Superconductivity in Strongly Correlated Electronic Systems and Confinement versus Deconfinement Phenomenon

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The supersymmetrical gauge theory of strongly correlated electronic systems, based on a geometrical approach to the quantization of the Hubbard model, is presented. It is shown that topological magnetic excitations induce a long-distance interaction between the charged particles. This interaction, depending on the statistics of the magnetic excitations, leads to the confinement or statistical transmutation phenomenon and finally to the superconductivity. The long-wave theory of the short-range-order antiferromagnetic insulator state is proposed.

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The discovery of novel superconductors has given rise to searches for new mechanisms of superconductivity characterized by Coulomb scales. The pioneer idea goes back to Anderson,[1] who pointed to the possibility of superconductivity in strongly correlated electron systems near the metal-insulator transition. In this Letter it is shown that in canonical Hubbard-type models one encounters an unusual, for condensed-matter physics, mechanism of attraction between charged particles (holes), which at low concentrations induces superconductivity. Strong attraction emerges when the Néel state of an insulator is destroyed by the quantum spin-wave interaction. The short-range-order state (SRO), depending on the statistics of topological excitations, falls into (i) *quantum paramagnetic* (QP) and (ii) *quantum spin-liquid* (QSL) states. In the first case excitations of the insulator are bosons with a gap. The holes embedded in this state are in a *confining potential.* In the second case, excitations are *neutral spin-$\frac{1}{2}$ fermions.*[1-4] The holes form bound states with neutral fermions and thenceforth change their spin and statistics. According to Ref. 4 both states correspond to collinear antiferromagnets with integer of half-integer spins, respectively. In the ordered Néel state there is short-range attraction. Thus, *the attraction between holes is a measure of quantum magnetic fluctuations.* I shall show that models of strongly correlated electron systems are supersymmetric compact [periodic, (i), or antiperiodic, (ii)] Abelian gauge theories, and the superconductivity is an analog of chiral symmetry breaking, known in QCD. In this Letter I shall concentrate on the problems of *heavy holes* (which have no effect on magnetic ordering) in the permanent confinement (QP) regime.

The model and supersymmetry.— Let us consider the Hubbard model with strong intra-atomic Coulomb repulsion, making use of atomic representation: Let $\{|a\rangle\} = \{|0\rangle, |\uparrow\rangle, |\downarrow\rangle\}$ be three possible states on a lattice site, corresponding to the empty site and spin-up (and spin-down) states. In terms of projection operators $\chi^{ab} = |a\rangle\langle b|$ the Hamiltonian reads

$$H = \sum_{\sigma,\sigma'=\uparrow\downarrow} \sum_{ij} (t_{ij}\chi_i^{0\sigma}\chi_j^{\sigma 0} + J_{ij}\chi_i^{\sigma\sigma'}\chi_j^{\sigma\sigma'}). \tag{1}$$

The operators χ^{ab} form a basis of the semisimple *doubly graded (supersymmetrical) Lie algebra* Spl(1,2) $\approx$ Osp(1,2$|C$)[5] given by the (anti)commutative relations

$$\{\chi_i^{ab},\chi_j^{cd}\}_{\mp} = (\chi_i^{ad}\delta^{bc} \mp \chi_j^{bc}\delta^{ad})\delta^{ij}, \tag{2}$$

where (+) should be used only in the case when both operators are fermionic.

Considering other representations or different superalgebras one gets a generalization of the model. For example, let the atomic shell be a half-filled orbital singlet which prohibits all ionization processes, except $|s\rangle \rightleftarrows |s-\frac{1}{2}\rangle$, thus establishing correlation of ions. Such a model corresponds to the spin-s representation of the algebra (2). In the realistic situation the hopping (t) and exchange (J) amplitudes are governed by different electronic processes and should be considered as independent energy scales. Generally, effective Hamiltonians of strongly correlated electron systems should be considered as *supergeneralization of magnetic Hamiltonians.* A hole is a superpartner of a spin excitation.

Quantization.— Here I propose the Feynman integral representation for Eq. (1) using the *geometrical quantization approach* (also called *coherent-state method*[6]) for quantum mechanics associated with a semisimple Lie algebra.[7] (See also Batyev,[8] where part of the results of this section have been obtained.) Assume that the Hamiltonian $H(\{\chi\})$ is an element of the group. Let g be a unitary irreducible representation of the group. Take a highest vector of the representation as a vacuum $|0\rangle$ and consider its orbit $Q(g) = gPg^{-1}$, $Q^2 = Q$, tr$Q = 1$, where $P = |0\rangle\langle 0|$ is the projection operator onto the vacuum.

Then the quantum thermodynamics is represented by the Feynman integral over trajectories of the orbit,

$$\mathrm{Tr}F(\{\chi\})\exp(iH/T) = \int \mathrm{tr}\{FQ(\tau)\}\exp\left[i\int_0^\beta L\{Q(\tau)\}d\tau\right]DQ(\tau).$$

Here $F(\{\chi\})$ is a quantum operator, tr is a Killing form,[7] and

$$L=-\mathrm{tr}[H(\{\chi\})Q]+S\int_0^1 du(\mathrm{tr}\{Q\partial_u Q\partial_\tau Q\}), \qquad (3)$$

where $Q(\tau,u)$ satisfies the conditions $Q(\tau,0)=\text{const}$, $Q(\tau,1)=Q(\tau)$.

Let us suppose that the classical ground state of the model is a two-sublattice collinear Néel state. Take this state as a vacuum $|0\rangle$: spins up in sublattice A and spins down in sublattice B, no holes. For spin $\frac{1}{2}$, the element of the orbit in each lattice site is

$$Q=\tfrac{1}{2}(1-\rho)(1+\mathbf{m}\cdot\boldsymbol{\sigma})_{\sigma\sigma'}\chi^{\sigma\sigma'}+\psi_\sigma\chi^{\sigma 0}+\chi^{0\sigma}\psi_\sigma^\dagger+\rho\chi^{00},$$

where $\rho=\sum\psi_\sigma^\dagger\psi_\sigma$; $\mathbf{m}=\mathbf{n}$ in sublattice A and $\mathbf{m}=-\mathbf{n}$ in sublattice B; $\mathbf{n}$ is a unit vector, $\mathbf{n}^2=1$; ψ_σ are Grassmann variables, obeying the constraint

$$(1+\mathbf{m}\cdot\boldsymbol{\sigma})\psi=0.$$

After some algebra one gets the Langrangean, which can now be written for arbitrary spin S,

$$L=\sum_{\sigma i}\psi_{\sigma i}^\dagger\partial_\tau\psi_{\sigma i}+\sum_{\sigma\langle i,j\rangle}t_{ij}\psi_{\sigma i}^\dagger\psi_{\sigma i}+L_M, \qquad (4)$$

where the magnetic part has the form[9,10]

$$L_M=S\int_0^1 du\sum_i(\mathbf{m}_i\cdot\partial_u\mathbf{m}_i\times\partial_\tau\mathbf{m}_i) -\sum_{ij}J_{ij}\mathbf{m}_i\cdot\mathbf{m}_j(S-\tfrac{1}{2}\rho_i)(S-\tfrac{1}{2}\rho_j), \qquad (5)$$

with the conditions $\mathbf{m}_i(\tau,1)=\mathbf{m}_i(\tau)$, $\mathbf{m}_i(\tau,0)=\text{const}$. The supersymmetry transformations now read $\delta\mathbf{m}=\psi\boldsymbol{\sigma}\epsilon$; $\delta\psi=\rho\epsilon$; $(1-\mathbf{m}\cdot\boldsymbol{\sigma})\epsilon=0$.

Local gauge invariance.—The constraint on ψ implies that the hole is described by one component of $\psi_{i\sigma}$ which is antiparallel to $\mathbf{m}_i$. This field can be singled out at the expense of emergence of local gauge invariance. First represent $\psi=\frac{1}{2}(1-\mathbf{m}\cdot\boldsymbol{\sigma})\eta$ thus solving the constraint, then let $\boldsymbol{\sigma}\cdot\mathbf{n}=g\sigma^3g^{-1}$, where $g\in SU(2)$. Now rotate the spinor to the z axis $(U,V)=g^{-1}\eta$. Then one has $\psi_\sigma=Uz_\sigma$ in sublattice A and $\psi_\sigma=Vz_\sigma$ in sublattice B, where $z_\sigma=g_{\sigma\uparrow}^{-1}$, $z_\sigma^*=g_{\sigma\downarrow}^{-1}$, $z_\sigma^*z_\sigma=1$. Inserting these terms into the fermionic part of Eq. (5), one gets

$$L=\sum_a U_a^\dagger(i\partial_\tau+A_\tau)U_a+\sum_b V_b^\dagger(i\partial_\tau-A_\tau)V_b +\sum_{ab}t_{ab}(A_{a\bar b}^+U_a^\dagger V_b+A_{ba}^-V_b^\dagger U_a)+L_M, \qquad (6)$$

where the vector gauge potentials are

$$A_\tau=\mathrm{tr}(\sigma^3g^{-1}\partial_\tau g)=z^*\partial_\tau z; \quad A_{a\bar b}^\pm=\mathrm{tr}(\sigma^\pm g_a^{-1}g_b). \qquad (7)$$

In such a form the theory becomes locally gauge invariant. The gauge transformation $g\to g\exp(i\phi\sigma^3/2)$, $z\to z\exp(i\phi/2)$ does not affect the vector $\mathbf{n}$, yet changes potentials and fermionic fields:

$$A_\tau\to A_\tau+\tfrac{1}{2}i\partial_\tau\phi;$$

$$A_{a\bar b}^\pm\to A_{a\bar b}^\pm\exp[\pm\tfrac{1}{2}i(\phi_a+\phi_b)];$$

$$U\to U\exp(\tfrac{1}{2}i\phi);\quad V\to V\exp(-\tfrac{1}{2}i\phi).$$

Note that U and V particles have different charges with respect to the "geometrical" gauge field A and henceforth are attracted. The magnetic part of the action can also be treated as a lattice gauge theory: $S(\mathbf{n},d\mathbf{n},d\mathbf{n})=2SdA$; $\mathbf{n}_a\cdot\mathbf{n}_b=A_{a\bar b}^+A_{b\bar a}^-$.

Long-wave action of antiferromagnets.—In the long-wave limit the ordered Néel state is conventionally described by the action

$$L=\tfrac{1}{2}\int d^2x\int_0^\beta d\tau\{J^{-1}(\partial_\tau\mathbf{n})^2+JS^2(\partial_\mu\mathbf{n})^2\}, \qquad (8)$$

where $\mathbf{n}^2$ ($\mathbf{n}=1$) is a soft variable, which is a direction of the local spin in one sublattice averaged over many ions. S is a magnitude of the spin of an ion. Note that the existence of local $\mathbf{n}$ does not necessarily imply long-range order. Therefore, at least phenomenologically, at sufficiently small S when fluctuations become important a phase transition into the SRO state becomes possible (the critical value of spin might be, however, unphysical, $S<\frac{1}{2}$, but the quantum fluctuations can be effectively increased by frustration of the lattice[1]).

In addition to Eq. (8) another term was suggested in Ref. 4, related to the Hopf winding number of the $\mathbf{n}$ field:

$$H=-\int d^3xA_\mu F_{\nu\lambda}e^{\mu\nu\lambda}, \qquad (9)$$

where

$$F_{\nu\lambda}=\partial_\nu A_\lambda-\partial_\lambda A_\nu=\frac{1}{4\pi}e^{abc}(n_a\partial_\nu n_b\partial_\lambda n_c).$$

Spin and statistics of topological excitations.—After $2\pi sH$ is added to Eq. (8), the topological excitations in the 2D antiferromagnet, the skyrmions,[1] acquire the spin s (Wilczek and Zee[11] have elegantly demonstrated the relation between spin and statistics of skyrmions). The skyrmion in an antiferromagnet is a cylindrical domain, i.e., the domain-wall loop bounding the Néel state, shifted by a half period. The interior of the domain wall is, roughly speaking, the insertion of the ferromagnetic state. The spin s of the skyrmion is thus formed by the spin of ions and should be proportional to the magnitude S of the ion spin. It is therefore natural to associate s with the spin S of the original Heisenberg model. In this case the cylindrical domain in a half-integer-spin antiferromagnet is a spin-$\frac{1}{2}$ neutral fermion.[4] In the SRO phase the fermions become gapless (this is most probably what is called the resonating-valence-bond state[1-3]). This phase will be referred to as a *quantum spin liquid*

(QSL). The SRO phase, where the topological term is irrelevant and the excitations are spinless bosons, is referred to as the *quantum paramagnetic* (QP) state. (Note that it differs from ordinary paramagnets by the violation of translational symmetry.)

Antiferromagnet as a gauge theory.—In the CP^{-1} representation the O(3) σ model, Eq. (8), becomes a U(1) gauge theory. Putting $JS=1$ one gets

$$L=\tfrac{1}{2}S\int d^2x\,|(i\partial_\mu+A_\mu)z|^2, \tag{10}$$

where now A_μ is an independent gauge field. The integration over z leads to a gauge effective action. The gauge bosons are massive in the Néel state which is the analog of the *Higgs phase* with $\langle F_{\mu\nu}(k)F_{\mu\nu}(k)\rangle \sim R_c^{D-1}k^2$ at $k\to 0$, whereas for the SRO quantum-paramagnetic phase

$$\langle F_{\mu\nu}(k)F_{\mu\nu}(k)\rangle_{k\to 0}=\text{const}\,R_c^{-D+3}, \tag{11}$$

where $R_c\sim J^{-1}$ is the dimensional scale of the system.

Global properties of the gauge group. Monopoles. —As a result of the U(1) gauge group, Polyakov-'t Hooft[12] monopoles emerge: $\int F_{\mu\nu}d\sigma_{\mu\nu}=2\pi q$. A monopole, having a hedgehog configuration in $\mathbf{n}$ representation, is a particle in 3D space. In the (2+1) dimension this is a space-time point where the history of skyrmion terminates. In the Higgs phase monopoles are confined in pairs, which is illustrated by the "length" law for the Wilson loop,

$$W(C)=\left\langle \exp i\oint A_\mu dx_\mu\right\rangle \sim \exp(-L/R_c).$$

It implies that external holes embedded in the Néel state are particles with a mass $\sim R_c^{-1}$, which are attracted by a short-range potential. Properties of the SRO state is a more subtle matter.

By virtue of the combined translational and time inversion symmetry, RT, of the Néel state, one should distinguish between the two possibilities: (i) the periodic compact, or, let us say, (ii) antiperiodic compact gauge group. Here it is understood that under the local transformation $A_\tau\to A_\tau+i\pi\delta(x)$ the long-wave functional does not or does acquire the sign factor $\exp i \times S(A)\to \pm\exp iS(A)$. It is most likely that these two possibilities correspond to integer or half-integer spins of the Heisenberg model. This is supported by the following consideration. In the time-aperiodic singular gauge transformation localized in one point,

$$\phi_i(\beta)-\phi_i(0)=2\pi\delta_{i0},$$

which affects the gauge field as $A_\tau\to A_\tau+i\phi\delta_{i0}$, the exponent of the lattice action (5) changes by the factor $(-1)^{2S}$. If the above hypothesis together with the hypothesis on the topological term (9) are accepted, one is led to the conclusion that the (2+1)-dimensional long-wave gauge action for an antiperiodic gauge theory (i.e., for the QSL state) includes the topological mass term (i.e., Chern-Simon secondary characteristic class[13]). This term is closely associated with the anomalous phenomenon of fermions in the gauge field.[14]

Confinement versus deconfinement.—Now we are ready to consider the behavior of external charges in the SRO phase. For the periodic compact (2+1) gauge theory it has been proven[12] that monopoles are in a plasma state. This implies the "area" law for the Wilson loop, $W(C)\sim\exp(-\text{area}/R_c^2)$, where area is the surface stretched upon the contour C. The quantum paramagnet is a *confinement phase.* In the (3+1)-dimensional theory it is known that the confinement arises only when the constant in Eq. (11) is large enough. The order parameter of the confinement-deconfinement transition[12] is the so-called Polyakov line $P(x)=\exp[i\int_0^\beta d\tau A_\tau(x,\tau)]$ which is not invariant under the *central group* [i.e., aperiodic global gauge transformations $\phi(x,\tau)=\phi(\tau)$]: It acquires the factor $\exp\{i/2[\phi(\beta)-\phi(0)]\}$. In the deconfinement phase, where $\langle P\rangle$ is nonzero, the central symmetry is broken.

As for the antiperiodic gauge theory associated with the QSL state, one can state that the central symmetry is always broken, i.e., even in the strong-coupling limit one still lives with deconfinement.

Superconductivity in quantum paramagnets.—In the limit of heavy holes, $t\ll J$, the confinement of holes, i.e., disappearance of charged fermions from the spectrum of the QP regime, occurs. In the representation of the Feynman path integral in terms of a sum over contours with length L, the contours with well-separated U and V are suppressed by the area law, while the long loops with a small separation between U and V are not suppressed. It reflects the world line of the meson $b_\mu=U(x) \times\exp(i\int_x^y A_\mu)V(y)$, which is a vector charged massive boson. For low hole concentration and high temperature one can get the estimation

$$\langle b^\dagger(x)b(0)\rangle \underset{x\to\infty}{\longrightarrow} \sum_L (t^2R_c/T)^L\exp(-L/R_c),$$

which points to the instability with respect to meson condensation: $\langle b^\dagger(x)b(0)\rangle\to|\Delta^2|$. This phase transition breaks down the electromagnetic gauge symmetry, i.e., leads to superconductivity. The effect is similar to the chiral-symmetry breaking known in QCD.

Enlargement of concentration of holes destroys the confinement in the QP state, whereas the QSL is, to this end, more stable.

Statistical transmutation in the QSL state. Scenario. —More detailed analysis of the QSL state will be presented elsewhere; here I only outline the qualitative picture of how holes do feel in it. It is particularly interesting in the (2+1)-dimensional space. Let us consider an external particle in the field of the solitary skyrmion and rotate the system. Then by virtue of the factor $(-1)^{2S}$ coming from the Hopf term (9) in the action, the boundary conditions for the wave function of a particle do change: $\psi(\phi+2\pi)=\psi(\phi)\exp[i2\pi(s+S)]$, where s is the vacuum spin of a particle and ϕ is the azimuthal angle. Therefore, at half-integer spin S the particle localized on the neutral fermion changes its spin and henceforth its statistics. Closely related phenomena were recently discussed by Wilczek[15] and Wu,[16] who have noticed the connection of spin and statistics with the topological terms. Polyakov[10] has recently produced an elegant proof of the statistical transmutation in the $(2+1)$-dimensional gauge theory with the Chern-Simons term. Qualitatively, this phenomenon can be visualized as follows: The interior of the domain wall (i.e., skyrmion) is a ferromagnetic loop. Hence, energy consideration does favor localization of a particle on this loop, along which it propagates as a free particle. It is a bound state of a "spinon," i.e., a neutral fermion spin soliton, and a hole which obviously is a charged spinless boson. Note that this state possesses the orbital momentum $l=1$. Besides, the alternative even-parity bosons, which are bound states of two holes on the skyrmion with twice the topological charge, are possible. A condensation of the latter states is not hindered which agrees with considerations in Ref. 2.

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