Symmetry and Static Correlation Function of Tilted Hexatic Liquid Crystals

In an interesting recent Letter,[1] Sprunt and Litster have reported a light-scattering study on a thick freely suspended film of tilted hexatic liquid crystal and they have suggested a theoretical modeling of their results. Here we point out that the formula for the static correlation function [Eq. (2) of Ref. 1] that they used to interpret their data is incorrect, and that their starting Eq. (1) does not reflect correctly the symmetry of the phases.

In their Eq. (2) Sprunt and Litster suggest that there are three contributions (three peaks) to the static correlation function, partly with a very unusual structure (e.g., the first one contains only $q_\perp$ but not $q_\parallel$). It is easy to see why Eq. (2) of Ref. 1 cannot be correct [even assuming the validity of their Eq. (1)]: (a) In Eq. (1) there are only two variables, and consequently there can be only two poles (two eigenmodes)[2,3] to comply with Goldstone's theorem. (Note that the number of independent elastic distortions can be larger than the number of poles, since the former are related to gradients of the variables, not to the variables themselves.) (b) Two of the three terms in Eq. (2) are structurally incompatible with Eq. (1), since they do not involve $q_\perp$ and $q_\parallel$ on an equal footing, as Eq. (1) does.

A straightforward calculation[4] of the static correlation function for ϕ, which follows from Eq. (1) of Ref. 1, yields the formula

$$\langle \delta\phi^2(q)\rangle / k_B T = [\bar{K}(1+b) + H(1+b)^2]^{-1} + [\bar{K}_A + \bar{K}]^{-1}, \qquad (1)$$

where $\bar{K}_{(A)} \equiv K_{(A)} q_\perp^2 + K'_{(A)} q_\parallel^2$ and $b = \bar{K}/\bar{K}_A$.

The structure of the present Eq. (1) reflects the nature of the two modes involved,[5] one is hydrodynamic and one is optical (and soft at the phase transition). That the authors of Ref. 1 were able to fit their data by their Eq. (2) or (3) suggests that the interplanar elastic constants $K'_{(A)}$ and the stiffness constant H may be small in the compound studied. However, a complete reanalysis of the data in the light of Eq. (1) as well as for the dynamics,[6,7] which is richer than assumed in Ref. 1, is inevitable.

In addition, already the starting point of Ref. 1 [their Eq. (1)] does not show the correct symmetry of tilted smectics, because isotropy inside the plane of the layers is assumed. However, tilted smectics have uniaxial symmetry within the layers (biaxial globally).[4,6,8,9] This is clear from the existence of the **c** vector[4] in both smectic *C* and smectic *I*. Also, the "hexagonal" bond-orientational order is hexagonal in the plane normal to the director only. Since in smectic *I* the director is tilted with respect to the layer planes, a projection of the "hexagonal" bond-orientational order into the layer planes results in a rectangular symmetry.

As a consequence all terms in Eq. (1) of Ref. 1 involving in-plane gradients have to be generalized to allow for two nonequivalent in-plane directions (e.g., parallel and perpendicular to **c**). In addition, cross-coupling terms of the form $(\nabla_i\theta)(\nabla_j\phi)$ and terms mixing in-plane and interplane gradients, like, e.g., $(\nabla_z \cdots)(\mathbf{c}\cdot\nabla \cdots)$, are missing (for the explicit form of the free energy cf. Ref. 6).

This makes the correct static correlation function much more complicated than Eq. (1) above, but the general structure of two peaks, one with and one without a gap, is preserved in the general (biaxial) case.

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[1] S. Sprunt and J. D. Litster, Phys. Rev. Lett. **59**, 2682 (1987).

[2] P. C. Martin, O. Parodi, and P. S. Pershan, Phys. Rev. A **6**, 2401 (1972).

[3] D. Forster, *Hydrodynamic Fluctuations, Broken Symmetries and Correlation Functions* (Benjamin, Reading, MA, 1975).

[4] P. G. de Gennes, *The Physics of Liquid Crystals* (Clarendon, Oxford, 1974).

[5] H. R. Brand and H. Pleiner, Phys. Rev. Lett. **59**, 2822 (1987).

[6] H. Pleiner and H. R. Brand, Phys. Rev. A **29**, 911 (1984).

[7] H. Pleiner, Mol. Cryst. Liq. Cryst. **114**, 103 (1984).

[8] P. A. C. Gane, A. J. Leadbetter, and P. G. Wrighton, Mol. Cryst. Liq. Cryst. **66**, 247 (1981).

[9] P. E. Cladis, H. R. Brand, P. Keller, and P. L. Finn, J. Phys. (Paris) **46**, 2151 (1985).