Effect of Parallel Magnetic Field Gradients on Absorption and Mode Conversion in the Ion-Cyclotron Range of Frequencies

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The nonlocal Vlasov-Maxwell dielectric response kernel, $\mathbf{K}(\mathbf{r},\mathbf{r}')$, is constructed by integration along particle orbits in a nonuniform magnetic field. The phase integrals comprising the usual plasma dispersion function are altered, and contain a parameter characterizing the parallel field gradient. We numerically solve a 1D integral wave equation, including a parallel field gradient, describing the propagation, mode conversion, and absorption physics. Significant changes in absorption are found for the small-$k_{\parallel}$ regime.

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In this Letter we address an issue of importance to ion-cyclotron-resonance heating in fusion experiments arising from a self-consistent treatment of the particle motion in magnetic fields possessing a parallel gradient. Such parallel gradients are manifest in magnetic-mirror confinement, but they also occur in toroidal devices by way of rotational transform from the poloidal field.[1] The physical implications of a parallel gradient derive from the fact that the guiding-center gyrofrequency, $\Omega_{\rm gc}$, is not constant along a particle orbit, resulting in a temporally and spatially localized resonance interaction.[2] Moreover, the screening effect, whereby the left-hand polarized wave fields are nulled by the resonant particle currents, is strongly modified by the localized character of the wave-particle interaction.

Previous analysis of the wave absorption was based on expansions of the rf conductivity with strictly perpendicular gradients, and led to the occurrence of the plasma dispersion function, $Z((\omega-n\Omega)/|k_{\parallel}|v_{\rm th})$ from uniform-warm-plasma theory ($v_{\rm th}$=thermal velocity, ω =angular frequency of the rf, $\Omega=qB/m$), in the coefficients of the resulting wave equations.[3-5] Since the argument of Z contains $k_{\parallel}$, the wave number along the magnetic field, use of this function implicitly requires uniformity along $\mathbf{B}$. In the presence of even small gradients along $\mathbf{B}$, this function must be viewed as an operator rather than a scalar.

In this Letter we generalize the mode-conversion and absorption physics to include parallel as well as perpendicular gradients by describing the wave propagation as an integral equation of the form

$$\nabla\times\nabla\times\mathbf{E}(\mathbf{r})-K_0^2\int d^3k\,e^{i\mathbf{k}\cdot\mathbf{r}}\mathbf{K}(\mathbf{r},\mathbf{k})\cdot\mathbf{E}(\mathbf{k})=i\omega\mu_0\mathbf{J}_{\rm ant}(\mathbf{r}) \qquad (1)$$

($k_0^2=\omega^2/c^2$) for the electric field $\mathbf{E}(\mathbf{r})$, or its Fourier transform, $\mathbf{E}(\mathbf{k})=(2\pi)^{-3}\int d^3r\,e^{-i\mathbf{k}\cdot\mathbf{r}}\mathbf{E}(\mathbf{r})$, arising from the antenna source current $\mathbf{J}_{\rm ant}(\mathbf{r})$, where $\mathbf{K}(\mathbf{r},\mathbf{k})$ is the dielectric kernel from the solution of the linerized Vlasov equation. Our analysis is based on a "quasilocal" precept: While the rf conductivity is definitely nonlocal in nature, the extent of the particles' phase memory encompasses less than a bounce time. This precept will be shown to be valid by the inclusion of realistic phase decorrelation mechanisms, thus providing an important improvement on earlier work[6] that used the quasilocal methodology.

Dielectric kernel and phase integral. —The derivation of the dielectric kernel follows from the linearized Vlasov equation for the perturbed distribution $f^{(1)}$,

$$[-i\omega+\mathbf{v}\cdot\nabla+\Omega(\mathbf{v}\times\hat{\mathbf{e}}_{\parallel})\cdot\nabla_v]f^{(1)}(\mathbf{r},\mathbf{v})=-(q/m)\nabla_v f^{(0)}\cdot[1+(1/i\omega)\mathbf{v}\times\nabla\times 1]\cdot\mathbf{E}(\mathbf{r}), \qquad (2)$$

where $\mathbf{E}(\mathbf{r})$ is to be specified in a given modal representation. We substitute plane-wave modes, $\mathbf{E}(\mathbf{r})\rightarrow\mathbf{E}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}}$, to get $f^{(1)}(\mathbf{r},\mathbf{k},\mathbf{v})$, the perturbed distribution at $\mathbf{r}$ from a single $\mathbf{k}$ mode. Integration along characteristics, followed by summation over species and velocity, gives the plasma current in the usual manner. Insertion of this current in Maxwell's equations, and use of linearity to sum over modes, gives wave equation (1). The mixed kernel $\mathbf{K}(\mathbf{r},\mathbf{k})$ strongly resembles the uniform-plasma result in $\mathbf{k}$ dependence:

$$\mathbf{K}(\mathbf{r},\mathbf{k})=1+\sum_s(4\pi q^2/i\omega^2 m)\int_0^\infty d\tau\int d^3v\,e^{i\phi}\mathbf{v}[1(\omega-\mathbf{k}\cdot\mathbf{v}^*)+\mathbf{v}^*\mathbf{k}]\cdot\nabla_{v^*}f^{(0)}(\mathbf{v}^*,\mathbf{r}^*), \qquad (3)$$

but contains position dependence through $f^{(0)}$, and the unperturbed orbits $\mathbf{r}^*$ and $\mathbf{v}^*$. $\phi=\omega\tau+\mathbf{k}\cdot(\mathbf{r}^*-\mathbf{r})$ is the particle-rf phase, where τ is reverse time along the orbits. Following our quasilocal assumption, the velocities $v_\parallel^*$ and $v_\perp^*$ are expanded about their end-point values to first order in the local parallel field gradient[6]:

$$v_\parallel^*(\tau)=v_\parallel+v_\perp^2\tau/2L_\parallel,\quad v_\perp^*(\tau)=v_\perp-v_\perp v_\parallel\tau/2L_\parallel. \tag{4}$$

The instantaneous gyrofrequency is $\Omega_{gc}^*(\tau)=\Omega_{gc}(1-v_\parallel\tau/L_\parallel-v_\perp^2\tau^2/4L_\parallel^2)$. These forms conserve energy and magnetic moment to order τ, and are valid when $\tau^2\ll 4L_\parallel^2/v^2$.

We adopt the usual ordering of second-harmonic/minority-ion cyclotron-resonance heating to second order in $k_\perp\rho_L$ (Refs. 3–5) and perform a spatial transformation to local circularly polarized directions. With the assumption of a local Maxwellian, the kernel becomes

$$\begin{aligned}\mathbf{K}(\mathbf{r},\mathbf{k})=1+i\sum_s(2\omega_p^2/\omega v_{\rm th}^5\pi^{1/2})\int_0^\infty d\tau\int_{-\infty}^\infty dv_\parallel\int_0^\infty v_\perp dv_\perp\exp(-v^2/v_{\rm th}^2)\\ \times\{\hat{\mathbf{e}}_+\hat{\mathbf{e}}_+[v_\perp^2(1-v_\parallel\tau/2L_\parallel)e^{i\psi_1}+(k_\perp v_\perp/4\Omega)^2e^{i\psi_2}-(k_\perp v_\perp^2\tau/2\Omega L_\perp)e^{i\psi_2}e^{-i\beta}]\\ +\hat{\mathbf{e}}_-\hat{\mathbf{e}}_-[v_\perp^2(1-v_\parallel\tau/2L_\parallel)e^{i\psi_{-1}}+(k_\perp v_\perp/4\Omega)^2e^{i\psi_{-2}}+(k_\perp v_\perp^2\tau/2\Omega L_\perp)e^{i\psi_{-2}}e^{i\beta}]\\ +2\hat{\mathbf{e}}_\parallel\hat{\mathbf{e}}_\parallel(v_\parallel^2+v_\parallel v_\perp^2\tau/2L_\parallel)e^{i\psi_0}\}.\end{aligned} \tag{5}$$

Equation (5) contains the odd-order-in-$k_\perp$ terms arising from a perpendicular field gradient, where $k_\perp\cos\beta$ is the projection of $\mathbf{k}$ onto $\nabla_\perp B$. Also included is the changing gyroradius size. The particle-rf harmonic phases, ψ_n, are

$$\psi_n=(\omega-n\Omega-k_\parallel v_\parallel)\tau+n\Omega v_\parallel\tau^2/2L_\parallel \tag{6a}$$

$$-v_\perp^2(k_\parallel\tau^2/4L_\parallel-n\Omega\tau^3/12L_\parallel^2) \tag{6b}$$

$$-k_y(v_\parallel^2\tau/\Omega R_c+v_\perp^2\tau/2\Omega L_{\nabla B}). \tag{6c}$$

Part (6a) contains the usual uniform plasma phase, and the dominant nonuniformity term arising from the particle's streaming along the field gradient. Part (6b) contains lesser nonuniformity terms arising from the particle's changing parallel velocity, and its higher-order effect on the instantaneous gyrofrequency. Part (6c) contains curvature and ∇B drifts, assumed to be in the y direction. Pressure drift terms, which are the effects of ∇n and ∇T, are not included but should be of the same order as the ∇B and curvature drifts.

The velocity integrals in Eq. (5) are simple moments of the Gaussian, resulting in straightforward algebraic expressions. The remaining integral over τ is the parallel-gradient generalization of the plasma dispersion function. Contributions to this integral will be limited by collisional phase decorrelation, which determines the maximum coherence time of the wave-particle interaction. In one analysis of this effect,[7] it has been concluded that the phase diffusion can be effectively modeled by the replacement of the quantity $e^{i\psi}$ with $\exp(i\psi-\frac{1}{2}\langle\delta\psi^2\rangle)$, where $\langle\delta\psi^2\rangle$ is the variance of the phase due to random collisions. We find pitch-angle scattering[8] to be the dominant collision mechanism, affecting ψ through the $k_\parallel v_\parallel\tau$ term, and we may estimate $\langle\delta\psi^2\rangle$ as follows. Suppose that at time $\tau_j=j\Delta\tau$, the parallel velocity is $v_{\parallel j}$, and let ξ_j be a small random change in pitch angle between times τ_{j-1} and τ_j. Then $v_{\parallel j}=v_{\parallel j-1}$

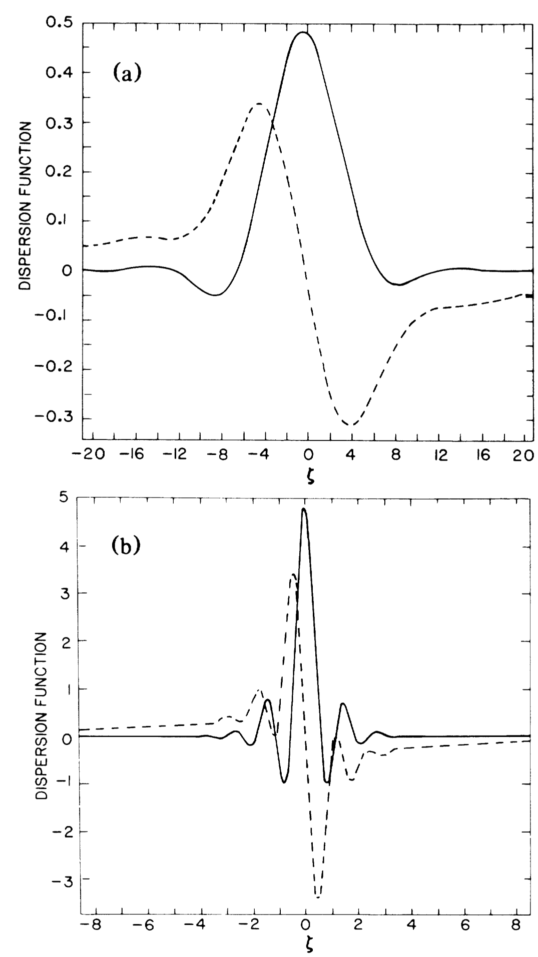

FIG. 1. Replacement Z function, $Z(\zeta,\alpha;\gamma)$ vs ζ, for (a) $\alpha=-10$, the small-$k_\parallel$ limit, and (b) $\alpha=0.5$, the small positive parallel-gradient limit.

$-v_\perp \xi_j$. The cumulation of the phase change over time gives

$$\delta\psi_j^2 = (k_\parallel v_\perp \Delta\tau)^2 \sum_{m=1}^{j} \sum_{n=1}^{j} mn\xi_{j-m}\xi_{j-n}. \tag{7}$$

The ensemble averages, with the assumption of no correlation between scattering events, are

$$\langle \xi_{j-m}\xi_{j-n}\rangle = \begin{cases} \nu\Delta\tau, & m=n, \\ 0, & m\neq n, \end{cases} \tag{8}$$

where ν^{-1} is the ion-ion deflection time. Taking the continuous limit $\Delta\tau \to 0$ while $j\Delta\tau$ remains constant gives

$$\langle \delta\psi^2 \rangle = \tfrac{1}{3} k_\parallel^2 v_\perp^2 \nu \tau^3. \tag{9}$$

Despite the typically small value of ν, the τ^3 dependence causes the collisional phase diffusion to exceed terms (6b) and (6c) for large τ. Hence, inclusion of collisional phase decorrelation permits us to treat these terms as insignificant.

In summary, Eq. (5) produces a form similar to the perpendicular-gradient analysis except for the replacement of the plasma dispersion function with the new quantity

$$Z(\zeta,\alpha;\gamma) \equiv i\int_0^\infty dx \exp[-x^2(1-\tfrac{1}{2}\alpha x)^2/4 + i\zeta x - \tfrac{1}{8}\gamma x^3], \tag{10}$$

where $\zeta = (\omega - n\Omega)/|k_\parallel| v_{\rm th}$ is the usual resonance parameter, $\alpha = n\Omega/k_\parallel |k_\parallel| L_\parallel v_{\rm th}$ is a new parameter characterizing the parallel gradient, and $\gamma = \nu/|k_\parallel| v_{\rm Th}$ is the phase diffusion parameter. We note agreement with the classical result at $\alpha=\gamma=0$. We also note agreement with Faulconer,[9] and taking the $|k_\parallel| \to 0$ limit we agree with Ref. 6.

The replacement Z function shows two new features not present in the uniform-plasma response. First, and most significant, is the presence of damping in the $k_\parallel \to 0$ limit, for which $Z \propto i|\alpha|^{-1/2}$. Figure 1(a) shows $Z(\zeta,\alpha;\gamma)$ vs ζ for $\alpha = -10$, which is within this limiting range. Effectively, the dispersion function broadens in scale length and decreases in size as $|\alpha|$ becomes large, consistent with the heuristic approach taken in other work.[10] The second new feature occurs as α approaches zero, the uniform-field limit, from positive values. There appears a marked oscillatory behavior near the resonance with average value equal to the uniform-field form. Figure 1(b) shows such behavior in $Z(\zeta,\alpha;\gamma)$ vs ζ for $\alpha = 0.5$. As α decreases, the oscillations first shorten in wavelength, and then weaken as α drops below some value determined by the collisional phase damping parameter γ. The physical origin of this behavior lies in the accumulation of phase change between particles nearly at resonance with the rf over long times in the weak gradient.[9]

1D wave scattering and absorption coefficients.—The phase integral is employed in wave equation (1) which we numerically solve as a convolution integral equation in k space. Our 1D geometry, and field and density profiles, taken from Ref. 1, have been chosen to represent the case of a tokamak with rotational transform in a simplified sense. x is the direction of nonuniformity, and **B** lies in the x-z plane, B_z being analogous to the toroidal

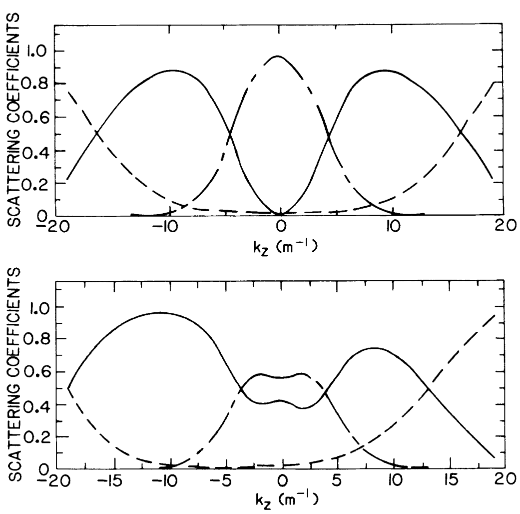

FIG. 2. Low-field incidence on a minority resonance without (top) and with (bottom) poloidal field. Transmission, dashed line; reflection, short and long dashed line; absorption, solid line.

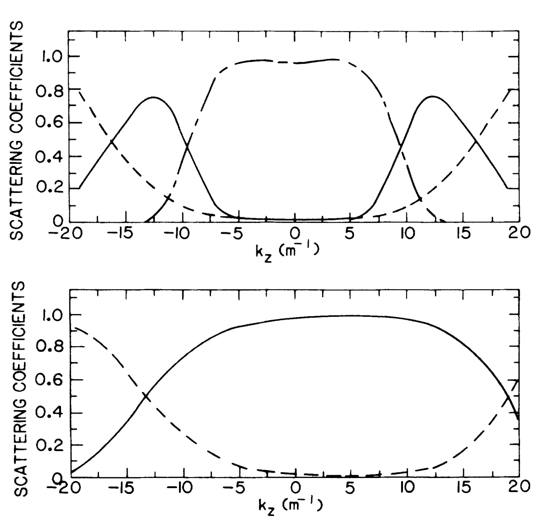

FIG. 3. High-field incidence on a minority resonance without (top) and with (bottom) poloidal field. Transmission, dashed line; mode conversion, short and long dashed line; absorption, solid line.

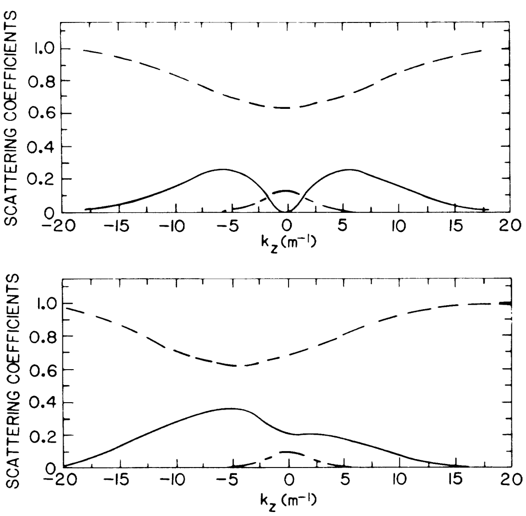

FIG. 4. Low-field incidence on a second-harmonic resonance without (top) and with (bottom) poloidal field. Transmission, dashed line; reflection, short and long dashed line; absorption, solid line.

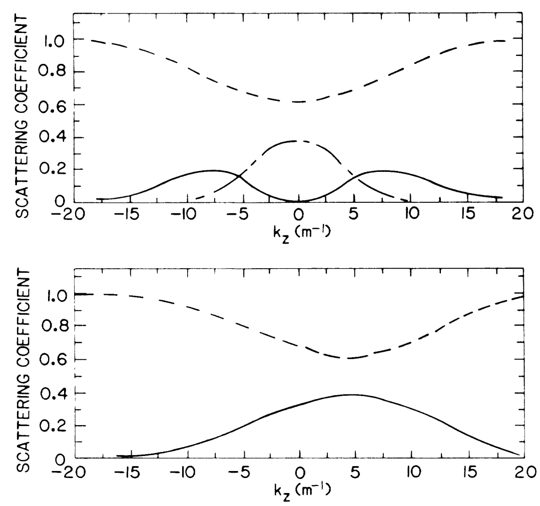

FIG. 5. High-field incidence on a second-harmonic resonance without (top) and with (bottom) poloidal field. Transmission, dashed line; mode conversion, short and long dashed line; absorption, solid line.

field. For rotational transform cases we have used $B_p/B=\frac{1}{10}$. Several series of runs were made for typical experimental conditions ($f=60$ MHz, $B_0=40$ kG, $n_0=4\times10^{13}$ cm^{-3}, $T=5$ keV, $k_y=0$ m^{-1}, $R_{\rm maj}=3$ m, $a_{\rm wall}=1$ m, 95% D–5% H minority heating, or 100% D second-harmonic heating), comparing the power transmitted, reflected, mode converted, and absorbed for plane-wave incidence on the resonance layer.

Power flow into and out of the resonance zone is calculated by our attaching uniform plasma regions to either end of the slab, wherein power on each of the various modes can be unambiguously determined. (No attempt to define or investigate "local" energies or power division among the various ion species is made in this Letter.) Figure 2 compares the scattering coefficients versus k_z of a minority-heating low-field launch case, with and without rotational transform, for the above parameters. Symmetry is clearly broken by the parallel gradient; transmission appears to be systematically shifted in k_z, which can be traced to the modal relation $k_\parallel=b_Tk_z+b_pk_x$ ($b_p=B_x/B$, $b_T=B_z/B$). Reflection and absorption are altered in a more fundamental way, with the anticipated absorption at $k_z\approx0$ being clearly evident. This result has the significant effect of enhancing the accessibility of the cyclotron layer for low-field incidence. Comparisons were also made for minority-heating high-field incidence, shown in Fig. 3. Here the rotational transform causes complete electron Landau damping of the ion Bernstein wave close to the cyclotron resonance. Results are also obtained for pure second-harmonic heating, shown in Figs. 4 and 5.

In summary, the implication from this analysis for experiments is that the ion-cyclotron resonance is significantly more absorptive at small toroidal wave numbers than predicted by gradient-free theory. The increased absorption arises from the nonlocal nature of the plasma current at a resonance, with subsequent limitation of the self-screening effect.

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