Exact Three-Body Calculation of *p-d* Polarization Observables

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Modified Faddeev-type three-nucleon equations that also include the Coulomb potential between the two protons are solved in momentum space. With a separable representation of the Paris potential for the *N-N* $^1S_0$, the coupled $^3S_1-{}^3D_1$, and the *P* states, we have calculated for the first time *p-d* polarization observables. Our *p-d* results at $E_p=2.5$ MeV are compatible with the measured *p-d* data and demonstrate that a proper treatment of the *p-d* Coulomb corrections is required.

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The nucleon-nucleon (*N-N*) system represents the prime source of information on the interaction between nucleons at energies relevant for nonrelativistic potential theory. However, some aspects of the interaction, such as the spin- (and to some extent also the isospin-) dependent part, are not easily accessible through *N-N* scattering, and the off-shell component of the interaction can practically not be obtained from a pure two-body system. In the system with three nucleons the interaction between two nucleons is distorted by the presence of the third nucleon, and consequently the off-shell component of the *N-N* interaction can be studied. In addition, spin-polarization effects in three-nucleon scattering are bigger than in *N-N* scattering by at least an order of magnitude and lend themselves even, at rather small laboratory energy, to better information on the spin dependence of the *N-N* interaction. Also, since an exact theory describing three nucleons interacting via strong two-body forces is available by the Faddeev equations, the situation seems to be quite favorable for improving our knowledge of the *N-N* interaction by means of the three-nucleon system.

Unfortunately the Coulomb force has spoiled real progress so far because it has an inverse impact on theory and experiment. For experiments, namely, spin-polarizaton experiments, mostly the reaction with electrically charged particles, i.e., proton-deuteron (*p-d*) scattering, has been studied,[1] whereas for the neutral-reaction counterpart, i.e., neutron-deuteron (*n-d*) scattering, data are sparse and are restricted to differential cross sections and neutron analyzing powers in elastic reactions.[2] What is advantageous for the experiment turns out to be a major problem for the theory, because originally the Faddeev equations were derived only for two-body potentials of sufficiently short range. To accomodate the infinite range of the Coulomb potential in the Faddeev equations is by no means trivial, and this has been the reason why data from *p-d* experiments have usually been analyzed on the basis of *n-d* calculations supplemented by approximate Coulomb corrections. The standard approximation[3] originates from the two-body Coulomb scattering between the proton and the electric charge of the deuteron located at its center of mass (c.m. scattering), whereas another approximation[4] tried, in addition, to describe in a two-body manner the distortion that the strong interaction experiences in the presence of the Coulomb force. The fact that *p-d* polarization measurements have been essentially compared with a *n-d* theory underlines the need for an exact description of *p-d* scattering. Successful attempts at an exact treatment of elastic *p-d* scattering have already been made (more than ten years ago[5]) within the quasiparticle concept applied to Coulomb-modified three-body scattering, but because of the complexity of the problem only *N-N S* waves were used. Later on, *p-d* configuration-space calculations,[6,7] again employing just *N-N S* waves, were published, stirring up a dispute on the low-energy scattering results of both approaches.[8-14] The problem seems to be settled now, with the validity of the configuration-space *p-d* calculation of Kuperin, Merkuriev, and Kvitsinskii[6] being questioned. However, still no *p-d* observables have yet been calculated, other than differential cross sections[5,13] and some second-order spin polarizations,[15] because only for these observables are the gross features of the angular distribution dominated by the *N-N S* waves, whereas all other spin-polarization observables are identical to zero as long as higher *N-N* partial waves are neglected. To establish the three-nucleon scattering system as a more stringent test for details of the *N-N* interaction, it is therefore mandatory to set up and solve *p-d* scattering equations that allow the calculation of polarization observables such as the first-order ones. Prior to such a theoretical study it is not really appropriate to discriminate among different *N-N* force models on the basis of *p-d* scattering data.

The quasiparticle method lends itself to the incorporation of the shielded Coulomb potential in the three-nucleon scattering equations.[5] The complication involved with considering higher *N-N* partial waves is, however, more aggravated than with non-Coulomb Faddeev equations. Apart from the significant increase of computer facility requirements linked to highly coupled integral equations in spin-isospin representation—with just *N-N S* waves the total orbital angular momentum remains conserved and the coupling of the integral equations is only threefold at most—one part of the three-body effective potential that contains the Coulomb potential in a nucleon-exchange-type expression requires special handling.[16] Nevertheless, the additional angular-momentum expansions which are necessary in this part of the calculation do converge sufficiently fast, as has already been shown in Ref. 16 for a bound-state calculation, and the three-body equations describing the strong and the screened Coulomb interaction in the p-d scattering system can be used. For such an application of the quasiparticle method, the Coulomb potential is typically taken to be exponentially screened,

$$v^R(r)=(e^2/r)\exp(-r/R), \tag{1}$$

where R is the radius of the screening.

For the N-N interaction, we have chosen to take a rank-one separable representation of the Paris potential[17] in all partial waves, because it provides a compromise between a reasonable representation of the N-N on-shell as well as off-shell interaction and also because of the necessity of keeping the potential to the lowest rank for reason of available computer facilities. Since we have considered the $^1S_0$, $^3S_1$-$^3D_1$, and all P waves of the N-N interaction, we are dealing with modified Faddeev scattering equations[5,15,16] with 10, 15, or 16 three-body channels depending on the total angular momentum and parity under consideration. Furthermore, we have consistently replaced the Coulomb T matrix of the p-p subsystem by the Coulomb potential, which should be acceptable away from breakup threshold.[18] Still, the calculation of the full screened on-shell T matrix T_R of elastic p-d scattering below breakup threshold required, for total angular momenta up to $J=\frac{13}{2}$ and both parities, about 600 h of CPU time on a VAX 8700 computer. The difference of the total p-d and the c.m. Coulomb p-d on-shell T matrix yields the Coulomb-modified strong p-d on-shell T matrix T_{SR} for a certain value of the screening radius. The unscreened T matrix T_{SC} which is necessary to find the physical p-d scattering amplitude can be recovered by a standard renormalization procedure; i.e., the Coulomb-modified screened strong p-d T matrix is calculated for a series of increasing values of the screening radius and then multiplied by a well-known renormalization factor.[5,13] In the limit of the screening radius tending to infinity, the unscreened Coulomb-modified strong p-d T matrix is obtained. The physical p-d scattering amplitude is finally given by the addition of the analytically known c.m. Coulomb amplitude.

The p-d observables are then studied as functions of the screening radius and we find that the screening and renormalization procedure is indeed a practical one. The screening limit is reached rather fast (which has been shown before in the case of phase shifts in Refs. 13 and 14), whereby stability is obtained first in the differential cross section, but in any case by around 300 fm for all observables at the given energy and an accuracy of roughly 1%.

The reasons why we have decided to calculate at $E_p=2.5$ MeV are as follows: First, the kinematic singularities and the singularitylike structure due to the screened Coulomb potential of the integral equation are less complicated below breakup threshold. Second, at this energy measurements of all first-order p-d polarizations are available.[19] In addition, a rather old but not outdated measurement of the differential cross section[20] exists. Furthermore, the higher partial waves not considered here (e.g., the small D waves) are supposed to have a negligible influence on the low-energy observables.

The results of our calculation are presented in Figs. 1 to 4. Both in the differential cross section and the first-order spin-polarization observables we show the results of the full p-d calculation (solid line), the corresponding n-d calculation (dotted line), and the n-d calculation where just the simplest pure Coulomb p-d corrections[3] were taken into account (dashed line). The dash-dotted lines in Fig. 4 show the results from full p-d calculations where just the $^1S_0$ and the $^3S_1$-$^3D_1$ N-N partial waves were considered for the strong interaction. In all figures the experimental data are also given for reference, but we have to be careful in drawing conclusions from the comparison with the measurements. Nevertheless, in all observables the data are compatible with our results from the full p-d calculation suggesting that the overall features of the N-N potential seem to be acceptable at this energy. Calculations with other interaction models will be necessary for a further discussion of this topic and also of a possible dependence of the Coulomb distortion effect on the choice of the N-N potential.

The exact Coulomb corrections are apparently essen-

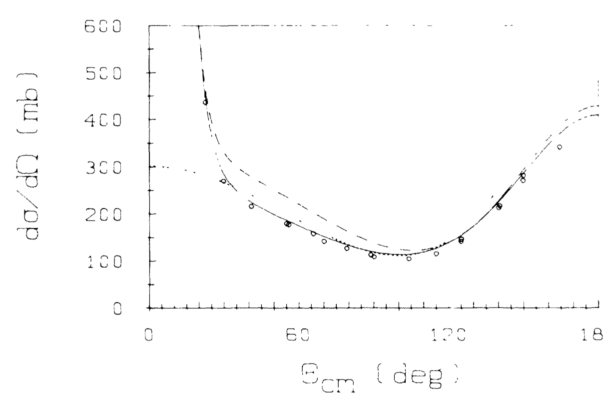

FIG. 1. Nucleon-deuteron differential cross section at $E_N=2.5$ MeV. The solid line represents the complete p-d calculation, the dotted line the corresponding n-d result, and the dashed line the n-d calculation with simple pure Coulomb corrections added. The p-d data are taken from Ref. 20.

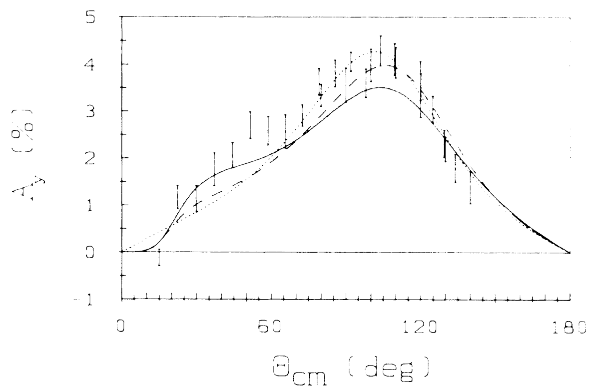

FIG. 2. Nucleon analyzing power at $E_N = 2.5$ MeV. The curves represent the same results as in Fig. 1. The *p-d* data are taken from Ref. 19.

tial for improving the agreement with the data, and the *n-d* calculations with the simple approximation that neglects the Coulomb distortion completely are found to be inadequate. The nucleon analyzing power is the quantity where one hopes to gain information on the *N-N P* waves and eventually on three-body forces.[21] As far as the proton analyzing power is concerned, our results show that Coulomb corrections contribute in the forward direction and in particular around the maximum value of this observable. This shift between the maxima of the neutron and proton analyzing powers is interesting because it has been established before by *n-d* and *p-d* measurements of comparable accuracy at energies above breakup threshold.[2,22] Both the shape and the magnitude of the observed difference cannot be explained by the simplest Coulomb corrections (dashed line in Fig. 2) whereas they may well be described by the exact full Coulomb contributions provided their percentage becomes smaller with increasing energy.[2,4,22]

Three-nucleon vector polarization seems to be governed by the *N-N P* states whereas tensor polarizations are supposed to be dominated by the *N-N* tensor force.[1] The validity of the latter statement can hardly be substantiated by the results of Fig. 4. An explanation could be that at rather small energies like $E_p = 2.5$ MeV the importance of the lower partial waves (*P* states) relative to the *D*-state admixture in the $^3S_1$-$^3D_1$ tensor state is enhanced whereas at energies such as 10 MeV the *D* state becomes more important. Although the study of T_{20} and T_{21} at lower energies cannot provide the desired test for the *N-N* tensor force, the tensor polarization T_{22} contains, at least, information on its asymptotic properties. Our finding that the Coulomb distortion is practi-

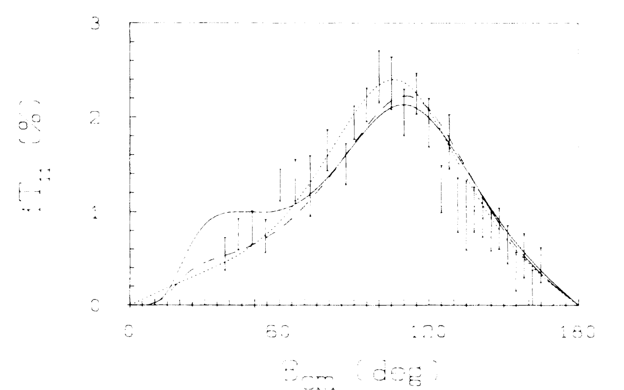

FIG. 3. Deuteron vector analyzing power at $E_d = 5.0$ MeV. The notation is the same as in Fig. 2.

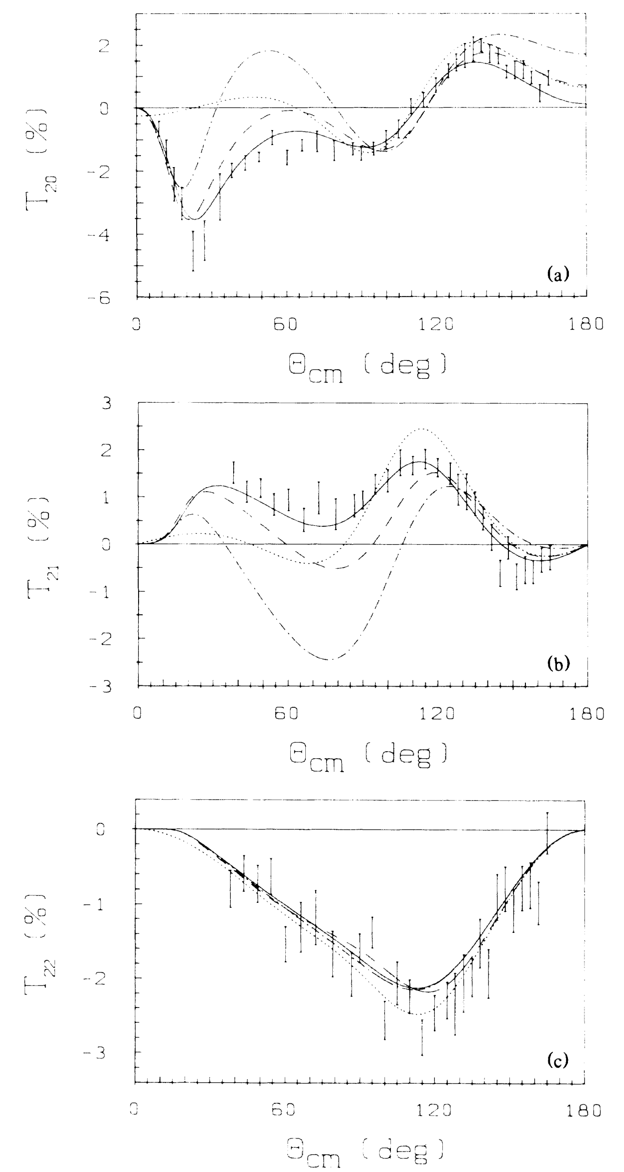

FIG. 4. Deuteron tensor analyzing powers at $E_d = 5.0$ MeV. Solid, dashed, and dotted lines are the same as in Fig. 1. The dash-dotted line shows the result from the full *p-d* calculation where just the $^1S_0$ and the $^3S_1$-$^3D_1$ *N-N* states were taken into account. The data are taken from Ref. 19.

cally negligible in T_{22} would favor the extrapolation of p-d data to extract the asymptotic normalization constants. Provided that the accuracy of the p-d measurement at low energies could be improved, the dominance of the $^3S_1$-$^3D_1$ tensor force in T_{22} together with the negligible role of the Coulomb distortion would make us hope that a significant constraint on the nonasymptotic part of the tensor force can also be found.[23]

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