ϵ'/ϵ and the Electric Dipole Moment of the Neutron in Left-Right-Symmetric Models

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(Received 23 May 1988)

We observe that the recent results on ϵ'/ϵ enable us to place a lower bound on the left-right mixing parameter ξ in pseudomanifest left-right-symmetric models: $\tan\xi \gtrsim 2.0\times 10^{-6}$. We give the complete expression for the neutron electric dipole moment D_n and show that it satisfies $D_n < 7.6\times 10^{-24}$ e cm. In a particular model, due to Chang, we can relate D_n to ϵ'/ϵ and place limits on D_n: $1.9\times 10^{-26} \gtrsim |D_n| \gtrsim 1.9\times 10^{-27}$ e cm.

PACS numbers 13.40.Fn, 11.30.Er, 12.15.Ji, 13.25.+m

The phenomena of CP nonconservation in left-right-symmetric models have been extensively discussed in the literature.[1-9] The reason for our revisiting this familiar ground is to make the observation that the recent experimental result from CERN,[10] viz.,

$$\epsilon'/\epsilon = (3.3\pm 1.1)\times 10^{-3}, \tag{1}$$

implies that

$$\epsilon'/\epsilon > 1.1\times 10^{-3} \tag{2}$$

at 95% confidence. Thus one can ask the question whether this result, which implies a finite value for left-right (L-R) mixing in the $SU(2)_L\otimes SU(2)_R$ models, is compatible with other known limits on the L-R mixing.

We show, without introducing special models for the CP-nonconserving phases in the L-R-symmetric model, that the result (2) gives the lower bound

$$\tan\xi > 2.0\times 10^{-6} \tag{3}$$

on the L-R mixing parameter. This is quite compatible with the upper bound[11]

$$\tan\xi \le 4\times 10^{-3}. \tag{4}$$

One may have expected that the bounds (3) and (4) would enable us to place both upper and lower bounds on the L-R model prediction of the neutron electric dipole moment. However, the parameters which enter ϵ' and D_n differ, so that it is necessary to use explicit models in order to estimate D_n—we obtain an estimate using a model due to Chang.[3]

The CP-nonconservation parameter may be written in a standard notation[12] as

$$\epsilon' = \frac{i}{\sqrt{2}} e^{i(\phi_2-\phi_0)} \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}(\tan\theta_2 - \tan\theta_0), \tag{5}$$

where δ_I are the $\pi\pi$ phase shifts in the state of isospin I, and

$$A_I = \langle(\pi\pi)_I|\mathcal{L}_w|K^0\rangle = |A_I| e^{\theta_I} \tag{6}$$

are the weak interaction amplitudes. We use the form of Eq. (5) because we assume that we can compute the phases of the $K^0\to\pi\pi$ amplitudes more accurately than their magnitudes, which are notoriously difficult to obtain.

In the L-R-symmetric model, the charged-current interaction in the quark sector has the form[1-9]

$$\mathcal{L} = \tfrac{1}{2}\sqrt{2}[g_L\bar{U}\gamma_\mu L U_L D W_L^\mu + g_R \bar{U}\gamma_\mu R U_R D W_R^\mu] + \text{H.c.}, \tag{7}$$

where $L=\frac{1}{2}(1-\gamma_5)$, $R=\frac{1}{2}(1+\gamma_5)$ are the chirality projections, $U=(u,c,\ldots)$, $D=(d,s,\ldots)$, U_L and U_R are the quark mixing matrices, and we allow W_L-W_R mixing so that the mass eigenstates are

$$W_1 = \cos\xi W_L + \sin\xi W_R, \quad W_2 = -\sin\xi W_L + \cos\xi W_R. \tag{8}$$

The effective four-fermion Hamiltonian responsible for ϵ' is then[6]

$$H_{\Delta s=1} = H(W\text{-exchange}) + H(\text{penguin}), \tag{9}$$

where

$$H(W\text{-exchange}) = a_1\left[\tfrac{1}{2}C_+^L O_+^{LL} + \tfrac{1}{2}C_-^L O_-^{LL}\right] + a_2\left[C'_+ O_+^{RL} + C'_- O_-^{RL}\right] + a_3\left[C'_+ + O_+^{LR} + C'_- O_-^{LR}\right] + a_4\left[\tfrac{1}{2}C_+^R O_+^{RR} + \tfrac{1}{2}C_-^R O_-^{RR}\right]. \tag{10}$$

The C_i are QCD correction factors and are given by

$$C_+^{\rm L}=[\alpha_s(\mu^2)/\alpha_s(m_1^2)]^{-2/b},\quad C_+^{\rm R}=[\alpha_s(\mu^2)/\alpha_s(m_2^2)]^{-2/b},\quad C_-^{\rm L}=[\alpha_s(\mu^2)/\alpha_s(m_1^2)]^{4/b},\quad C_-^{\rm R}=[\alpha_s(\mu^2)/\alpha_s(m_2^2)]^{4/b},$$
$$C'_+=[\alpha_s(\mu^2)/\alpha_s(m_1^2)]^{-1/b},\quad C'_-=[\alpha_s(\mu^2)/\alpha_s(m_1^2)]^{8/b}, \tag{11}$$

with $b=11-\frac{2}{3}n_f$, and n_f the number of light quarks. Using conventional values of the parameters, we have

$$C_+\sim 0.6, C_-\sim 2.6,\ C'_+\sim 0.8,\ C'_-\sim 6.8. \tag{12}$$

Furthermore,

$$\begin{pmatrix} O_\pm^{\rm LL} \\ O_\pm^{\rm RR}\end{pmatrix}=\bar d\gamma^\mu\begin{pmatrix} L\\ R\end{pmatrix}u\bar u\gamma_\mu\begin{pmatrix} L\\ R\end{pmatrix}s\pm\bar d\gamma^\mu\begin{pmatrix} R\\ L\end{pmatrix}s\bar u\gamma_\mu\begin{pmatrix} L\\ R\end{pmatrix}u,$$
$$\begin{pmatrix} O_+^{\rm LR} \\ O_+^{\rm RL}\end{pmatrix}=\bar d\gamma^\mu\begin{pmatrix} L\\ R\end{pmatrix}u\bar u\gamma_\mu\begin{pmatrix} R\\ L\end{pmatrix}s+\tfrac{2}{3}\bar d\begin{pmatrix} R\\ L\end{pmatrix}s\bar u\gamma_\mu\begin{pmatrix} L\\ R\end{pmatrix}u,\quad \begin{pmatrix} O_-^{\rm LR} \\ O_-^{\rm RL}\end{pmatrix}=-\tfrac{2}{3}\bar d\begin{pmatrix} L\\ R\end{pmatrix}s\bar u\begin{pmatrix} R\\ L\end{pmatrix}u, \tag{13}$$

and

$$a_1=\tfrac{1}{2}g_{\rm L}^2(\cos^2\xi/m_1^2+\sin^2\xi/m_2^2)\sin\theta_{\rm L}\cos\theta_{\rm L},\quad a_2=\tfrac{1}{2}g_{\rm L}g_{\rm R}\sin\xi\cos\xi(1/m_1^2-1/m_2^2)\sin\theta_{\rm L}\cos\theta_{\rm R}e^{-i(\gamma-\delta_2)},$$
$$a_3=\tfrac{1}{2}g_{\rm L}g_{\rm R}\sin\xi\cos\xi(1/m_1^2-1/m_2^2)\sin\theta_{\rm R}\cos\theta_{\rm L}e^{i(\gamma-\delta_1)},\quad a_4=\tfrac{1}{2}g_K^2(\sin^2\xi/m_1^2+\cos^2\xi/m_2^2)\sin\theta_{\rm R}\cos\theta_K e^{i(\delta_2-\delta_1)}. \tag{14}$$

In the above expression we have used the parametrization for $U_{\rm L}$ and $U_{\rm R}$ appropriate to two-generation mixing:

$$U_{\rm L}=\begin{pmatrix}\cos\theta_{\rm L} & \sin\theta_{\rm L}\\ -\sin\theta_{\rm L} & \cos\theta_{\rm L}\end{pmatrix}, \tag{15}$$

$$U_{\rm R}=e^{i\gamma}\begin{pmatrix}\cos\theta_{\rm R}e^{-i\delta_2} & \sin\theta_{\rm R}e^{-i\delta_1}\\ -\sin\theta_{\rm R}e^{i\delta_1} & \cos\theta_{\rm R}e^{i\delta_2}\end{pmatrix}. \tag{16}$$

We take the point of view that since the coupling of the first and second generations to the third is small, it is a good approximation to neglect the third-generation effects when a finite contribution occurs at the level of the first two generations.

With QCD corrections, the operators appearing in H(penguin) will mix with operators in H(W-exchange). To simplify the problem, we follow Ref. 6 and use the one-loop result for H(penguin) and normalize the standard-model penguin to the full-QCD-corrected result. In this way, one obtains an effective $\alpha_s\simeq 0.6$ which we then insert into the L-R penguin. We have

$$H(\text{penguin})=\frac{G_{\rm F}}{\sqrt{2}}\sin\theta_{\rm L}\cos\theta_{\rm L}\left[\cos^2\xi+\sin^2\xi\left(\frac{m_1}{m_2}\right)^2\right]\frac{\alpha_s}{6\pi}\ln\frac{m_u^2}{m_c^2}\bar d\gamma_u L\lambda^a s\bar q_i\gamma^a q_i$$
$$+\frac{G_{\rm F}}{\sqrt{2}}\frac{g_{\rm R}^2}{g_{\rm L}^2}\sin\theta_{\rm R}\cos\theta_{\rm R}\left(\frac{m_1}{m_2}\right)^2 e^{i(\delta_2-\delta_1)}\left[\sin^2\xi+\cos^2\xi\left(\frac{m_1}{m_2}\right)^2\right]\frac{\alpha_s}{6\pi}\ln\frac{m_u^2}{m_c^2}\bar d\gamma_u L\lambda^a s\bar q_i\gamma^u\gamma^a q_i$$
$$+\frac{G_{\rm F}}{\sqrt{2}}\frac{g_{\rm L}g_{\rm R}}{g_{\rm L}^2}\sin\xi\cos\xi\frac{m_c\alpha_s}{2\pi k^2}\bar d i\sigma_{\mu\nu}k^\nu(\sin\theta_{\rm R}\cos\theta_{\rm L}e^{-i(\gamma+\delta_1)}L+\sin\theta_{\rm L}\cos\theta_{\rm R}e^{-i(\gamma+\delta_2)}R)$$
$$\times\gamma^a s\bar q_i\gamma^\mu\lambda^a q_i+\text{H.c.}, \tag{17}$$

where the first term is the penquin contribution from exchange of $W_{\rm L}$, the second is from exchange of $W_{\rm R}$, and the third is from $W_{\rm L}$-$W_{\rm R}$ mixing. In the model with pseudomanifest L-R symmetry, so that $g_{\rm L}=g_{\rm R}$, $\sin\theta_{\rm L}=\sin\theta_{\rm R}$, we have

$$\epsilon'=e^{i(\phi_2-\phi_0+\pi/2)}\frac{{\rm Re}A_2}{\sqrt{2}\,{\rm Re}A_0}[\tan\theta_2-\tan\theta_0], \tag{18}$$

with

$$\tan\theta_2-\tan\theta_0\simeq\tan\xi\left\{[\sin(\gamma-\delta_2)+\sin(\gamma-\delta_1)]\left[\frac{\langle I=2|P_2|\bar K^0\rangle}{\langle I=2|P_1|\bar K^0\rangle}-\frac{\langle I=0|P_2|\bar K^0\rangle}{\langle I=0|P_1|\bar K^0\rangle}\right]\right.$$
$$\left.-[\sin(\gamma+\delta_2)+\sin(\gamma+\delta_1)]\left[\frac{\langle I=2|P_3|\bar K^0\rangle}{\langle I=2|P_1|\bar K^0\rangle}-\frac{\langle I=0|P_3|\bar K^0\rangle}{\langle I=0|P_1|\bar K^0\rangle}\right]\right\}, \tag{19}$$

where

$$\langle I=0,2|P_1|\bar{K}^0\rangle=2\langle I=0,2|C_+O_+^{RR}+C_-O_-^{RR}|\bar{K}^0\rangle+(\alpha_s/12\pi)\ln(m_u^2/m_c^2)\langle I=0,2|\bar{d}\gamma_\mu\gamma_5 s\bar{q}_i\gamma^\mu\lambda^a q_i|\bar{K}^0\rangle,$$
$$\langle I=0,2|P_2|\bar{K}^0\rangle=4\langle I=0,2|C'_+O_+^{RL}+C'_-O_-^{RL}|\bar{K}^0\rangle, \tag{20}$$
$$\langle I=0,2|P_3|\bar{K}^0\rangle=(\alpha_s m_c/4\pi k^2)\langle I=0,2|\bar{d}i\sigma_{\mu\upsilon}k^\upsilon\gamma_5\lambda^a s\bar{q}_i\gamma_\mu\lambda^a q_i|\bar{K}^0\rangle.$$

We estimate the matrix element of the four quark operators in the factorization approximation[13] and obtain

$$\epsilon'=\frac{1}{\sqrt{2}}\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}e^{i(\phi_2-\phi_0+\pi/2)}\{-19.5[\sin(\gamma-\delta_2)+\sin(\gamma-\delta_1)]+1.8[\sin(\gamma+\delta_2)+\sin(\gamma+\delta_1)]\}\tan\xi. \tag{21}$$

Expressions similar to (21), with different estimates of the matrix elements, have also been obtained by other authors.[3,6,9]

We can readily estimate ϵ, following the methods of Beal, Bander, and Soni, and others,[7,14] but keeping the lowest-order contribution from L-R mixing. The calculations in these papers are not gauge invariant and this particular problem has been pointed out and corrected in Ref. 15. Fortunately, the numerical result is changed very little.[15] We extend the result in Refs. 7 and 14 to obtain

$$\epsilon=\frac{1}{2\sqrt{2}}\left[430\left(\frac{m_1}{m_2}\right)^2\sin(\delta_2-\delta_1)e^{i\pi/4}-i\left(\frac{m_1}{m_2}\right)^2\sin(\delta_2-\delta_1)+i\tan\xi\{26[\sin(\gamma-\delta_2)+\sin(\gamma-\delta_1)]\right.$$
$$\left.-1.8[\sin(\gamma+\delta_2)+\sin(\gamma+\delta_1)]\}+O\left(\tan^2\xi,\tan\xi\left(\frac{m_1}{m_2}\right)^2\right)\right]. \tag{22}$$

It is clear from Eq. (22) that one cannot accommodate the experimental value of ϵ without attributing the bulk of ϵ to the phase $\delta_2-\delta_1$; i.e., the L-R mixing term proportional to $\tan\xi$ gives negligible contribution to ϵ. Then from the limit[14] on the right-handed gauge-boson mass from the K_S-K_L mass difference, $(m_1/m_2)^2\leq\frac{1}{430}$, we obtain

$$\sin(\delta_2-\delta_1)>2\sqrt{2}|\epsilon|. \tag{23}$$

Combining Eq. (21), $\mathrm{Re}A_2/\mathrm{Re}A_0\simeq\frac{1}{22}$, and the observed value of ϵ, we obtain

$$\left|\frac{\epsilon'}{\epsilon}\right|=276\tan\xi|\sin(\gamma-\delta_2)+\sin(\gamma-\delta_1)-0.1[\sin(\gamma+\delta_2)+\sin(\gamma+\delta_1)]|\leq550\tan\xi. \tag{24}$$

Then the observed lower bound on $|\epsilon'/\epsilon|$ quoted in Eq. (2) yields

$$\tan\xi\geq2.0\times10^{-6}. \tag{25}$$

We now turn to the neutron electric dipole moment D_n where a nonzero value of ξ can play a role. The neutron electric dipole moment D_n has been calculated by several authors in the valence-quark-dipole-moment approximation.[4,16] We have examined possible additional contributions[17]; including them, we can write

$$D_n=D_v+D_c+D_h, \tag{26}$$

where D_v is the valence-quark model contribution, D_c is the contribution from the neutron wave-function correction due to the color-dipole moment of quarks, and D_h is due to hadron loops. Combining all these contributions, which are described individually in more detail elsewhere,[17] we have

$$D_n=10^{-23}\sin2\xi\{4.5\sin(\gamma-\delta_2)+74\sin(\gamma+\delta_1)-1.1\sin(\gamma-\delta_1)+16\sin(\gamma+\delta_2)\}\ e\ \mathrm{cm}. \tag{27}$$

Unfortunately, the angles γ,δ_1 occur differently in ϵ' and D_n, so that we cannot estimate a lower bound for D_n without additional assumptions. However, an upper bound follows immediately from Eq. (4):

$$|D_n|\leq7.6\times10^{-24}\ e\ \mathrm{cm}. \tag{28}$$

In a particularly simple model, due to Chang, the phases γ,δ_1,δ_2 are given by

$$\delta_1\simeq-\frac{3}{2}\left(\frac{K'}{K}\right)\frac{m_c}{m_s}\sin2\alpha,\quad \delta_2\simeq-\frac{1}{2}\left(\frac{K'}{K}\right)\frac{m_c}{m_s}\sin2\alpha,$$
$$\gamma\simeq2\alpha+\frac{1}{2}\left(\frac{K'}{K}\right)\frac{m_c}{m_s}\sin2\alpha, \tag{29}$$

where K, K', and α occur in the CP-nonconserving vacuum expectation value of a Higgs ϕ transforming as (2,2,0) under $SU(2)_L\otimes SU(2)_R\otimes U(1)$:

$$\langle\phi\rangle=e^{i\alpha}\begin{pmatrix}K & O\\ O & K'\end{pmatrix}. \tag{30}$$

Equation (22) then shows that, unless $(m_1/m_2)^2\ll\frac{1}{430}$, $K'/K\ll1$, and so γ is the dominant phase. With this approximation of neglecting δ_1 and δ_2 compared to γ,

$$|\epsilon'/\epsilon|=497\tan\xi\sin\gamma, \tag{31}$$

$$D_n=1.8\times10^{-21}\sin\xi\sin\gamma\ e\ \mathrm{cm}, \tag{32}$$

and

$$|D_n| = 3.6\times 10^{-24}|\epsilon'/\epsilon|\ e\ \mathrm{cm} \tag{33}$$

in this model. Equation (33) is the relationship between ϵ'/ϵ and D_n that we promised earlier. Now we can use the result (2) or the lower bound for $|\epsilon'/\epsilon|$ to obtain

$$|D_n| \geq 4.0\times 10^{-27}\ e\ \mathrm{cm}$$

in this model; we can also sharpen the upper bound to

$$|D_n| \leq 1.9\times 10^{-26}\ e\ \mathrm{cm}.$$

Thus we are able in this special case of the pseudomanifest L-R symmetric model to bound $|D_n|$ within 1 order of magnitude,

$$1.9\times 10^{-26} \geq |D_n| \geq 4.0\times 10^{-27}\ e\ \mathrm{cm}. \tag{34}$$

If, on the other hand, $(m_1/m_2)^2 \ll 1$, then the experimental value of ϵ can be obtained in Chang's model with $K'/K \gg 1$ and $\sin 2\alpha \ll 1$, in which case δ_1 and δ_2 remain the same but γ simplifies to

$$\gamma = -\delta_2 = -\frac{1}{3}\delta_1 = \frac{1}{2}\left(\frac{K'}{K}\right)\frac{m_c}{m_s}\sin 2\alpha,$$

leading to

$$|D_n| \simeq 1.7\times 10^{-24}|\epsilon'/\epsilon|, \tag{35}$$

instead of Eq. (33). This implies the bounds

$$0.9\times 10^{-26} \geq |D_n| \geq 1.9\times 10^{-27}\ e\ \mathrm{cm},$$

which is less than a factor of 3 variation in the bounds on $|D_n|$. If the value of ϵ'/ϵ is made more precise, these limits can be narrowed down even further. Measurements of ϵ'/ϵ and D_n with slightly improved sensitivity will constrain pseudomanifest left-right-symmetric theories of CP nonconservation tightly and can verify or rule out some models.

This work was supported in part by the Australian Research Grants Committee, and by the Research and Graduate Studies Committee of The University of Melbourne, and in part by the U.S. Department of Energy under Contract No. DE-AN03-76SF00235.

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