# Measurement of the Resistivity in a Partially Degenerate, Strongly Coupled Plasma

R. L. Shepherd[(a)]
*Department of Nuclear Engineering, University of Michigan, Ann Arbor, Michigan 48109*

and

D. R. Kania[(b)] and L. A. Jones
*Physics Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*
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Plasmas with densities of $6\times10^{22}$ $e$/cm$^3$ and temperatures of 10 eV have been created by means of capillary discharge. These parameters indicate that the plasmas are partially degenerate and strongly coupled. By measuring the size of the plasma, the current, and the voltage, one can infer the resistivity of the plasma and compare it with calculated values from various transport theories. The results of this experiment show that theories that do not accurately model the complete electron-ion interaction within this regime can be inaccurate by as much as a factor of 200.

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Most experiments in plasma physics are done in a regime where the interaction potential energy of the ions is small compared to their kinetic energy (weak coupling). Furthermore, the electrons in the plasma are nondegenerate (Boltzmann distribution). A useful parameter for measuring the effects of the coupling in singly ionized plasma is

$$\Gamma = e^2/r_i k_B T, \tag{1}$$

where

$$1/n_i = (4\pi/3) r_i^3, \tag{2}$$

$e$ is the elementary charge, $k_B$ is the Boltzmann constant, $T$ is the ion temperature, and $n_i$ is the ion density. When $\Gamma$ (the strong-coupling parameter) is of order 1, the plasma is said to be strongly coupled. Also, the Fermi degeneracy parameter (used to measure the level of degeneracy) is

$$\theta = \frac{2m_e k_B T}{\hbar}(3\pi^2 n_e)^{-2/3}, \tag{3}$$

where $m_e$ is the electron mass and $n_e$ is the electron density. When $\theta$ is much less than 1, the plasma is degenerate. In the absence of anomalous resistivity, for weak coupling ($\Gamma \ll 1$) and nondegeneracy ($\theta \gg 1$) the electron conduction can be described by the Spitzer resistivity.[1] As the temperature of the plasma decreases and the density increases, both degenerate and strong-coupling effects become significant and the Spitzer theory is no longer valid.

Numerous theoretical models have been presented to describe the physics of strongly coupled plasmas.[2-4] Although the theoretical research has been intense, there have been few measurements performed in this regime.[5-7] We present in this Letter a measurement of the low-frequency electrical resistivity in a plasma which is partially degenerate and strongly coupled. Comparisons with theory indicate that accurate modeling of the electron-ion interaction potential is critical to properly calculating the low-frequency resistivity.

The plasmas are created by discharge of a 600-kV pulse across a 1.7-cm-long, 20-$\mu$m-diam hole (capillary) in polyurethane[8] (see Fig. 1) resulting in a peak current of 500 kA in 300 ns.[9] When the pulsed power system is discharged, the inner walls of the capillary are heated to conduction temperatures, accreting material from the walls and filling the initial void with plasma. As the current rises, the plasma expands at a velocity of $6\times10^5$ cm/s by ionization of the surrounding cold material. The particle loss is small, and results only from particle loss out of the anode end of the capillary. Thus, once the plasma volume is large compared to the initial capillary volume, the plasma density is essentially at solid density. The result is an $\approx$10-eV, solid-density ($\approx 6\times10^{22}$ $e$/cm$^3$) plasma for the first 100 ns of the discharge.

The resistivity is determined by measurement of the lumped parameters of the plasma circuit. To achieve this, the diagnostics used on the experiment were focused on four parameters: temperature, channel size, voltage, and current. The temperature was measured by comparison of the relative continuum intensities of the extreme ultraviolet to soft x-ray radiation emission. In analyzing these data, the plasma was assumed to be a blackbody radiator with a spatially uniform temperature. On the average, the temperature of the plasma was $\approx 10\pm3$ eV during 10 to 100 ns into the discharge.

The channel radius as a function of time was measured both along the $z$ axis and perpendicular to the $z$ axis. The channel radius was measured perpendicular to the plasma's axis with use of a visible streak camera and schlieren photography. A streak camera was used to measure the size of the visible light front as a function of

 

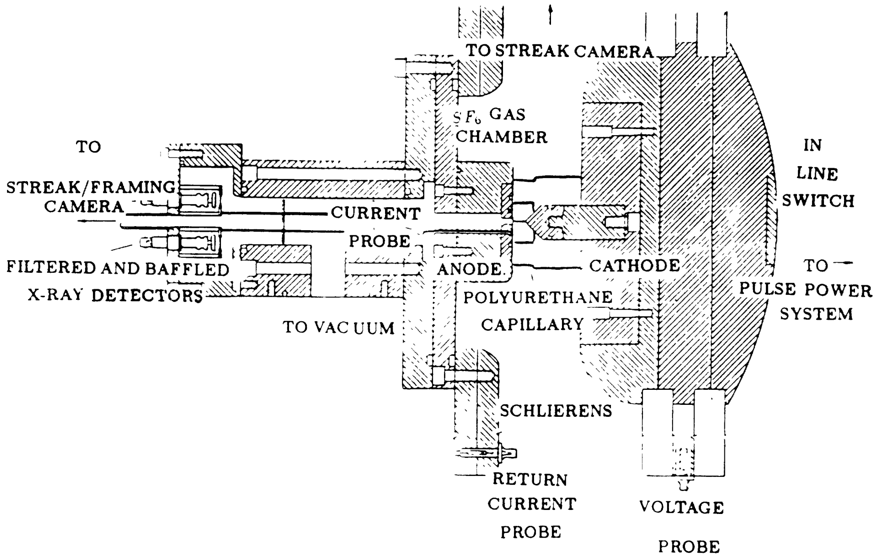


FIG. 1. Experimental apparatus.

time both axially and radially, while the schieren photography was used to measure the spatial growth in the critical density as a function of time.

The radial size as a function of time was also measured axially to ensure that the channel expansion was uniform along the channel length. The time-dependent radial size measurements (axial and radial) are fitted well by a constant expansion of $6\times10^5$ cm/s.

In comparison of the time-dependent volume of the plasma (determined by the radial size diagnostics) to the initial void, the initial capillary volume was ignored and particle conservation was used to determine the density. Calculations of end losses support this approach.[9]

The voltage was measured by a voltage probe capacitively coupled to the chamber anode with water as the dielectric medium. The current was measured at the face of the capillary with a Rogowski coil. The $dI/dt$ data were integrated numerically to generate the current for each shot.

The time-dependent resistivity can be inferred by our combining the measurements of the voltage, current, and radius. By subtracting out the inductive contribution to the voltage, one can deduce a resistance for the plasma. Further, with the assumption that the distribution is uniform, the resistivity is found to be

$$\eta_0=\frac{\pi r_1^2}{I l_p}\left[V_p-\frac{l_p}{c^2}\left\{\frac{dI}{dt}\left[1+2\ln\left(\frac{r_0}{r_1}\right)\right]-2I^2\frac{dr_1}{dt}\right\}\right], \tag{4}$$

where $r_1$ is the plasma radius, $r_0$ is the return current radius, $l_p$ is the plasma length, $V_p$ is the voltage drop across the plasma, and $I$ is the current through the plasma. Magnetic field effects were assumed to be negligible because the electron collision frequency ($\approx 3.3\times10^{15}$ $s^{-1}$) was much larger than the electron cyclotron frequency ($\approx 7.6\times10^{13}$ $s^{-1}$). The current density was assumed to be uniform on the basis of on skin-depth calculations. A plot of resistivity versus time is displayed in Fig. 2 for nine experiments.

The error in the resistivity measurement is characterized by the spread in the data in Fig. 2 (random error) and the error propagated in the calculation by uncertainties in data tables (systematic error). The total systematic error was estimated to be on the order of $\pm50\%$. The random error can be estimated from the nine shots displayed in Fig. 2. Including these errors, the resistivity is $\approx(2.5^{+3.5}_{-1.75})\times10^{-3}$ Ω cm.

In order to compare the measured resistivity to the values calculated from the relevant theories, it is neces-

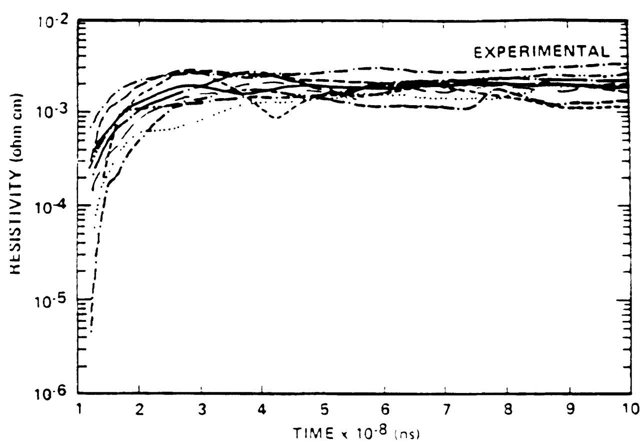


FIG. 2. Resistivity vs time for nine shots.

sary to calculate the Coulomb coupling and Fermi degeneracy parameters from experimental values. For our conditions, $0.6 \leq \Gamma \leq 1.8$ and $1.2 \leq \theta \leq 1.9$, which implies that the plasma is partially degenerate and in the intermediate- to strong-coupling regime.

Comparisons were made to resistivities calculated with the Spitzer, Ichimaru-Tanaka,[10] and Rinker[11] theories. The calculation of the Spitzer resistivity is[1]

$$\eta_S = 1.03 \times 10^{-2} \bar{Z} \ln\Lambda / T^{3/2} \ \Omega\ \text{cm}, \tag{5}$$

where $T$ is the temperature in eV, $\bar{Z}$ is the average degree of ionization, and $\ln\Lambda$ is the Coulomb logarithm (usually defined as the logarithm of the ratio of the Debye shielding length to the distance of closest approach). For conditions in the experiment, this definition of $\ln\Lambda$ yields a value less than 0.1.[12] An effective Coulomb logarithm of 0.6 has been calculated from a kinetic-theory derivation of the interparticle collision frequencies using plasma correlation functions accurate for a fully ionized plasma.[13] The average ionization $\bar{Z}$ is provided by the theoretical calculation in Ref. 9 (the underlying principles used in this calculation are stated later in this paper in the discussion of Rinker's resistivity calculation). With a temperature of 10 eV, a $\bar{Z}$ of 0.3, and a density of $6 \times 10^{22}$ cm$^{-3}$, the Spitzer resistivity was calculated for two cases (1) $\ln\Lambda = 0.1$ (the Coulomb logarithm) and (2) $\ln\Lambda = 0.6$ (the effective Coulomb logarithim), and displayed in Table I.

A more general model for the resistivity of a plasma has been proposed by Ichimaru and Tanaka. The model assumes a fully ionized plasma and is based on a quantum-statistical derivation of the transport equation for the electrons in the plasma using fluctuation-dissipation theory.[10,14] The derivation further uses an effective Born approximation to calculate the scattering cross section.

Using the classical definition of $\ln\Lambda$, the Spitzer resistivity underestimates the measured value by a factor of 200. Calculations based on Spitzer theory with an effective $\ln\Lambda$ are still a factor of 42 less than the measured value. This is to be expected because of the assumptions that Spitzer theory is based on breakdown in plasmas with a level of degeneracy and strong coupling such as that estimated in the capillary experiment. The Ichimaru-Tanaka calculation corrects the resistivity for the level of degeneracy and strong coupling in the plasma. However, the assumption of a fully ionized plasma simplifies the more complex electron-ion interaction which exists in the partially ionized capillary plasmas. This fact is suspected to be the cause of the poor agreement between the Ichimaru-Tanaka calculation and the measured value (the theory predicts values approximately a factor of 10 less than the measured value).

TABLE I. Resistivity calculated with various theories.

| Theory | Resistivity ($\Omega$ cm) | Measured/theory |
|---|---|---|
| Spitzer | $9.8 \times 10^{-6}$ | 256 |
| Modified Spitzer | $5.9 \times 10^{-5}$ | 42 |
| Ichimaru-Tanaka | $2.5 \times 10^{-4}$ | 10 |
| Rinker | $1.8 \times 10^{-3}$ | 1.4 |
| Measured value | $2.5 \times 10^{-3}$ | ... |

The calculation that produces the best agreement with the measurement is a calculation by Rinker. This calculation was done for a $C_2H_3$ mixture by combination of the resistivity for carbon and hydrogen. The resistivity for carbon and hydrogen are calculated independently and then combined with a method successfully employed in the study of binary liquid metals.[15]

Resistivities calculated by this method involved the following steps. (1) For each temperature and material density of interest, a potential was constructed from Dirac-Fock-Slater and Thomas-Fermi-Dirac theories. The carbon potentials were constrained to approximate the electronic shell structure of diamond at room temperature.[16] (2) A complete partial-wave analysis of electron states was carried out to yield (a) scattering cross sections, and (b) densities of states, from which the chemical potential and ionization state $\bar{Z}$ were obtained by standard methods. (3) These were combined in the Ziman formula with a one-component plasma structure factor to give the resistivity at that temperature and density. The resistivities computed from the above theories are displayed in Table I.

In summary, the plasmas created in the capillary experiment had a strong-coupling parameter ($\Gamma$) between approximately 0.6 and 1.8 and a Fermi degeneracy parameter ($\theta$) between 1.9 and 1.2. Thus, the plasmas were partially degenerate and intermediate to strongly coupled. Three theories for the resistivity were used for comparison in this parameter regime (Spitzer, Ichimaru-Tanaka, and Rinker). An effective Spitzer theory is approximately a factor of 42 less than the measured resistivity, suggesting that plasmas in this $\Gamma$-$\theta$ regime, as expected, have a non-Spitzer electron conduction. The measurement is approximately a factor of 10 greater than the Ichimaru-Tanaka calculation. This fact along with the *good* agreement between the measured resistivity and the Rinker calculation suggest the importance of the modeling of the electron-ion interaction potential in electron transport in dense plasmas.

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[(a)]Present address: Physics Division, Lawrence Livermore National Laboratory, Livermore, CA 94550.

[(b)]Present address: Laser Experiments, Lawrence Livermore National Laboratory, Livermore, CA 94550.

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