Limits on the Variability of G Using Binary-Pulsar Data

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One of the few experimental handles on unified theories of gravity with other interactions comes from possible time variation of coupling constants over the Hubble time: $H_0^{-1} \simeq (7.67\times 10^{-11}\ \mathrm{yr}^{-1})^{-1}$. We present a new theory-independent estimate (consistent with zero) of the time variation of Newton's gravitational constant derived from the timing of the binary pulsar PSR 1913+16: $\dot{G}_0/G_0 = (1.0\pm 2.3)\times 10^{-11}\ \mathrm{yr}^{-1}$. We anticipate that this estimate will become sharper as more data are acquired.

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The time variability of the gravitational "constant" G is a fascinating theoretical possibility, which was raised long ago by Dirac[1] on the basis of his large-number hypothesis. It has been recently revived in the context of Kaluza-Klein and superstring theories which naturally predict a variability of coupling constants $G, \alpha, G_F, \dots$, on the Hubble time scale.[2-4] The observation of these variabilities constitutes, at present, one of our very few hopes of testing directly the existence of more dimensions,[2] and may help to discriminate between different shapes of the effective potential which fixes, in superstring theories, the size and dynamics of the internal space.[3] In view of the fundamental interest of such links between cosmology and particle physics, it is important to assess all the existing experimental limits on $\dot{G}, \dot{\alpha}, \dots$, and to extract new limits from other sources of data.

At present, one has in hand several very restrictive upper limits on the variation of the electromagnetic and strong (and to a lesser degree weak) coupling constants.[2,5-7] However, there are only relatively "poor" limits on the variation of the gravitational constant, poor meaning not small compared to the present Hubble expansion rate, say $H_0 = h_{75}\times 75$ km/s Mpc $= h_{75}\times 7.67\times 10^{-11}\ \mathrm{yr}^{-1}$. Until recently the best limit came from the comparison between classical astronomical observations, using ephemeris time, and atomic-time observations of the secular acceleration in mean longitude of the Moon. This yields[8] an upper limit of $|\dot{G}_0/G_0| \lesssim 3\times 10^{-11}\ \mathrm{yr}^{-1}$. This method, however, is marred by many uncertainties, and it is very difficult to exclude the presence of systematic errors (in fact, the dispersion between individual results is high and has been interpreted[9] as evidence for a nonzero $\dot{G}_0$). More recently, the acquisition, through the Viking landers, of high-quality radar ranging data between the Earth and Mars, raised the hope of getting much better limits on $\dot{G}_0/G_0$. This hope was supported by the analysis of the Viking data by Hellings *et al.*[10] that gave the estimate $\dot{G}_0/G_0 = (0.2\pm 0.4)\times 10^{-11}\ \mathrm{yr}^{-1}$. However, an independent analysis[11] of the same data yielded only the upper limit $|\dot{G}_0/G_0| < 3\times 10^{-11}\ \mathrm{yr}^{-1}$. This discrepancy in the interpretation of the same data seems to come not from the quality of the data themselves (which is high) but rather from a different appraisal of the uncertainty due to the difficulty in modeling the dynamical effects of the asteroids. One should also mention the limit based on primordial nucleosynthesis.[12] However, on the one hand this test is strongly dependent on many simplifying assumptions that enter the standard big-bang model, and, on the other hand, it does not restrict the present value of $\dot{G}/G$, but rather the ratio $\xi^2 = G_{\text{nucleosynthesis}}/G_{\text{now}}$. The results are that ξ^2 must stay within 20% of unity, which yields an upper limit for the *average*,

$$(|\dot{G}/G|)_{\text{avg}} \sim |\Delta G|/G\,\Delta t \lesssim 0.2 t_0^{-1} \simeq 2\times 10^{-11}\mathrm{yr}^{-1},$$

if $t_0 \simeq 10^{10}$ yr.

One sometimes sees in the literature statements to the effect that present limits on the Jordan-Brans-Dicke parameter $(\omega > 500)$[7] provide corresponding limits on $\dot{G}_0/G_0 \sim H_0/\omega$. However, this is only in the context of a *particular* theory, and indeed, in a separate paper, we will construct a model in which ω is large while at the same time $\dot{G}_0/G_0$ is not small compared to H_0. We have felt it important in this paper to concentrate on what can be said about $\dot{G}/G$ in a model-independent fashion. In particular, the purpose of this Letter is to point out that timing of the orbital dynamics of the binary pulsar PSR 1913+16 (Ref. 13) provides a new test of the variability of G, which (i) rests on a simple, and theoretically well understood, dynamical model; (ii) already yields an interesting estimate (consistent with zero) of the present value of $\dot{G}/G$,

$$\dot{G}_0/G_0 = (1.0 \pm 2.3)\times 10^{-11}\,\mathrm{yr}^{-1}; \tag{1}$$

and (iii) in the coming years will yield increasingly accurate estimates for $\dot{G}$ as new data are continuously acquired. The Viking data set, by contrast, is limited to the duration of that mission (1976–1982), and only a reanalysis of the existing data set, with all the uncertainties concerning asteroids taken into account, could yield an improved limit. As well as the high quality of the observational data continuously acquired[13] since its discovery in 1974, two other features make the binary pulsar especially suited for testing any variability of G: (a) the "cleanness" of this astrophysical system for which all the observational evidence convincingly indicates that one is dealing with a simple dynamical system of two gravitationally condensed bodies which have swept out any surrounding matter, and (b) the recent development of a complete general relativistic theory of the motion[14,15] and of the measurement, via pulsed electromagnetic signals,[16–18] of such a system of two strongly self-gravitating objects.

The problem of extracting an evaluation of $\dot{G}_0/G_0$ from the binary-pulsar timing data can be split into three steps. The first step consists in our computing the effects of a slow variation $G(t)$ on the *coordinate motion.* We assume that the equations of motion of a binary system of condensed bodies have the form ($a,b=1,2$; $a\neq b$)

$$d^2\mathbf{x}_a/dt^2 = -G(t)m_b(\mathbf{x}_a-\mathbf{x}_b)/|\mathbf{x}_a-\mathbf{x}_b|^3+\mathbf{A}_a^{\rm GR}(G(t),\mathbf{x}_a-\mathbf{x}_b,\mathbf{v}_a,\mathbf{v}_b,\mathbf{S}_a,\mathbf{S}_b), \tag{2}$$

where $\mathbf{v}_a = d\mathbf{x}_a/dt$ denote the orbital velocities, and $\mathbf{S}_a$ the spins of the bodies, and where

$$\mathbf{A}_a^{\rm GR} = c^{-2}\mathbf{A}_a^{\rm 1PN}+c^{-4}\mathbf{A}_a^{\rm 2PN}+c^{-4}\mathbf{A}_a^{\rm SO}+c^{-5}\mathbf{A}_a^{\rm reac}$$

includes all the general relativistic contributions to the equations of motion up to the gravitational radiation-reaction level inclusively. All the terms making up $\mathbf{A}^{\rm GR}$, though numerically small compared to the "Newtonian" acceleration, are important in that they contribute significantly to the final timing formula which is used to interpret the observations. In the usual case of a constant G, see Refs. 17 and 18 for the contribution of the orbital first post-Newtonian term, $A^{\rm 1PN}\sim(v^{\rm orbit}/c)^2A^{\rm Newt}$, Ref. 19 for the contributions of the second post-Newtonian, $A^{\rm 2PN}\sim(v^{\rm orbit}/c)^4A^{\rm Newt}$, and spin-orbit terms, $A^{\rm SO}\sim(v^{\rm orbit}/c)^3(v^{\rm spin}/c)A^{\rm Newt}$, and Ref. 14 for the contributions of the gravitational radiation-reaction term, $A^{\rm reac}\sim(v^{\rm orbit}/c)^5A^{\rm Newt}$. Let us now assume in Eq. (2) a slowly changing $G(t)$,

$$G(t)=G_0+\dot{G}_0t+\ldots = G_0(1-\dot{G}_0t/G_0)^{-1}+\ldots,$$

where the unwritten terms of order $\ddot{G}_0t^2$ or $\dot{G}_0^2t^2/G_0$ are supposed to be negligible, and t is counted from some epoch near the present and within the span of observations, say Julian Day 2444000. We then need to solve Eq. (2) by keeping the same $(v/c)^5$ relativistic accuracy as in the $G=$const case but by treating $\dot{G}_0/G_0$ as a small parameter. A simply way to achieve this is to draw on the exact knowledge of the effect of a $G(t)$ varying as the inverse of a linear function of t on the *Newtonian* dynamics of two-body (Ref. 20) or N-body (Ref. 21) systems. In particular, the result of Ref. 21 suggests the following change of space and time variables:

$$\mathbf{x}'_a=(1-\dot{G}_0t/G_0)^{-1}\mathbf{x}_a(t),\qquad t'=(1-\dot{G}_0t/G_0)^{-1}t. \tag{3}$$

A simple calculation allows one to check that the equations of motion for $\mathbf{x}'_a(t')$ have the form of the usual general relativistic equations, i.e., Eq. (2) with $G=$const, modulo terms of order

$$[(\dot{G}_0t/G_0)^2+\ddot{G}_0t^2/G_0+(v^{\rm orbit}/c)^2\dot{G}_0t/G_0]A^{\rm Newt}.$$

These extra terms are completely negligible in the equations of motion, except for the terms $\sim(v^{\rm orbit}/c)^2\dot{G}_0t/G_0$, coming from the time variation of G in the first post-Newtonian acceleration. As we shall point out later, these terms, however, do contribute negligibly to the secular effects that we are interested in. Thus the coordinate motion $\mathbf{x}_a(t)$ of a relativistic binary system when G varies, i.e., the solution of Eq. (2), is simply obtained by our applying the transformations (3) to the solution $\mathbf{x}'_a(t')$ of the usual, $G=$const, equations of motion. The latter is fully known from the investigations of Refs. 14–19 and references therein. Retaining in the solution $\mathbf{x}'_a(t')$ only those terms that are within the reach of the present (and foreseeable) observational accuracy, we can write,[15] using polar coordinates r'_a,θ' in the plane of the motion, the following parametric representation of the motion in the center-of-mass frame:

$$C_0+\frac{2\pi}{P_0}\left(t'-\frac{1}{2}\frac{\dot{P}_0^{\rm GR}}{P_0}t'^2\right)=U-e_t\sin U, \tag{4a}$$

$$r'_a=a'_a(1-e_a\cos U),\quad \theta'=\omega_0+(1+k)A_{e\theta}(U), \tag{4b}$$

where $A_e(U)\equiv 2\arctan\{[(1+e)/(1-e)]^{1/2}\tan U/2\}$. In

Eqs. (4), $\dot{P}_0^{\rm GR}$ denotes the secular acceleration of the mean orbital motion induced by the gravitational radiation-reaction terms in Eq. (2), which a direct dynamical calculation[14] has shown to be equal to

$$\dot{P}_0^{\rm GR} = -(64\pi/15\sqrt{3})x_1x_2k^{5/2}(1-e^2)^{-1}\left[1+\tfrac{73}{24}e^2+\tfrac{37}{96}e^4\right], \tag{5}$$

where $x_a = m_a/(m_1+m_2)$, e is any of the (slightly different) relativistic eccentricities appearing in Eqs. (4), and where the relation, at the second post-Newtonian plus spin-orbit level, among the dimensionless periastron advance parameter $k = \langle\dot{\omega}\rangle P_0/2\pi$, the masses, and the "Newtonian" parameters will be found in Ref. 19.

Using the transformation (3) we then easily find that the motion $\mathbf{x}_a(t)$ in presence of a varying G can be written as

$$C_0+\frac{2\pi}{P_0}\left[t-\frac{1}{2}\frac{\dot{P}_0^{\rm tot}}{P_0}t^2\right] = U - e_t\sin U, \tag{6a}$$

$$r_a = a_a(1-e_a\cos U), \quad \theta = \omega_0+(1+k)A_{e\theta}(U), \tag{6b}$$

where the only changes from Eqs. (4) are (i) a secular dilation (for $\dot{G}_0<0$) of the size of the orbits,

$$a_a(t) = (1-\dot{G}_0t/G_0)a_a', \tag{7}$$

and (ii) a small modification of the coordinate-time secular acceleration,

$$\dot{P}_0^{\rm tot} = \dot{P}_0^{\rm GR} - 2P_0\dot{G}_0/G_0. \tag{8}$$

We have checked directly, by "variation of the second post-Newtonian (2PN) elements,"[14] that the terms $\sim(v^{\rm orbit}/c)^2\dot{G}_0t/G_0$ in the equations of motion mentioned above contribute negligibly to Eqs. (7) and (8). The influence of these terms is most conveniently obtained with the recently worked out, adiabatically invariant, 2PN Delaunay action variables.[19] This confirms that they make no appreciable contributions to Eqs. (7) and (8), while they induce only unobservably small secular effects in the periastron advance and eccentricities. In other words, we see that our consistent relativistic approach gives, to lowest order, the same result as a heuristic calculation consisting simply of adding onto the general relativistic motion the Newtonian effects of a time variation of G.

Having in hand the effects of $\dot{G}$ on the coordinate motion, we need now to investigate the corresponding "timing formula" giving the observed arrival times on Earth of each pulsar signal as a function of the number of the pulse. In keeping with our *Ansatz* (2) for the effect of a varying G, we also assume that the space-time metric is "adiabatically" obtained by inserting a time-varying $G(t)$ in the general relativistic metric describing in Einstein's theory the space-time associated with two condensed bodies. It then can be checked, by going through all the steps of the derivation of the usual timing formula,[16-18] that the nonconstancy of G in the space-time metric introduces only negligible contributions compared to the coordinate-motion effects (7) and (8) [for instance, it induces an extra contribution in $\dot{P}^{\rm observed}$, but only of the order of $(v^{\rm orbit}/c)^2P_0\dot{G}_0/G_0$]. One then finds that the formula giving the pulse arrival times, τ, at the barycenter of the solar system, as a function of the number, N, of the pulse, can be written in the following parametric form:

$$\tau = T + x(T)[\sin\omega(\cos u - e_1)+(1-e_\theta^2)^{1/2}\cos\omega\sin u] + \gamma\sin u + \Delta_s + \Delta_A, \tag{9a}$$

$$N = N_0 + \nu_pT + \tfrac{1}{2}\dot{\nu}_pT^2 + \tfrac{1}{6}\ddot{\nu}_pT^3, \tag{9b}$$

$$C_0+\frac{2\pi}{P_0}\left[T-\frac{1}{2}\frac{\dot{P}_0^{\rm tot}}{P_0}T^2\right] = u - e_T\sin u, \tag{9c}$$

where, corresponding to Eq. (7), the orbit-crossing light-time parameter is

$$x(T) = (1-\dot{G}_0T/G_0)x^{\rm usual}, \tag{10}$$

and where[22] $\dot{P}^{\rm tot}$ is again given by Eq. (8) (investigations of other possible sources of apparent orbital period change[7] have shown them to be negligible, $\dot{P}/P \lesssim 2\times10^{-13}\ {\rm yr}^{-1}$, or unlikely).

Now, the measurement of the timing parameters $P_0 = 2\pi/n_0$, e_T, γ, and k [which enters $\omega = \omega_0 + kA_{e\theta}(u)$] allows one to determine the mass ratios,

$$x_2 \equiv 1 - x_1 = \tfrac{1}{2}\{[1+12n_0\gamma/ke_T(1-e_T^2)]^{1/2}-1\}. \tag{11}$$

Hence $\dot{P}_0^{\rm GR}$, Eq. (5), can be expressed in terms of P_0, e_T, γ, and k, so that Eq. (8) gives directly $\dot{G}_0/G_0$ in terms of observed timing parameters of the binary pulsar:

$$\dot{G}_0/G_0 = (2P_0)^{-1}[\dot{P}_0^{\rm GR}(P_0,e_T,\gamma,k) - \dot{P}_0^{\rm tot}]. \tag{12}$$

Using the experimental values quoted in the most recent entry in Ref. 13, notably $\gamma = 4.302\pm0.024$ ms, $k = (1.038\,240\pm0.000\,025)\times10^{-5}$, and $\dot{P}_0^{\rm tot} = (-2.419\pm0.040)\times10^{-12}$, we obtain Eq. (1) above, in which the dominant source of uncertainty in $\dot{G}_0/G_0$ is that of $\dot{P}_0^{\rm tot}$. A detailed discussion of the experimental uncertainties (statistical plus possible systematic errors) will be given elsewhere.[23] Let us note here only that the quoted experimental uncertainties are twice the formal statistical standard deviations, and that they are furthermore thought to be conservative.

As for the $\dot{G}$-induced secular variation, Eq. (10), of the orbit-crossing light-time parameter x, it is so small $[|\dot{G}_0 T/G_0| \lesssim 3\times 10^{-10} T/(10 \text{ yr})]$ compared to the present observational precision $(\delta x/x \sim 10^{-5})$ that there is no hope of our making use of it for estimating $\dot{G}$ [also one must first detect the secular shrinking of x, of order $\dot{P}_0^{\mathrm{GR}} T/P_0 \sim -10^{-8} T/(10 \text{ yr})$, induced by gravitational radiation-reaction effects]. The same remarks apply *a fortiori* to the even smaller secular effects, proportional to $(v^{\mathrm{orbit}}/c)^2 \dot{G}_0/G_0$, induced in the periastron advance and the eccentricities.

Finally, one should also mention the influence of a time variation of G on the proper spin-down rate of any pulsar[24]:

$$\dot{P}_p/P_p = (\dot{P}_p/P_p)^{G=\mathrm{const}} - \kappa \dot{G}_0/G_0, \tag{13}$$

where the internal-structure-dependent dimensionless coefficient κ measures the relative increase of the moment of inertia of a neutron star under a relative decrease of G. The prospects for getting an interesting limit on $\dot{G}_0/G_0$ from the effect (13), however, are low since the longest known pulsar spin-down age is that of PSR 0655+64, for which[25] $\dot{P}_p/P_p \simeq 10^{-10}\ \mathrm{yr}^{-1}$.

[1]P. A. M. Dirac, Nature (London) **139**, 323 (1937), and Proc. Roy. Soc. London A **165**, 199 (1938).

[2]W. J. Marciano, Phys. Rev. Lett. **52**, 489 (1984).

[3]Y.-S. Wu and Z. Wang, Phys. Rev. Lett. **57**, 1978 (1986).

[4]K. Maeda, Mod. Phys. Lett. A **3**, 243 (1988).

[5]A. Shlyakhter, Nature (London) **264**, 340 (1976).

[6]J. M. Irvine, Philos. Trans. Roy. Soc. London A **310**, 239 (1983).

[7]C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge Univ. Press, Cambridge, 1981), and Phys. Rep. **113**, 345 (1984).

[8]J. G. Williams, W. S. Sinclair, and C. F. Yoder, Geophys. Res. Lett. **5**, 943 (1978).

[9]T. C. Van Flandern, Astrophys. J. **248**, 813 (1981).

[10]R. W. Hellings *et al.*, Phys. Rev. Lett. **51**, 1609 (1983).

[11]R. D. Reasenberg, Philos. Trans. Roy. Soc. London A **310**, 227 (1983).

[12]J. D. Barrow, Mon. Not. Roy. Astron. Soc. **184**, 677 (1978); J. Yang *et al.*, Astrophys. J. **227**, 697 (1979); T. Rothman and R. Matzner, Astrophys. J. **257**, 450 (1982).

[13]J. H. Taylor and J. M. Weisberg, Astrophys. J. **253**, 908 (1982); J. M. Weisberg and J. H. Taylor, Phys. Rev. Lett. **52**, 1348 (1984); J. H. Taylor, in *General Relativity and Gravitation,* edited by M. A. H. MacCallum (Cambridge Univ. Press, Cambridge, 1987), p. 209, and in "Timing Neutron Stars", edited by H. Ogelman and E. P. J. van den Heuvel (Reidel, Dordrecht, to be published).

[14]T. Damour, in *Gravitational Radiation,* edited by N. Deruelle and T. Piran (North-Holland, Amsterdam, 1983), p. 59, and Phys. Rev. Lett. **51**, 1019 (1983), and in *Proceedings of Journées Relativistes 1983,* edited by S. Benenti *et al.* (Pitagora Editrice, Bologna, 1985), p. 89.

[15]T. Damour and N. Deruelle, Ann. Inst. Henri Poincaré **43**, 107 (1985).

[16]R. Blandford and S. A. Teukolsky, Astrophys. J. **205**, 580 (1976).

[17]R. Epstein, Astrophys. J. **216**, 92 (1977), and **231**, 644 (1979); M. P. Haugan, Astrophys. J. **296**, 1 (1985).

[18]T. Damour and N. Deruelle, Ann. Inst. Henri Poincaré **44**, 263 (1986).

[19]T. Damour and G. Schäfer, Nuovo Cimento **101B**, 127 (1988).

[20]J. P. Vinti, Mon. Not. Roy. Astron. Soc. **169**, 417 (1974).

[21]D. Lynden-Bell, Observatory **102**, 86 (1982).

[22]The definitions of the other quantities appearing in Eqs. (9), and the justification for our having replaced the Doppler factor D by 1, will be found in Ref. 18.

[23]J. H. Taylor and J. M. Weisberg, to be published.

[24]C. C. Counselman and I. I. Shapiro, Science **162**, 352 (1968); V. N. Mansfield, Nature (London) **261**, 560 (1976).

[25]J. H. Taylor and R. J. Dewey, Astrophys. J. (to be published).