Model-Independent Constraints on Possible Modifications of Newtonian Gravity

C. Talmadge
Physics Department, Purdue University, West Lafayette, Indiana 47907

and

J.-P. Berthias, R. W. Hellings, and E. M. Standish
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109
(Received 10 June 1988)

New model-independent constraints on possible modifications of Newtonian gravity over solar-system distance scales are presented and their implications discussed. The constraints arise from the analysis of various planetary astrometric data sets. The results of the model-independent analysis are then applied to set limits on a variation in the $1/r^2$ behavior of gravity, on possible Yukawa-type interactions with ranges of the order of planetary distance scales, and on a deviation from Newtonian gravity of the type discussed by Milgrom.

PACS numbers: 04.80.+z, 04.90.+e, 96.35.Tf

We present here the results of an analysis of high-precision solar-system data in which we test for an apparent variation of the effective value of $\mu_\odot = GM_\odot$ with distance, where G is the Newtonian constant of gravity and $M_\odot$ is the mass of the Sun. We also present new limits on an anomalous precession of the perihelia of Mercury and Mars, obtained by studying how closely the orbits of these planets complied with the predictions of general relativity. Both of these results are then interpreted in terms of constraints on a deviation of the law of gravity from Newton's $1/r^2$ behavior, on possible Yukawa-type intermediate-range interactions, and on a modification of nonrelativistic gravitation of the type suggested by Milgrom.

One of the consequences of a variation of $\mu_\odot$ with distance is a modification of Kepler's third law:

$$a_p^3 = \mu_\odot(a_p)(T_p/2\pi)^2, \tag{1}$$

where T_p is the period and a_p is the physically measured semimajor axis of the orbit of planet p. Given a set of values for a_p and T_p, Eq. (1) can be used to determine $\mu_\odot(r)$. However, the period of a planet has historically been determined much more accurately than has the semimajor axis. For this reason the standard method of analysis of solar-system astrometric data[1] has been to define $\mu_\odot$ to have a particular (constant) value, $\mu_\odot(r) = \mu_\odot(a_\oplus) \equiv \kappa^2$, where κ is Gauss's constant (0.017 202 098 95 $\mathrm{AU}^{3/2}$/day, where AU denotes the astronomical unit), and to derive a "semimajor axis parameter" via

$$\tilde{a}_p^3 = \mu_\odot(a_\oplus)(T_p/2\pi)^2. \tag{2}$$

This procedure gives the semimajor axis relative to a standard orbit at 1 AU, and this is all that can be determined with period data alone. However, for several planets there are also range data available—either planetary radar or spacecraft tracking to a planetary orbiter, lander, or flyby. In these cases it is possible to measure a_p directly (and to thereby determine the size of the AU in kilometers).

If $\mu_\odot$ is a function of distance, then the scale of the semimajor axes will be different for each planet, and we can combine Eqs. (1) and (2) to yield

$$\left(\frac{a_p}{\tilde{a}_p}\right) \equiv (1+\eta_p) = \left[\frac{\mu_\odot(a_p)}{\kappa^2}\right]^{1/3}. \tag{3}$$

Thus, the signature for a variation of $\mu_\odot$ with distance is a disparity η_p in the conversion from AU's to kilometers appropriate to each planet. We note that Eq. (3) assumes no particular functional form for $\mu_\odot(r)$, except that $\mu_\odot(r)$ varies such that η_p may be treated as a constant over the orbit of planet p.

In addition to the prediction of a variation of $\mu_\odot$ from planet to planet, the various modifications of Newtonian gravity that have been suggested also predict an anomalous secular precession of perihelion within the orbit of a single planet. As is well known, most relativistic theories of gravity also predict a secular perihelion precession. Although the dynamical effects both of relativistic gravity and of these modifications of Newtonian gravity are not exactly described as simple perihelion precessions, the precession is still the dominant term. Since this is the case, and since we would like to present results in as model independent a way as possible, we have chosen to ignore the small periodic terms and to model the orbital effects of these theories in an approximate way by assuming that these effects are identical to the usual relativistic perihelion precession. The level of approximation involved in this approach is the same as that obtained by representation of the dynamical effects of relativity as a pure secular precession.

In the parametrized post-Newtonian formalism[2] of metric theories of gravity, the relativistic planetary perihelion precession is proportional to $(2-\beta+2\gamma)/3$, where β and γ are parametrized post-Newtonian parameters (both equal to 1 in general relativity). Unlike γ, the first-order effect of β in planetary data only enters through this dynamical term. Hence, if we determine β separately for each planet, we can determine the anomalous (non-Einsteinian) precession via

$$(\delta\phi_a)_p \equiv (\delta\phi)_p - (\delta\phi_0)_p = (\delta\phi_0)_p(1-\beta_p)/3, \tag{4}$$

where β_p is that value of β obtained from only the data relevant to planet p and $(\delta\phi_0)_p$ is the general-relativistic perihelion precession for planet p.

The result computed in Eq. (4) for $(\delta\phi_a)_p$ may be compared to that predicted by various theoretical models via the following prescription. If we have a potential $V_a(r)$ which deviates from the Newtonian potential only slightly (as it must to be consistent at all with experiment), then it can be shown that the anomalous precession predicted by $V_a(r\equiv 1/u)$ is given approximately by

$$(\delta\phi_a)_p = +\pi u \left(\frac{\partial^2 V_a}{\partial u^2}\right)\left(\frac{\partial V_a}{\partial u}\right)^{-1}\Bigg|_{u=1/a_p}. \tag{5}$$

In the studies testing for a variable $\mu_\odot$, a scaling factor η_p was assumed in the range data for each planet except the Earth (η_p is zero for the Earth by definition). This parameter was then added to the list of parameters which are usually determined in adjusting the solar-system model to fit the data, and least-squares fits for all parameters and all data sets were performed. The planets for which η_p values could be determined were those for which good physical distance measurements have been made, namely, Mercury (radar range and two ranges to Mariner 10 during Mercury flybys), Venus (radar range), Mars (ranging to the Mariner 9 orbiter and, especially, to the Viking landers), and Jupiter (ranging to Voyagers I and II during Jupiter flybys). The results of our analyses are shown in Table I. All of our quoted values for η_p are less than 1σ from 0, except for the Venus datum, which is a 1.6σ result. Since these results are consistent with a null hypothesis at the 49% confidence level, we conclude that we find no evidence for a variation of $\mu_\odot$ over the planetary scales under study.

TABLE I. Results of an analysis of planetary data searching for an anomalous variation of $\mu_\odot = GM_\odot$ with distance (given in column two in terms of η, as discussed in the text) and an anomalous perihelion shift $\delta\phi_a$ (given in column three in nanoradians per orbit).

Planet	η (units of 10^{-10})	$\delta\phi_a$ (nrad/orbit)
Mercury	$+40 \pm 50$	-80 ± 210
Venus	-55 ± 35	...
Mars	-0.9 ± 2.8	-130 ± 180
Jupiter	$+200 \pm 400$	...

The perihelion-precession analysis was done separately from the study of the variation of $\mu_\odot$. A value for β_p was determined for Mercury and Mars in similar least-squares fits to all data and all parameters. Equation (4) was then used to infer the values quoted in Table I for $(\delta\phi_a)_p$. As with the determinations of η_p, we find no evidence for any deviation from Newtonian gravity. Comparing these results to previous studies,[3,4] we see that the new result for Mercury has improved only marginally from the $\delta\phi_{a,\text{Mercury}} = 130 \pm 250$ nrad quoted by these previous authors, while the value for Mars has improved by a factor of 30 over the previous result $\delta\phi_{a,\text{Mars}} = 0 \pm 5600$ nrad.

We turn now to the applications of these results to several particular models of non-Newtonian gravity. We first consider the case of a simple excursion from the inverse-square law of the form

$$g(r) = \frac{\mu_\odot(a_\oplus)a_\oplus^\delta}{r^{2+\delta}}, \tag{6}$$

where $g \equiv |\nabla V|$ is the magnitude of the gravitational field due to the Sun. We then have

$$\mu_\odot(r) = \mu_\odot(a_\oplus)(a_\oplus/r)^\delta, \tag{7}$$

and

$$(\delta\phi_a)_p = \pi\delta. \tag{8}$$

Fitting of Eqs. (7) and (8) to the data from Table I gives $\delta = (-1.8 \pm 3.6)\times 10^{-10}$.

Recently, there has been much interest generated by a reanalysis[5] of the experiment of Eötvös, Pekár, and Fekete.[6] The original experiment was designed to test the equivalence principle between inertial and gravitational mass. The recent reanalysis pointed out that there was evidence of a hitherto unnoticed correlation of the results of Eötvös, Pekár, and Fekete to baryon number. This suggested the possibility of a new intermediate-range interaction coupling either to baryon number or hypercharge. Such a coupling could be described by a potential energy between two point sources 1 and 2 of the form

$$V_5(r) = -\frac{\alpha G_\infty m_1 m_2 e^{-r/\lambda}}{r}, \tag{9}$$

where V_5 is the potential associated with the new interaction, G_∞ is the Newtonian gravitational constant measured at $r \to \infty$, m_1 and m_2 are the masses of the attracting bodies, and α and λ characterize the strength and range of the proposed new coupling.

The variable-$\mu_\odot$ and anomalous-perihelion-precession results may also be applied to such a coupling, with solar-system-scale λ's. Starting from Eq. (9), we write the net gravitational acceleration felt by a planet as a sum of the usual Newtonian part and an anomalous V_5

part:

$$\mathbf{g}(r) = -\hat{\mathbf{r}}\frac{G_\infty M_\odot}{r^2}\left[1+\alpha\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda}\right]$$
$$\equiv -\hat{\mathbf{r}}\mu_\odot(r)/r^2. \tag{10}$$

We see from Eq. (10) that the principal effect of $V_5(r)$ is to induce an apparent variation of $\mu_\odot$ with distance. For a planet in a perfectly circular orbit, this will be the only effect. However, if the orbit of the planet is elliptical, then the gravitational field will no longer appear Newtonian, and one would expect additional effects on the orbit of the planet. The most significant such effect is an anomalous precession of the perihelion of the orbit, given by a rate of presession per orbit $\delta\phi_a$, where

$$\delta\phi_a \cong +\pi\alpha(a/\lambda)^2 e^{-a/\lambda}. \tag{11}$$

Here a is the mean value of the semimajor axis of the planetary orbit, and the effect of V_5 has been assumed to be small compared to Newtonian gravity.

We plot in Fig. 1 the 2σ constraints arising from various experimental systems over 16 orders of magnitude in λ. Our new results are presented in the curves labeled "Planetary $G(r)$" and "Planetary Precession." The "Laboratory $G(r)$" curves refer to laboratory Cavendish $G(r)$ tests,[7,8] and the "Geophysics" curves refer to the results of Holding, Stacey, and Tuck.[9] The curves labeled "Earth-Lageos-Lunar" refer to the comparison of $GM_\oplus$ inferred from Earth-surface gravity measurements to the values inferred from the study of the motions of the LAGEOS satellite and Moon, respectively. This analysis was first performed by Rapp,[10] but we use the somewhat more conservative limit quoted by Holding, Stacey, and Tuck.[9] Further details on these other constraints may be found in Ref. 11.

Finally, Milgrom has proposed a modification of Newtonian dynamics of the form[12]

$$m\kappa(a/g_0)\mathbf{a} = \mathbf{F}, \tag{12}$$

where the function κ is defined such that $\kappa(x \gg 1) \approx 1$ and $\kappa(x \ll 1) \approx x$, with g_0 set at $\approx 2\times 10^{-10}$ m s^{-2} in order to explain the galactic "missing-matter" problem. For solar-system dynamics, $x \equiv a/g_0 \gg 1$ and we have only a small modification of ordinary Newtonian mechanics. Following Milgrom, we write

$$\kappa(x) = 1 - \kappa_0 x^{-n}, \tag{13}$$

for $x \gg 1$. In this case, Eq. (12) leads to ordinary Newtonian dynamics, but with a modified gravitational field $g(r)$ of the form

$$g(r) = g_N[1+\kappa_0(g_N/g_0)^{-n}]. \tag{14}$$

This is equivalent to Newtonian gravity with a variable $\mu_\odot(r)$,

$$\mu_\odot(r) \cong \mu_\odot[1+\kappa_0(\mu/r^2 g_0)^{-n}], \tag{15}$$

and leads to a perihelion shift

$$(\delta\phi_a)_p = -2\kappa_0 n\pi(\mu/r^2 g_0)^{-n}. \tag{16}$$

Fitting Eq. (15) to the η_p data, for $n=1$ we find $\kappa_0 = (-2\pm 6)\times 10^{-8}$. Similarly, for $n=2$, we find $\kappa_0 = (-5\pm 15)\times 10^{-6}$, and for $n=3$, we find $\kappa_0 = (-1 \pm 4)\times 10^{-3}$.

Additionally, we mention that the results of this

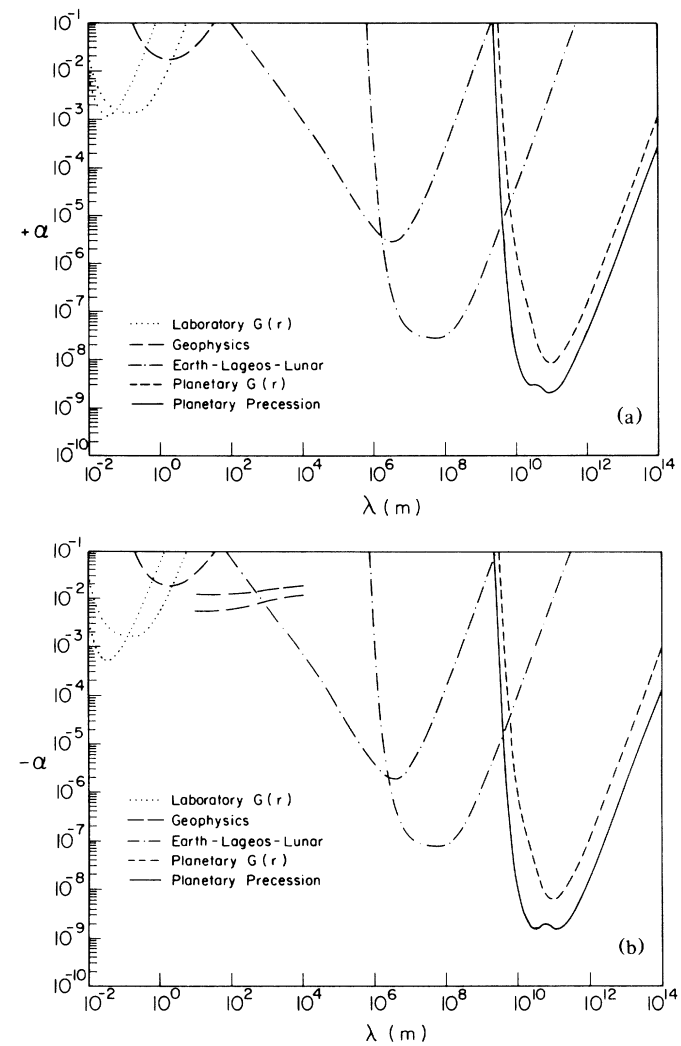

FIG. 1. Estimated 2σ-constraint curves arising from various experimental systems. For each curve the region above that curve is ruled out at the 95.5% confidence level. The two constraints "Planetary $G(r)$" and "Planetary Precession" are new constraints arising from the analysis presented in this paper. Constraints from experiments testing for composition dependence are not shown, since these depend on the specific model for the composition dependence of the proposed interaction. (a) Constraints for positive values of α, and (b) constraints for negative values α.

analysis could be used to set limits on several possible distributions of dark matter trapped in the vicinity of the Sun. While a detailed treatment of this idea is beyond the scope of this paper, we would suggest that Eq. (15) provides a possible formulation of the effect of matter distributed around the Sun with a spherically symmetrical monotonically decreasing density, and that the limits quoted above would therefore apply to such distributions as well.

This work was supported in part by the United States Department of Energy and in part by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration. One of the authors (C.T.) would like to thank the Jet Propulsion Labortory for their courtesy and generous support while part of this work took place.

[1]J. G. Williams and E. M. Standish, in "Reference Systems," edited by J. Kovalevsky, I. Mieller, and B. Kolaczek (Reidel, Dordrecht, to be published).

[2]C. M. Will. Phys. Rep. **113**, 345 (1984); *Theory and Experiment in Gravitational Physics* (Cambridge Univ. Press, Cambridge, 1981).

[3]D. R. Mikkelsen and M. J. Newman, Phys. Rev. D **16**, 919 (1977); see also I. I. Shapiro *et al.*, Phys. Rev. Lett. **28**, 1594 (1972).

[4]A. De Rújula, Nature (London) **323**, 760 (1986), and Phys. Lett. B **180**, 213 (1986).

[5]E. Fischbach, D. Sudarsky, A. Szafer, C. Talmadge, and S. H. Aronson, Phys. Rev. Lett. **56**, 3, 1427(E) (1986).

[6]R. v. Eötvös, D. Pekár, and E. Fekete, Ann. Phys. (Leipzig) **68**, 11 (1922); R. v. Eötvös, D. Pekár, and E. Fekete, *Roland Eötvös Gesammelte Arbeiten,* edited by P. Selényi (Akadémiai Kiado, Budapest, 1953), pp. 307-372.

[7]Y. T. Chen, A. H. Cook, and A. J. Metherell, Proc. Roy. Soc. London, Ser. A **394**, 47 (1984).

[8]J. K. Hoskins, R. D. Newman, R. Spero, and J. Schultz, Phys. Rev. D **32**, 3084 (1985).

[9]S. C. Holding, F. D. Stacey, and G. J. Tuck, Phys. Rev. D **33**, 3487 (1986).

[10]R. H. Rapp, Geophys. Res. Lett. **14**, 730 (1987).

[11]C. Talmadge and E. Fischbach, in "Searches for New and Exotic Phenomena," Proceedings of the Seventh Moriond Workshop, Les Arcs, France, 23–30 January 1988, edited by O. Fackler and J. Tran Thanh Van (Editions Frontières, Gif-sur-Yvett, France, to be published).

[12]M. Milgrom, Astrophys. J. **270**, 365, 371, 384 (1983).