$I_t = J_t$ Rule: A New Large-N_c Selection Rule for Meson-Baryon Scattering

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Quasielastic meson-baryon scattering is reformulated in t-channel language (meson exchange) instead of s-channel language (baryon-resonance formation), by use of skyrmion methods. A new selection rule emerges, valid to leading order in large N_c: The isospin of the exchanged state must equal its total angular momentum (spin+orbital). Differences between the two-flavor and the three-flavor approach are discussed.

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In this paper, we introduce a new large-N_c selection rule, the $I_t = J_t$ rule, governing meson-baryon quasielastic scattering. The derivation given here uses skyrmion methods, which are particularly well suited to large-N_c calculations. In a parallel paper, one of us (M.P.M.) establishes the rule in the more conventional context of one-boson exchange.[1]

As the reader probably knows, the skyrmion approach to hadron physics entails viewing the nucleon not as a bound state of three quarks, nor as a fundamental particle, but rather as a soliton in the effective theory of the low-lying mesons.[2-4] This approach, which is motivated by large-N_c arguments,[3,5] yields a surprising bonus: the emergence of a new symmetry, "**K** spin," which is the vector sum of isospin and angular momentum:

$$\mathbf{K} = \mathbf{I} + \mathbf{J}. \tag{1}$$

K conservation arises because the topologically nontrivial meson configurations of minimum energy can always be chosen so that spin and isospin indices are dotted together, yielding a **K**, although not an **I** or **J** singlet (e.g., the hedgehog configuration in the pion field).

A relation such as (1) involves specifying a conventional orientation between spatial and isospin axes. Of course, physical quantities cannot depend on this convention. Only by integration over all possible relative orientations can physical quantities be defined. As a result of this integration, though, manifest **K** symmetry is lost, while **I** and **J** conservation are regained. It is one of the primary tasks of the Skyrme modeler to think of ways in which the **K** symmetry of the underlying soliton can nevertheless be exposed experimentally. Such tests are purely group theoretic in nature, as they do not depend on the details of the effective meson Lagrangian. They can therefore be considered direct tests of large N_c.

Meson-baryon quasielastic scattering has proved to be a particularly fruitful place to search for "hidden" **K** symmetry. Much of the work thus far has focused on obtaining model-independent relations among the s-channel partial-wave amplitudes.[6-10] But recently, Donohue, in a beautiful series of papers,[11,12] has adapted the skyrmion formalism to describe helicity amplitudes and differential cross sections. To his surprise, he observed that his predictions simplified dramatically when he crossed from an s-channel to a t-channel description of total isospin. Motivated by this finding, in the present paper we revisit the subject of partial-wave amplitudes, but cross to the t channel in *both* isospin and angular momentum. A remarkably elegant selection rule emerges:

The only t-channel partial-wave amplitudes that survive in the large-N_c limit are those in which the isospin of the exchanged state equals its total angular momentum (spin+orbital).

This $I_t = J_t$ rule not only allows for an easy rederivation of previously known s-channel results, it also enables us to obtain predictions for more complicated processes (such as those involving tensor mesons) for which the s-channel approach proves cumbersome. We shall see that the rule only holds in two-flavor, and not in three-flavor, skyrmion models.

To derive our result, we first briefly review the s-channel formalism, for which we need some notation. We shall be considering processes of the type

$$\phi + B \rightarrow \psi + B', \tag{2}$$

where ϕ (ψ) denotes a nonstrange meson of arbitrary spin S_ϕ (S_ψ) and isospin I_ϕ (I_ψ), and B (B') stands for either a nucleon, with spin and isospin $S_B = \frac{1}{2}$ ($S_{B}' = \frac{1}{2}$), or a Δ, with spin and isospin $S_B = \frac{3}{2}$ ($S_B' = \frac{3}{2}$). A physical partial-wave amplitude in the s channel is specified by the quantum numbers $\{L, L', S, S', I_s, J_s\}$, where L and S (L' and S') are the initial (final) orbital angular momentum and total spin, and $\mathbf{I}_s$ and $\mathbf{J}_s$ are total s-channel isospin and angular momentum. In order to shorten our equations, we will let $[S]$ stand for $2S+1$, etc.

To leading order in large N_c, the following expression can be derived for the physical T matrix describing the scattering process (2) [8]:

$$T_{LL'SS'I_sJ_s}=\sum_{K,\tilde K,\tilde K'}[K]([S_B][S_B'][S][S'][\tilde K][\tilde K'])^{1/2}\times\begin{Bmatrix}L & I_\phi & \tilde K\\ S & S_B & S_\phi\\ J_s & I_s & K\end{Bmatrix}\begin{Bmatrix}L' & I_\psi & \tilde K'\\ S' & S_B' & S_\psi\\ J_s & I_s & K\end{Bmatrix}\tau_{K\tilde K\tilde K'LL'}. \quad (3)$$

Here τ, which, like **T**, is a function of energy, denotes the scattering matrix in the *unphysical* frame in which **K**, but not $\mathbf{I}_s$ or $\mathbf{J}_s$, is conserved—i.e., before the integration over orientations mentioned earlier. **K** conservation in this unphysical frame is evident in the fact that the same K appears in both $9j$ symbols, of which the first contains the quantum numbers of the initial state and the second, those of the final state. Equation (3) also introduces the hybrid quantum numbers $\tilde K$ and $\tilde K'$, defined by $\tilde{\mathbf{K}}=\mathbf{I}_\phi+\mathbf{L}$, $\tilde{\mathbf{K}}'=\mathbf{I}_\psi+\mathbf{L}'$. Thus $\mathbf{K}=\tilde{\mathbf{K}}+\mathbf{S}_\phi=\tilde{\mathbf{K}}'+\mathbf{S}_\psi$. The summation on $\tilde K$, $\tilde K'$, and K extends over all values consistent with the $9j$ symbols; that is, the quantities in each row and column must sum to integers, and obey the triangle inequality.

Although the τ's can be extracted numerically from any given effective meson Lagrangian, it is the **T**'s that are measured experimentally. Fortunately, for most two-body processes, there will be more **T**'s than τ's for a given choice of L and L'. Hence Eq. (3) implies a set of nontrivial energy-independent linear relations purely among the **T**'s, the experimental validity of which serves as a test of both large N_c and hidden **K** conservation. Finding these relations, however, has up to now been rather difficult, since explicit formulas for $9j$ symbols are unwieldy. Consequently the program has been carried out for only a handful of processes: $\pi N\to\pi N$,[6,7] $\pi N\to\pi\Delta$,[7] $\pi N\to\omega N$,[8,11,12] and $\pi N\to\rho N$.[8,11,12] In all but the last, both $9j$ symbols actually collapse to more tractable $6j$ symbols.

We seek to reformulate Eq. (3) with the quantum numbers appropriate to the t-channel process

$$\phi+\bar\psi\to\bar B+B'. \quad (4)$$

The procedure for crossing from s-channel isospin I_s to t-channel isospin I_t is well known, and involves a $6j$ symbol.[13] In order to derive the analogous prescription for angular momentum, it is helpful to think of the scattering (2) as occurring in the extreme large-N_c limit, in which the baryons are much heavier than the mesons, and the center-of-mass frame coincides with the rest frames of the initial and final baryon. In this limit, the orbital quantum numbers L and L' can be thought of as belonging exclusively to the mesons ϕ and ψ, and we can define the total meson angular momenta $\mathbf{J}_\phi=\mathbf{S}_\phi+\mathbf{L}$ and $\mathbf{J}_\psi=\mathbf{S}_\psi+\mathbf{L}'$. $\mathbf{J}_\phi$ and $\mathbf{J}_\psi$ can, in turn, be combined to form $\mathbf{J}_t$, the total angular momentum of the exchanged state. The allowed values of J_t are, of course, tightly constrained by the static baryons: if B and B' are both nucleons, then $J_t=0$ or 1; if B is a nucleon and B' is a Δ, or vice versa, then $J_t=1$ or 2; while if B and B' are both Δ's, $J_t=0$, 1, 2, or 3. With proper attention paid to the phase factors introduced when changing bras to kets,[14] one finds that recoupling the angular momenta in this manner is straightforward, and involves a product of three $6j$ symbols.

Altogether, the full $I_s\to I_t$, $J_s\to J_t$ crossing relations are given by

$$\mathbf{T}_{LL'J_\phi J_\psi I_tJ_t}=\sum_{S,S',I_s,J_s}[I_s][J_s]([J_\phi][J_\psi][S][S'])^{1/2}(-1)^{I_t+I_s+J_t+J_s+S_\phi+S_\psi+S+S'+J_\phi+I_\psi}\times\begin{Bmatrix}S_B' & S_B & I_t\\ I_\phi & I_\psi & I_s\end{Bmatrix}\begin{Bmatrix}S_B' & S_B & J_t\\ J_\phi & J_\psi & J_s\end{Bmatrix}\begin{Bmatrix}J_s & J_\phi & S_B\\ S_\phi & S & L\end{Bmatrix}\begin{Bmatrix}J_s & J_\psi & S_B'\\ S_\psi & S' & L'\end{Bmatrix}\mathbf{T}_{LL'SS'I_sJ_s}. \quad (5)$$

We should emphasize that this formula has nothing to do with the Skyrme model *per se*; it simply summarizes the Clebsch-Gordan manipulations involved in passing back and forth from an s-channel to a t-channel description of the collision, in the limit that the baryons are considered infinitely heavy.

If we apply Eq. (5) to the Skyrme-model expression, Eq. (3), and carry out the indicated sums with the help of some standard identities, we then obtain our main result[15]:

$$\mathbf{T}_{LL'J_\phi J_\psi I_tJ_t}=\delta_{I_tJ_t}\{([S_B][S_B'][J_\phi][J_\psi])^{1/2}/[I_t]\}\times\sum_{K,\tilde K,\tilde K'}(-1)^{J_t+J_\psi-I_\psi+K+\tilde K+\tilde K'+L+L'}[K]([\tilde K][\tilde K'])^{1/2}\times\begin{Bmatrix}J_\phi & I_\phi & K\\ I_\psi & J_\psi & J_t\end{Bmatrix}\begin{Bmatrix}J_\phi & I_\phi & K\\ \tilde K & S_\phi & L\end{Bmatrix}\begin{Bmatrix}J_\psi & I_\psi & K\\ \tilde K' & S_\psi & L'\end{Bmatrix}\tau_{K\tilde K\tilde K'LL'}. \quad (6)$$

The Kronecker δ embodies the $I_t=J_t$ selection rule stated earlier.

We make the following remarks.

(i) Although, for mathematical elegance, Eq. (6) has been cast in terms of t-channel quantities, the skyrmion approach can only be justified in the kinematic regime appropriate to the s-channel process (2), in which a meson of energy $O(N_c^0)$ scatters quasielastically from a massive $[m=O(N_c)]$, nonrelativistic baryon. Indeed, the t-channel process

(4), which manifestly requires meson energies of $O(N_c)$, is exponentially suppressed in large N_c (see Sec. 8.3 of Ref. 5). Furthermore, Eq. (6), like Eq. (3), is totally invalid in the "soft-pion" energy regime, and can only be expected to hold for higher energies (see Sec. 2 of Ref. 7).

(ii) If we plug specific values of $I_t \neq J_t$ into Eq. (5), and use our new found result that the left-hand side vanishes, then the vanishing of the right-hand side reproduces—and gives an explicit, universal formula for—the model-independent linear relations among s-channel partial-wave amplitudes familiar from previous work.[6-8] For example, for $\pi N \to \rho N$, there are eight *a priori* independent amplitudes for each choice of $L = L' \geq 1$, specified by $I_s = \frac{1}{2}$ or $\frac{3}{2}$, $S' = \frac{1}{2}$ or $\frac{3}{2}$, and $J_s = L \pm \frac{1}{2}$ in the s channel, or alternatively by $(I_t, J_t, J_\psi) = (0,0,L)$, $(1,1,L-1)$, $(1,1,L)$, $(1,1,L+1)$, $(1,0,L)$, $(0,1,L-1)$, $(0,1,L)$, or $(0,1,L+1)$ in the t channel. The vanishing of the latter four, which have $I_t \neq J_t$, yields relations among the s-channel amplitudes equivalent to those given in Eq. (15) of Ref. 8, where they were derived (much more painfully) from a direct analysis of Eq. (3). The simpler example of $\pi N \to \pi N$ is worked out in detail in Ref. 1.

(iii) When all other quantum numbers are equal, the amplitudes for $\phi N \to \psi N$, $\phi N \to \psi \Delta$, $\phi \Delta \to \psi N$, and $\phi \Delta \to \psi \Delta$ are predicted by Eq. (6) to be proportional to each other, with proportionality constants simply given by $([S_B][S'_B])^{1/2}$.

(iv) Donohue's remarkable conclusion[12] that the η does not couple to the nucleon follows trivially from the $I_t = J_t$ rule. To see this, consider a process (2) to which single-η exchange ostensibly contributes. Since the η is a spinless isosinglet, the $I_t = J_t$ rule dictates that it be in an S wave. But an S wave η has negative parity, and so cannot couple to a static nucleon.

(v) Equation (6) can actually be derived directly, without invoking the s-channel result (3), by the same sorts of manipulations carried out in Refs. 7 and 8.

The preceding has relied on a two-flavor skyrmion analysis. Inclusion of a third light flavor turns out to be relatively straightforward.[16] Here, because of space restrictions, we merely state the main findings; the detailed derivations will be presented elsewhere.[17]

Let the four particles B, B', ϕ, and ψ be in $SU(3)_{\text{flavor}}$ representations R, R', R_ϕ, and R_ψ, which we can imagine combining into pure s-channel or t-channel representations R_s or R_t. The three-flavor analogs of Eqs. (3) and (6), for s-channel and t-channel scattering, respectively, are then given by

$$\begin{aligned}
T_{LL'SS'R_s\gamma_s\gamma'_sJ_s} = \sum_{II'I''Y} (-1)^{I+I'+Y}
&\times \left(\begin{matrix} R & R_\phi \\ S_B 1 & IY \end{matrix}\middle|\begin{matrix} R_s\gamma_s \\ I'', Y+1 \end{matrix}\right)\left(\begin{matrix} R_s\gamma'_s \\ I'', Y+1 \end{matrix}\middle|\begin{matrix} R' & R_\psi \\ S'_B 1 & I'Y \end{matrix}\right) \\
&\times \sum_{K\tilde{K}\tilde{K}'} \frac{[I''][K]\{(\dim R)(\dim R')[S][S'][\tilde{K}][\tilde{K}']\}^{1/2}}{\dim R_s} \\
&\times \begin{Bmatrix} L & I & \tilde{K} \\ S & S_B & S_\phi \\ J_s & I'' & K \end{Bmatrix}\begin{Bmatrix} L' & I' & \tilde{K}' \\ S' & S'_B & S_\psi \\ J_s & I'' & K \end{Bmatrix} \tau^{\{II'Y\}}_{K\tilde{K}\tilde{K}'LL'}
\end{aligned} \tag{7}$$

and

$$\begin{aligned}
T_{LL'J_\phi J_\psi R_t\gamma_t\gamma'_tJ_t} = \{(\dim R)(\dim R')[J_\phi][J_\psi]\}^{1/2}/\dim R_t
&\times \sum_{II'Y} \left(\begin{matrix} R_\phi & R_\psi^* \\ IY & I', -Y \end{matrix}\middle|\begin{matrix} R_t\gamma_t \\ J_t 0 \end{matrix}\right)\left(\begin{matrix} R_t\gamma'_t \\ J_t 0 \end{matrix}\middle|\begin{matrix} R^* & R' \\ S_B, -1 & S'_B 1 \end{matrix}\right) \\
&\times \sum_{K\tilde{K}\tilde{K}'} (-1)^{(Y+1)/2+S_B+J_t+J_\psi+K+\tilde{K}+\tilde{K}'+L+L'} [K]\{[\tilde{K}][\tilde{K}']\}^{1/2} \\
&\times \begin{Bmatrix} J_\phi & I & K \\ I' & J_\psi & J_t \end{Bmatrix}\begin{Bmatrix} J_\phi & I & K \\ \tilde{K} & S_\phi & L \end{Bmatrix}\begin{Bmatrix} J_\psi & I' & K \\ \tilde{K}' & S_\psi & L' \end{Bmatrix} \tau^{\{II'Y\}}_{K\tilde{K}\tilde{K}'LL'}.
\end{aligned} \tag{8}$$

The new symbols that appear in these expressions are SU(3) isoscalar factors, tabulated by deSwart.[18] The quantities γ_s and γ_t (primed for final, unprimed for initial state) serve merely to distinguish the two **8**'s that appear in **8**×**8**. The outer summations extend over all values consistent with the isoscalar factors. Note that the three-flavor τ's have more structure than their two-flavor counterparts.

By SU(3) invariance, Eqs. (7) and (8) are supposed to hold for any choice of $(I_s, I_{sz}, Y_s) \in R_s$ or $(I_t, I_{tz}, Y_t) \in R_t$, respectively. Evidently the $I_t = J_t$ rule no longer applies—even for processes involving only nonstrange particles—although a relic of it appears in the first isoscalar factor in Eq. (8). (It is generally true that three-flavor

Skyrme-model predictions do not precisely reproduce their two-flavor counterparts when restricted to nonstrange systems.) Experimental deviations from the $I_t = J_t$ rule for nonstrange processes might therefore be partially ascribed to the presence of a third light flavor!

The t-channel expression is nevertheless simpler than the s-channel one, as it involves one fewer index of summation. Once again, when all other t-channel quantum numbers are held fixed, the amplitudes for processes involving decuplet baryons are directly proportional to the ones corresponding to octet baryons, the constants of proportionality being given this time by

$$(-1)^{S_B}[(\dim R)(\dim R')]^{1/2}\left(\begin{matrix} R_t\,\gamma_t' \\ J_t 0 \end{matrix}\middle|\begin{matrix} R^* & R' \\ S_B, -1 & S_B' 1 \end{matrix}\right).$$

For instance, when $J_t = 1$, the amplitudes for $\bar{K}^- p \to K^+\Xi^-$ and $\bar{K}^- p \to K^+\Xi^{*-}$, which are pure **27**'s in the t channel, are predicted to be in the ratio $(8/\sqrt{5}){:}2$.

Further discussions of our two-flavor and three-flavor findings will be given in due course.

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[15]We known of several ways to prove the equivalence of Eqs. (3) and (6). The simplest is to apply the *inverse* of Eq. (5) to Eq. (6); one then recovers Eq. (3) using standard $6j$ symbol identities [Eqs. (C.45d) and (C.41) of A. Messiah, *Quantum Mechanics* (Wiley, New York, 1958), Vol. 2, appendix C].

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