Quantum Statistics of a Squeezed-Pump Laser

M. A. Marte and H. Ritsch[(a)]
Physics Department, University of Waikato, Hamilton, New Zealand

and

D. F. Walls[(b)]
Optical Sciences Center, University of Arizona, Tucson, Arizona 85721
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A laser with squeezed-pump fluctuations is found to oscillate with one of two macroscopically distinct phases. The phase diffusion rate is reduced below that of the usual laser and the output light may have amplitude fluctuations reduced below the vacuum level.

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There have now been several experiments which generate squeezed states of light.[1] Squeezed light has less fluctuations than the vacuum in one quadrature phase of the electromagnetic field.[2] These demonstrations of squeezed-light generation are all in passive systems where there is no pumping of the optical medium.

In contrast to this there has been the approach taken by Yamamoto, Machida, and Nilsson[3] who have analyzed the consequences of pump-noise suppression in a laser. They show that the photon-number fluctuations of the laser output can be reduced compared to the usual incoherently pumped laser, if the amplitude fluctuations of the pump are reduced. Machida and co-workers[4] have recently demonstrated the operation of a semiconductor laser pumped with an electron beam with reduced fluctuations and shown that the output light has a 7.3% reduction in number fluctuations below the standard quantum limit.

A related effect is found in theoretical studies of the micromaser, where single excited atoms are fed into the laser cavity.[5,6] If a constant velocity distribution of the atoms and hence a constant interaction time is assumed, the output of the micromaser is predicted to have sub-Poissonian statistics.

Other techniques have been suggested to reduce the quantum noise of the laser, either by feedback methods[7] or by squeezing of the vacuum fluctuations entering the laser cavity.[8]

In this paper we shall squeeze the pump fluctuations in the laser. The laser is modeled as an ensemble of N two-level atoms interacting with a single-mode unidirectional field tuned to the atomic resonance. This is a standard model used for the quantum statistics of the laser,[9] and is appropriate for a homogeneously broadened solid-state laser. The fluctuations in the laser arise from the pumping and spontaneous emission of the atoms and the vacuum fluctuations entering the cavity mode. We shall consider an optically pumped laser where the pump fluctuations are reduced by pumping the laser medium with squeezed light. In an actual system this would require a three-level medium where two of the transitions are pumped with correlated light beams as, for example, from the output of a nondegenerate parametric oscillator or four-wave mixer.[10] In our theoretical model an effective two-level medium interacts with a squeezed pump. The squeezed light is assumed to be broadband, a condition that requires the cavity linewidth of the parametric oscillator to exceed by an order of magnitude the natural linewidth of the atoms in the laser medium. The squeezing of the pump is characterized by an effective squeeze parameter including the spontaneous-emission contribution which reduces the effective squeezing of the pump. The spontaneous-emission contribution becomes less important as the laser goes higher above threshold.

In a recent paper[11] we considered the atoms to be pumped with a squeezed vacuum (for example, the output of a parametric oscillator below threshold). In this paper we shall consider the atoms to interact with a squeezed field with a coherent amplitude $\epsilon=|\epsilon|e^{i\psi}$ (e.g., the output of a parametric oscillator below threshold). Depending on the relative phase ψ of the driving field and the squeezed quadrature in the bath, one is able to describe a pumping process where the pump itself displays sub- or super-Poissonian statistics.

The effect of irradiating the atoms with squeezed light is to modify the decay rates of the atomic polarization.[12] The Maxwell-Bloch equations of the atoms and field variables (in a frame rotating at the rate of the atomic transition frequency) in the semiclassical limit are given by

$$\begin{aligned} \dot{\alpha}_x &= -\kappa\alpha_x + g v_x, \quad \dot{\alpha}_y = -\kappa\alpha_y + g v_y, \\ \dot{v}_x &= -\gamma_\perp(1+M)v_x + gD\beta_x, \\ \dot{v}_y &= -\gamma_\perp(1-M)v_y + gD\beta_y, \\ \dot{D} &= -\gamma_\parallel D + \gamma N - 4g(v_x\beta_x + v_y\beta_y). \end{aligned} \tag{1}$$

Here $\alpha=\alpha_x+i\alpha_y$ is the amplitude of the laser mode, $v=v_x+iv_y$ is the atomic polarization, and D is the atomic inversion of the N atoms. $\beta=\alpha+\epsilon^*/g$, where ϵ is a coherent field driving the atoms. g stands for the atomic dipole coupling and κ is the cavity decay rate. $\gamma_\parallel$ denotes the decay rate of the atomic inversion, which is given by $\gamma_\parallel=\gamma(2n+1)$, where γ is the natural linewidth of the atoms. The polarization decay involves two time scales,

$$\gamma_x=\gamma_\perp(1+M), \quad \gamma_y=\gamma_\perp(1-M), \tag{2}$$

where

$$M=[n(n+1)]^{1/2}/(n+\tfrac{1}{2})\in(0,1) \tag{3}$$

is a parameter describing the amount of squeezing, n being the mean photon number in the squeezed bath coupled to the atoms. Elimination of the atomic variables adiabatically from Eq. (1) yields the following set of equations:

$$\dot\alpha_x=\kappa[cR^{-1}(1-M)\beta_x-\alpha_x]:=A_x(\alpha_x,\alpha_y),$$
$$\dot\alpha_y=\kappa[cR^{-1}(1+M)\beta_y-\alpha_y]:=A_y(\alpha_x,\alpha_y), \tag{4}$$

where

$$c=g^2D_0/\gamma_\perp\kappa, \tag{5}$$

and D_0 is the steady-state inversion below threshold. The saturation denominator reads

$$R\equiv R(\beta_x,\beta_y)$$
$$=(1-M^2)+\frac{\beta_x^2}{n_0}(1-M)+\frac{\beta_y^2}{n_0}(1+M), \tag{6}$$

with

$$n_0=\gamma_\parallel\gamma_\perp/4g^2. \tag{7}$$

In the case of a squeezed-vacuum pump the stable steady-state solutions were found to be in phase with the low-noise quadrature in the squeezed bath.[11] If one includes a classical driving field (assumed to be either purely real or purely imaginary) the semiclassical stationary points are easily seen to be the roots of a polynomial factorizing into a quadratic and a cubic.[13] The situation may be illustrated by means of the semiclassical potential $\Phi(\alpha_x,\alpha_y)$, defined by[14]

$$\partial\Phi(\alpha_x,\alpha_y)/\partial\alpha_x=-A_x(\alpha_x,\alpha_y),$$
$$\partial\Phi(\alpha_x,\alpha_y)/\partial\alpha_y=-A_y(\alpha_x,\alpha_y) \tag{8}$$

[with A_x and A_y given in Eq. (4)], which gives

$$\Phi(\alpha_x,\alpha_y)=\tfrac{1}{2}\kappa[\alpha_x^2+\alpha_y^2-cn_0\ln R(\beta_x,\beta_y)]. \tag{9}$$

The results for the atoms interacting with a squeezed vacuum ($\epsilon=0$) have been discussed in Ref. 11. We plot the potential in Fig. 1(a), in order to demonstrate that the phase symmetry of the usual laser is broken for nonzero M, even in the case of a zero driving field ϵ!

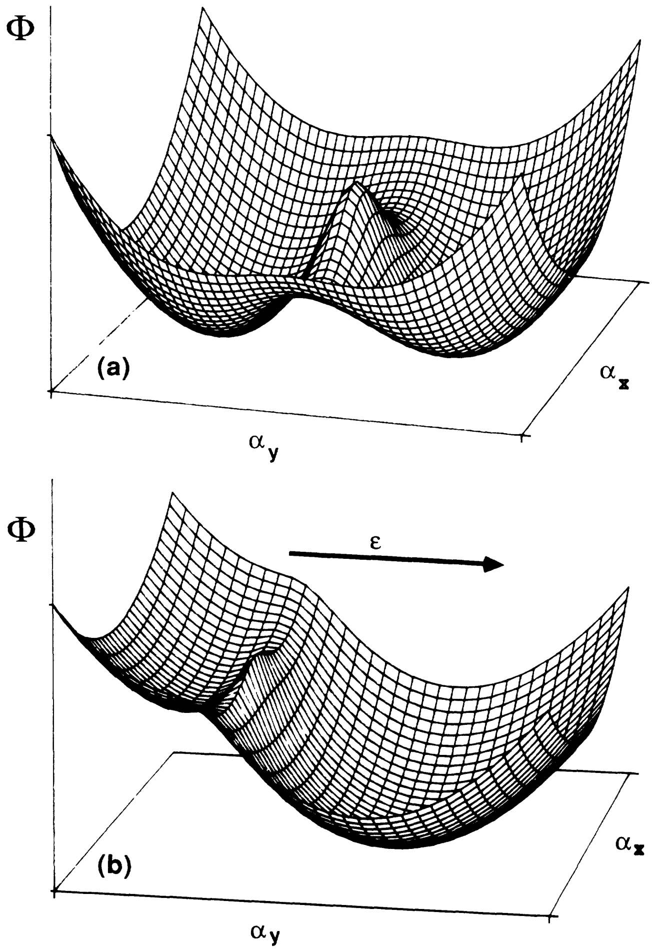

FIG. 1. The potential function $\Phi(\alpha_x,\alpha_y)$ of the squeezed-pump laser above threshold. (a) Squeezed-vacuum pump, $M=0.6$, $c=2$. (b) Antibunched input, $M=0.9$, $c=10$, $|\epsilon/g|=4$.

The squeezed bath acts to imprint a particular phase onto the steady state as is seen from the existence of two local minima on the imaginary axis. The two stable solutions with phase differing by π are in phase with the low-noise quadratures of the squeezed bath. We show in our discussion of the laser gain given by Eq. (11) that the gain is greatest for these phases ($\phi=\pm\pi/2$). This is different from the symmetry breaking which occurs in the laser with injected signal and also from the micromaser where only one phase is stable. The squeezed-pump laser will oscillate with a macroscopic coherent amplitude and may have a phase of either $+\pi/2$ or $-\pi/2$. This suggests the intriguing possibility of observation of macroscopic quantum coherence[15] in this optical system.

If ϵ is chosen nonzero and purely imaginary and M is assumed real and positive, which corresponds to a pump with reduced amplitude fluctuations, the coherent field with phase $\psi=\pi/2$ destroys the "degeneracy" in Fig.

1(a) by making the potential well along $\pi/2$ deeper than the one along $-\pi/2$. Thus the laser oscillates with a preferred phase of $\pi/2$, that is, in phase with the coherent field.

For a field with reduced phase fluctuations, that is, ϵ purely real, however, there is a tradeoff between a tendency to lock the laser phase ϕ to the phase $\psi=0$ of the driving field and a tendency to lock to $\phi=\pm\pi/2$ as a result of the phase-dependent gain brought about by the squeezed bath. This results in two stable steady states in the first and fourth quadrants which lie symmetrically about the real axis.

In a fully quantum mechanical treatment the atomic and field variables are described by Langevin equations which add fluctuating forces to the Maxwell-Bloch equations (1). If one adiabatically eliminates the atomic variables [assuming $\kappa \ll \gamma_\perp(1\pm M)$] from these equations, one finds the following Langevin equation for the field amplitude in the case of a squeezed-vacuum pump:

$$\dot{\alpha}=G(\alpha,\alpha^*)\alpha-\alpha_{\rm nl}+\Gamma_\alpha^{\rm tot}, \qquad (10)$$

where

$$G(\alpha,\alpha^*)=\kappa\left[-1+\frac{c}{1-M^2}(1-e^{2i\phi}M)\right], \qquad (11)$$

for $\alpha=\sqrt{I}e^{-i\phi}$ and

$$\alpha_{\rm nl}=\kappa c\alpha\frac{1-e^{2i\phi}M}{(1-M^2)^2}\left[\frac{|\alpha|^2}{n_0}-M\left(\frac{\alpha^2}{2n_0}+\frac{\alpha^{*2}}{2n_0}\right)\right]. \qquad (12)$$

Note that Eqs. (10) and (12) constitute a generalization of our previous results,[11] where we assumed weak squeezing and linear M dependence. The correlations of the total stochastic force including noise stemming from the atomic variables (cf. Haken's "noisy slave"[9]) read

$$\langle\Gamma_{\alpha^*}^{\rm tot}(t)\Gamma_\alpha^{\rm tot}(t')\rangle=\left(\frac{g}{\gamma_\perp(1-M^2)}\right)^2\{(1+M^2)\gamma(n+1)N-2\gamma_\perp M^2(N+D)\}\delta(t-t'),$$
$$\langle\Gamma_\alpha^{\rm tot}(t)\Gamma_\alpha^{\rm tot}(t')\rangle=\left(\frac{g}{\gamma_\perp(1-M^2)}\right)^2\{\gamma_\perp M(1+M^2)(N+D)-2M\gamma(n+1)N\}\delta(t-t'). \qquad (13)$$

Equation (10) is a rotating-wave van der Pol oscillator equation with a phase-dependent gain. It is clear that the gain G is largest for $\phi=\pm\pi/2$; hence a steady-state field with either of these phases is built up above threshold.

Converting the Langevin equation (10) to intensity and phase variables, we can derive the phase diffusion coefficient evaluated about the stable steady-state solution I with phase $\phi=\pm\pi/2$. We find

$$D_{\phi\phi}(\bar{I},\bar{\phi})=(\kappa c/2\bar{I})(1+n)/(1+M). \qquad (14)$$

For $M=0$, this expression reduces to the usual phase diffusion rate due to spontaneous-emission noise,[9] which may be written as

$$D_{\phi\phi}=(1/2\bar{I})g^2\bar{N}_2/\gamma_\perp, \qquad (15)$$

N_2 being the number of excited atoms in the steady state.

In Fig. 2 the phase diffusion coefficient $D_{\phi\phi}$ is plotted against the photon number n in the squeezed bath for the

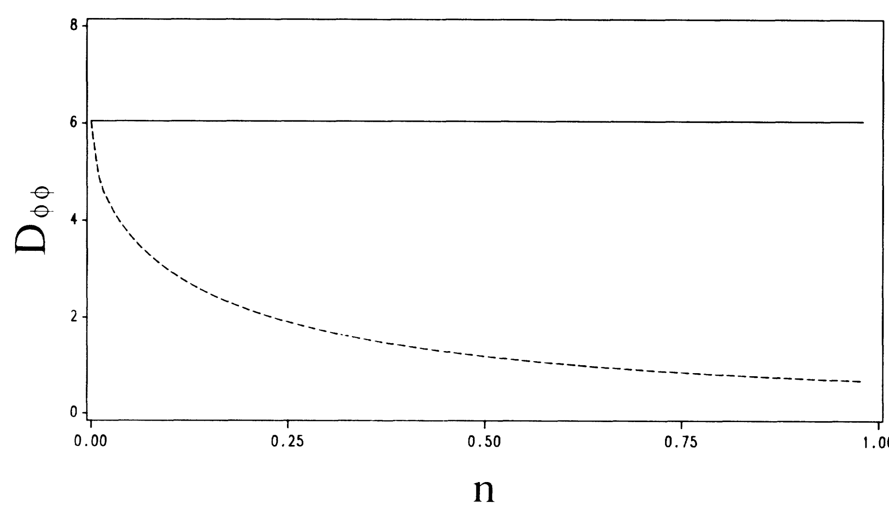

FIG. 2. The phase diffusion coefficient $D_{\phi\phi}$ for the laser at 10% above threshold. Solid line: usual laser ($n=0$); dashed line: squeezed-pump laser ($\epsilon=0$).

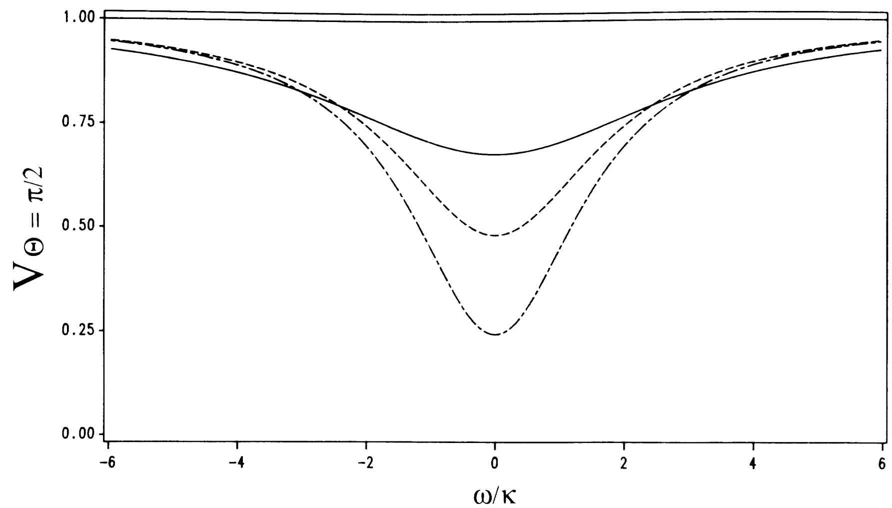

FIG. 3. Spectrum of fluctuations in the amplitude quadrature with antibunched pump for fixed output intensity. Solid line: $M = 0.47$; dashed line: $M = 0.70$; dash-dotted line: $M = 0.93$.

laser operating 10% above threshold. We see that the effect of the squeezed pump is to reduce the phase diffusion coefficient—and hence the laser linewidth—significantly below the level of an ordinary laser. We note that a similar effect has been found by Gea-Banacloche[8] for a laser with a squeezed vacuum entering the cavity. In this case a reduction of the phase diffusion coefficient by a factor of 2 may be achieved, although this occurs about an unstable state.

The quantum statistics of the output light may be calculated from the Langevin equation (10). In particular, we have calculated the spectrum of fluctuations in the quadrature phases of the output light[16]:

$$V(X_\Theta,\omega) = \int_{-\infty}^{\infty} e^{i\omega\tau} \langle X_\Theta(\tau), X_\Theta(0)\rangle d\tau, \qquad (16)$$

where X_Θ is a quadrature-phase operator. This spectrum for a laser with a squeezed pump corresponding to reduced amplitude fluctuations is plotted in Fig. 3. We see that for increasing values of the squeeze parameter M, there is a reduction in the output fluctuations below the level of the vacuum fluctuations. This leads to a laser with a sub-Poissonian output, a feature similar to that achieved in a semiconductor laser with suppressed pump fluctuations.[3,4]

We have investigated a new class of active lasing systems which oscillate with a finite coherent amplitude and may exhibit quantum coherence between two macroscopically distinct phases. The linewidth of these lasers are substantially reduced and, for a pump with reduced amplitude fluctuations, the output photon statistics may be sub-Poissonian.

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[(a)]Permanent address: Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria.

[(b)]Permanent address: Physics Department, University of Auckland, Auckland, New Zealand.

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