Effect of Dissipation on Thermal Activation in an Underdamped Josephson Junction: First Evidence of a Transition between Different Damping Regimes

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Measurements of the lifetime of the zero-voltage state in a single hysteretic Josephson junction show clear evidence of two different damping regimes. Results are obtained for a wide range of damping within the underdamped region because the damping resistance depends exponentially on inverse temperature as does the junction's quasiparticle resistance. Thus a further insight is also given on the thorny problem of the damping resistance for the resistivity shunted junction model.

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Intrinsic thermal noise is one of the mechanisms responsible for the decay of the supercurrent in Josephson tunnel junctions. This problem is often described in terms of the escape of a Brownian particle from a metastable state in a washboardlike potential. In the study of such a process, different regimes with respect to the damping coefficient η are typically considered. Kramers[1] obtained the expression of the lifetime τ of the metastable states with the hypothesis of a large energy barrier E_b, compared to k_BT. The comparison of η with the attempt frequency ω results in overdamping if $\epsilon \equiv \eta/\omega \gg 1$ and underdamping for $\epsilon \ll 1$. Here $\omega = \omega_J(1-\alpha^2)^{1/4}$, where ω_J is the Josephson plasma frequency and $\alpha \equiv I/I_c$ is the ratio of bias current I to I_c, the critical current in the absence of fluctuations.

Kramers recognized two distinct regimes within the underdamped ($\epsilon \ll 1$) range. For $\epsilon > \epsilon_c \equiv k_BT/2\pi E_b$, Kramers found intermediate damping which results in the transition-state theory for the escape rate:

$$\tau_{TR}^{-1} = (\omega/2\pi)\exp(-E_b/k_BT) \quad (1)$$

(TR denoting the thermal regime), whereas, for extremely underdamped systems ($\epsilon < \epsilon_c$) Kramers found

$$\tau_{LD}^{-1} = (\eta E_b/k_BT)\exp(-E_b/k_BT) \quad (2)$$

(LD for low damping). Note that for Josephson junctions,

$$E_b = (hI_c/e)\{-\pi\alpha + 2\alpha \sin^{-1}\alpha + 2(1-\alpha^2)^{1/2}\}.$$

Recently, several authors have tried to generalize Kramers's theory to provide an expression valid in the full damping range[2,3] and without[4] the restriction $E_b \gg k_BT$.

In the present work, we have experimentally investigated a wide range of η within the underdamped region of Josephson junctions and have made the first observation of the transition between the TR and LD limits [Eqs. (1) and (2)]. This was possible because η is given by $1/RC$, where R and C are the effective resistance and capacitance of the junction, respectively, and we show that R exhibits, for an unshunted tunnel junction, an exponential T dependence.[5] Our measurements of the switching-current distributions also provide excellent agreement with theoretical expressions in these two damping limits.

These results also provide insight into the relevant R for Josephson tunnel junctions and its T dependence. In all expressions for τ, the assumed resistively shunted junction model implies that a single effective resistance exists which correctly describes the switching dynamics. However, fluctuations of the Brownian particle in its potential well correspond to fluctuations of the phase difference, and therefore voltage, across the junction. Since the measured junction resistance is highly voltage dependent (see inset of Fig. 2), an important theoretical[6] and experimental problem arises as to the correct R for the resistivity shunted junction model. Most experiments[7-10] concerning the supercurrent decay in Josephson junctions deal with only the Kramers intermediate-damping regime [Eq. (1)], and thus no information can be obtained on the correct R, since in the thermal regime,[11] τ_{TR} is independent of R. On the other hand, R appears in τ_{LD}, so that experiments in the extremely low-damping regime can help in this problem. This question becomes particularly relevant in the interpretation of macroscopic quantum tunneling (MQT), since R appears as an exponential factor.[12]

Measurements used a Nb-oxide-Pb Josephson tunnel junction, which exhibited low subgap leakage current and a good diffraction pattern in an external magnetic field. The critical current density was $\simeq 3$ A/cm^2 at $T = 4.2$ K and the area 100×50 μm^2. The normal resis-

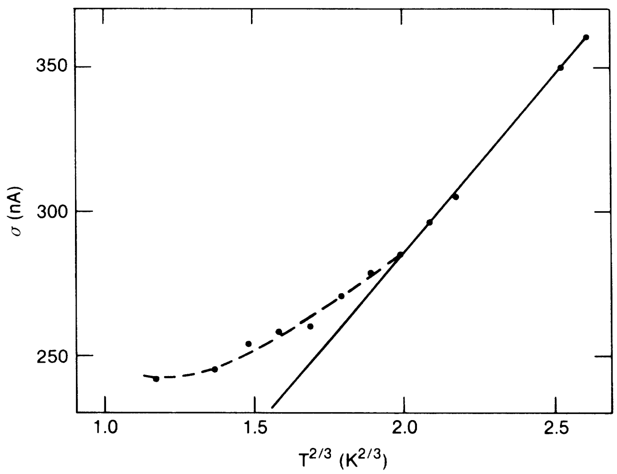

FIG. 1. Temperature dependence of the switching distribution width obtained from our experimental data (circles), the transition-state theory (solid line), and the LD regime (dashed line), which is based on Eq. (3) for $R(T)$.

tance, R_N, was 8.0 Ω and the capacitance was estimated to be 500 ± 100 pF from the Fiske step voltage, with use of the sum of the penetration depths of Pb and Nb obtained from the magnetic field diffraction pattern. The sample was mounted inside a massive copper block immersed in liquid helium. In order to reduce the external noise all the connections to room temperature went through filters located close to the junction in the helium bath, and the amplifiers and the sweep were run on batteries. A standard technique to obtain the experimental switching distributions has been used. The junction was biased through a 100-kΩ limiting resistor with use of a trapezoidal-shaped wave form at a frequency of about 50 Hz. The junction current also passed through a resistor whose voltage was amplified and used to synchronize the trigger for the multichannel analyzer with the beginning of the current ramp. The junction voltage was amplified and provided a pulse at the time of switching out of the $V=0$ state. This pulse was sent to the stop-time input of the analyzer so that a count is assigned to the channel corresponding to the switching current. About 20000 events were recorded in about 6 min. For a valid comparison with theory, a careful calibration was made. After each measurement, the current ramp was visualized on the analyzer with the same time scale as the distribution (i.e., the channel scale), so that the analyzer measures the bias current I corresponding to the channel N. This calibration was good[5] to better than 1% with a current resolution of 20 nA/channel.

Examples of these current distributions are reported in Ref. 5. If Eq. (1), i.e., the TR theory, is assumed true, then the experimental switching current distribution width, $\sigma \equiv \{\langle I^2\rangle - \langle I\rangle^2\}^{1/2}$, is expected to follow a $T^{2/3}$ dependence.[7,8] Our experimental $\sigma(T)$ are shown in Fig. 1 and agree with the TR dependence at high temperature as shown by the solid line. A deviation of the experimental points from such a behavior occurs at $T < 2.6$ K, and we will show that it is due to a transition into the LD regime. The agreement of τ^{-1} with theoretical expectations at high temperatures signifies the absence of a significant level of external noise. Because of the importance of this aspect, a series of fits, using the χ^2 test in the LD limit (i.e., at $T < 2.6$ K), were made leaving T as a free parameter, $T_{\rm fit}$. External electrical noise should simulate $T_{\rm fit} > T$. The free parameters are then $T_{\rm fit}$, the effective resistance R, and the zero-noise critical current I_c. The results of these fits are reported in Table

TABLE I. Results of the analysis based on χ^2 tests. For each test we report the values of the free parameters obtained by minimization of the χ^2 variables defined as $\chi^2 = \sum (P_i - P_{\rm th})^2/e_i^2$, where P_i and $P_{\rm th}$ are the experimental and the theoretical distributions and e_i is the statistical error on P_i. For χ^2 minimization we used a multivariables minimization program. The χ^2 values divided by the numbers of degrees of freedom N_L are shown in the last column.

	Fits using TR theory $T_{\rm fit}=T$		Fits using LD theory			
T (K)	I_c (μA)	χ^2/N_L	$T_{\rm fit}$ (K)	I_c (μA)	R (Ω)	χ^2/N_L
4.2	140.47	36.9/69				
4.0	144.81	47.9/64				
3.2	156.55	80.8/65				
3.0	157.64	86.1/68				
2.8	159.55	78.6/66				
2.6	161.23	78.8/72	2.570	161.24	8.63	66.9/70
2.4	162.62	89.1/68	2.398	162.58	12.58	80.0/66
2.2	161.70	173.0/62	2.199	161.55	20.00	67.7/60
2.0	162.63	491.0/64	2.000	162.56	41.38	74.1/62
1.8			1.795	162.91	123.7	81.4/67
1.6			1.622	161.7	265.7	77.9/63
1.3			1.300	162.09	2021.0	68.3/64

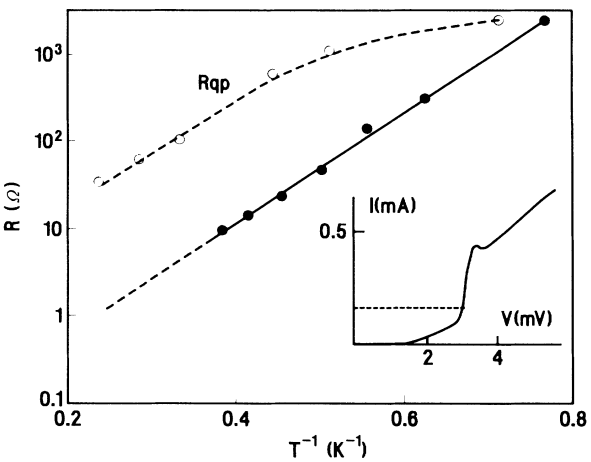

FIG. 2. Experimental values (solid circles) of the effective resistance obtained from the fits of the data in the LD limit; the full line is the least-squares fit of Eq. (3). The open circles and dashed line are the differential quasiparticle resistances R_{qp}, measured at zero bias in a small field. Inset: Typical $I(V)$ curve for $T=1.3$ K.

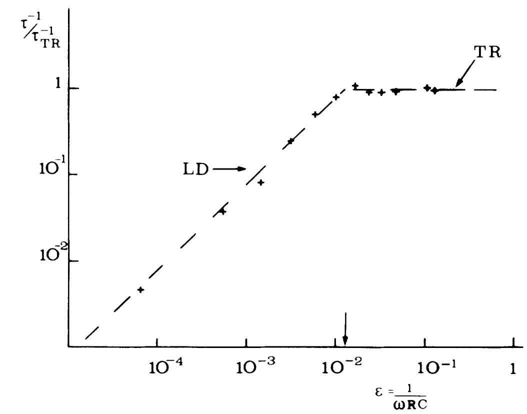

FIG. 3. Escape rate τ^{-1} in units of the TR result, τ_{TR}^{-1}, as a function of the dimensionless damping coefficient ϵ. For each point I is chosen such that $E_b/k_BT=12$. The crosses represent the experimental values and the dashed lines are the Kramers-theory dependences on ϵ for TR and LD regimes. The arrow indicates the crossover between regimes for Kramers's theory.

I. We first wish to point out that the χ^2 values show statistical compatibility between theoretical and experimental distributions. Note also the excellent agreement between the measured T and T_{fit}, providing further evidence that no external noise is present.

The resistances R, obtained from the fits in the LD regime, are plotted against T^{-1} in Fig. 2, and they show the dependence of the thermally excited quasiparticle density, N_{qp}, as previously observed[5]:

$$R(T)=R_0\exp(\Delta/k_BT), \tag{3}$$

where $\Delta=1.24$ meV and $R_0=3.15\times10^{-2}$ Ω are obtained from a least-squares fit (solid line). Also shown is the quasiparticle resistance at $V=0$, R_{qp}, which is measured by the application of a small magnetic field to suppress the dc Josephson current. Ideally, $R_{qp}\propto N_{qp}$, and this is found at high T before R_{qp} saturates, perhaps because of leakage currents. Although the magnitude of R_{qp} depends critically on the smearing of the reduced densities of states of Pb and Nb, the values reported in Fig. 2 are quite reasonable compared to calculations.[13] However, the large difference in magnitude between R_{qp} and R indicates that, while the intrinsic damping is certainly due to quasiparticles, the mechanism is unlikely to be tunneling. Note also that the effective R in other experimental situations may be limited by any parallel internal[14] or external resistive shunt or load line,[10] which was 100 kΩ in our case. For losses external to the junction, the ω-dependent complex impedance is relevant.[11,15]

It is the exponential T dependence of R that causes the change in damping regime shown in Fig. 1. This change is also seen in Fig. 3, in which the experimental switching rate τ^{-1}, normalized to the transition-state value τ_{TR}^{-1}, is plotted against the dimensionless damping parameter $\epsilon\equiv(\omega RC)^{-1}$, with R given by Eq. (3). The dependence of τ on I is overcome by our always choosing I such that E_b/k_BT is a constant (12 in this figure). In this case, Kramers's theory predicts $\tau_{LD}^{-1}/\tau_{TR}^{-1}\propto\epsilon$. The data of Fig. 3 show a clear crossover from the TR to the LD regime, and follow the expected behavior over a significant range of ϵ for each. Recent extensions of Kramers's theory (e.g., Ref. 2) predict a gradual crossover for τ^{-1}/τ_{TR}^{-1} between regimes. We cannot distinguish this, because *very* small *systematic* deviations of the actual I_c from the fit values used in Fig. 3 could emulate the results of Ref. 2. However, the occurrence of the transition is clear, and the results shown in Figs. 1 and 2 are affected much less by possible small systematic I_c variations. The crossover in Fig. 3 agrees with that of Fig. 1 and occurs for ϵ very close to its expected value of $\epsilon_c\equiv k_BT/2\pi E_b\sim0.0133$ for our choice of $E_b/k_BT=12$ (see arrow). Note that the crossover to MQT occurs at about 40 mK for our junction, so that even at our lowest T, the rate of switching due to MQT is calculated to be more than 10 orders of magnitude smaller than our measured rate.

These results for thermal activation, where the theoretical framework is better established, can be useful for interpreting experiments in the quantum case, since the correct R should be the same for both and is still an open question in MQT experiments. For example, the authors of Ref. 8 conclude that "the relevant damping resistance is the low-voltage quasiparticle resistance," even if they could not determine it directly. In Ref. 9, on the contrary, the authors conclude that only the T-independent normal-state resistance, R_N, fits the theory. This latter conclusion can be compatible with our interpretation only at low temperatures if an external shunt with effective resistance $\sim R_N$ were present in the experiment.

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