(T) \rightarrow 0$. It is possible that T_c is actually higher by an amount $\sim \delta T_c$. If mean-field theory were to break down, it is restricted to the region $\epsilon = \delta T_c / T_c$, conservatively estimated to be $< 5 \times 10^{-3}$, except for thin films where phase fluctuations are more important. In some of our earlier films, we found anomalously *larger* exponents, close to $\beta = 1$, and that the slope $(-d\lambda^{-1}/dT)$ is generally a decreasing function of δT_c. The larger exponent can be explained by the proximity-coupled *S-N-S* array model, with thin normal-metal weak links. An additional possibility is that texture in these films mixes supercurrent flow along the *c* axis, which dominates because the impedance is much higher, and that $\beta = 1$ arises from *S-N-S*-like behavior along *c*.

At the phase transition of the 2D *XY* model or Kosterlitz-Thouless-Berezinskii theory, the renormalized coupling constant K_R asymptotically takes on a universal value, given as[2]

$$K_R = \frac{d}{\lambda^2} \frac{\hbar^2 c^2}{16\pi e^2 k T_{KT}} = \frac{2}{\pi}, \tag{3}$$

where T_{KT} is the transition temperature. Theory predicts $K_R \rightarrow 0$ for $T > T_{KT}$. In practice, the transition is broadened, according to the dynamical theory of Ambegaokar *et al.*[10] and K_R is replaced by $2\pi^{-1}\{1 + [\ln(14D/\omega\xi^2)]^{-1}\}$. From the vortex diffusivity $D = 2 \times 10^{-4}$ cm^2 s^{-1}, determined from the flux-flow resistance in a magnetic field at $T = T_{KT}$,[11] the correction term turns out to be 6.5%. The locus of Eq. (3) is plotted as dashed curves for two selected d values in Fig. 2, and for $d = 500$ Å in Fig. 3. If we take into account the dynamical correction, the KT transition shown in Fig. 3 is at $T_{KT} = 88.4$ K, which is 1.0 K below our estimate of the mean-field BCS $T_c = 89.4$ K. A second lower-temperature transition, associated with the 11.7-Å spacing of CuO_2 double planes along c in $YBa_2Cu_3O_7$, is not so obvious, absent any dramatic drop in λ^{-2} at the appropriate point (see Fig. 2).

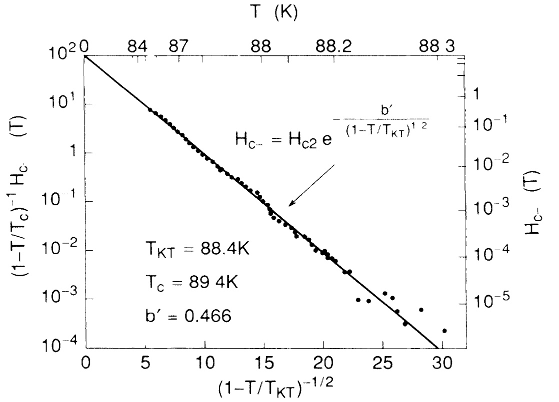

FIG. 4. Dependence of the scaled 2D critical field $H_{c-}(T)$ as a function of the inverse square-root deviation from T_{KT}, with experimental temperature and field scales are shown. The curve is the theoretical function.

Comparison of the unrenormalized BCS curve drawn in Fig. 3 for the 500-Å film with the dynamically corrected value of $\lambda^{-1}(T_{KT})$, which lies 3% above the dashed line in Fig. 3, yields the renormalization factor for K_R (i.e., of λ^{-2}). This is the dielectric constant $\epsilon_c = 4.6$ and is surprisingly large, since previous work on indium oxide found values of ϵ_c in the range 1.2–1.8.[3] The dirty-limit formula[12] $T_c/T_{KT} = 1 + 0.173\epsilon_c e^2 \times \hbar^{-1} R_N$, with $\epsilon_c = 4.6$ and $R_N = 27$ Ω/□, predicts $T_c - T_{KT} = 0.46$ K, about half the value indicated by our fit. The temperature dependence of $\lambda^{-1}(T)$ below T_{KT} is also rather peculiar, since it extrapolates linearly to the mean-field T_c. Film inhomogeneity might possibly explain this, although it seems inconsistent with the sharp transition and the good BCS behavior at low temperatures. We believe that the superfluid density is additionally renormalized by vortex-pair fluctuations in the CuO_2 planes, which remain bound to higher temperatures because of interplanar magnetic coupling.

A fundamental prediction of the renormalization-group theory is the algebraic decay with distance of superfluid density correlations for temperatures below T_{KT}.[2] Equivalently, λ^{-2} decays with length scale r as $(r/\xi)^{-\nu}$, where the exponent $\nu = b(1 - T/T_{KT})^{1/2}$ displays the characteristic cusp at T_{KT}, and $b \sim 1$. We examined this prediction by applying external magnetic fields at temperatures below T_{KT} so as to maintain the measured penetration depth at a constant value 3% above λ_{KT}. In this way $r = (\phi_0/H)^{1/2}$ and is identified with a correlation length $\xi_-(T)$, the maximum separation between bound vortex pairs in a field. The applied field is thus a 2D critical "quenching" magnetic field $H_{c-}(T)=\phi_0/2\pi\xi_-^2(T)$. Since $(\xi_-/\xi)^{-\nu}$ is kept constant, it immediately follows that

$$H_{c-}(T)=H_{c2}(T)\exp[-b'(1-T/T_{\mathrm{KT}})^{-1/2}],$$

where $b'\sim 1$.

The results of this experiment are shown in Fig. 4, where $H_{c-}(T)$ is scaled by $(1-T/T_c)$, the mean-field temperature dependence for $H_{c2}(T)$. We have throughout $H_{c-}(T)\ll H_{c2}(T)$, so T_{KT} is assumed to be independent of H. For temperatures within 4% of T_{KT}, the predicted behavior is observed over a 5-decade range in scaled field (10 mG $\lesssim H_{c-}\lesssim 4$ kG). The results of the fit shown in the figure are $b'=0.466$ and an intercept $H_{c-}(0)$ which is interpreted as $(-T_c dH_{c2}/dT)_{T_c}=100\pm 5$ T. Figure 4 constitutes the first measurement of the algebraic-decay law based on direct measurement of the superfluid density in a superconductor. Previous studies were more circumspect, such as the technique of breaking vortex pairs with a transport current.[3,13]

We conclude from measurements of the in-plane penetration depth of superconducting $\mathrm{YBa_2Cu_3O_7}$ films, that the order parameter obeys mean-field theory with the exponent $\beta=\frac{1}{2}$, to within $\delta T_c/T_c\lesssim 5\times 10^{-3}$ of the critical point, except for the phase fluctuations in thin films. The temperature dependence is consistent with s-wave pairing, weak coupling $\Delta(0)/kT_c=1.76$, and $\lambda(0)=0.15$ μm. Enhancement of λ by Josephson coupling is also observed. In thin 500-Å films, the Kosterlitz-Thouless transition is observed and the predicted square-root singularity in the algebraic-decay exponent for superfluid correlations is observed by a magnetic field quenching technique. Renormalization of the underlying superfluid appears to be stronger than for the low-T_c oxides, and it is suggested that bound vortex-pair excitations within the $\mathrm{CuO_2}$ planes might be the reason.

It is a pleasure to express our appreciation to R. H. Eick for contributions to the experiment, and J. R. Kwo, A. F. J. Levy, M. L. Mandich, and C. E. Rice, for providing films grown by a variety of techniques, results for which will be reported elsewhere.

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