New Phase of Quantum Electrodynamics: A Nonperturbative Fixed Point in Four Dimensions

J. B. Kogut and Elbio Dagotto
Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801

and

A. Kocic
Department of Physics, University of Arizona, Tucson, Arizona 85721
(Received 23 November 1987)

Finite-size studies of noncompact quantum electrodynamics with four species of almost massless fermions suggest that there is an ultraviolet-stable, nonperturbative fixed point in four dimensions which describes an interacting field theory of bound states. This theory may be relevant to grand unification schemes, technicolor, and recent anomalous heavy-ion experiments.

PACS numbers: 12.20.Ds, 11.15.Ha, 11.30.Qc, 11.30.Rd

There are very few candidate field theories which are believed to have sensible infrared and ultraviolet behavior and which, in addition, describe interacting physical systems. Quantum chromodynamics is one such candidate. It is asymptotically free at short distances and presumably describes interacting hadrons at large distances. In this Letter we shall describe numerical evidence that quantum electrodynamics possesses a new, strong-coupling phase where chiral symmetry is broken. We present arguments using the size dependence of simulation data that the phase transition is associated with an ultraviolet-stable, nonperturbative fixed point which describes an interacting theory of meson bound states. If our speculative interpretation of these numerical studies is true, then a rich family of field theories in which there is dynamical symmetry breaking, anomalous scaling laws at short distances, and composite low-energy states exists.

Our results should be assimilated into a classification of ultraviolet-stable fixed points in four dimensions. Our bare Lagrangean is conventional quantum electrodynamics, but the renormalized Lagrangean at the fixed point may bear little resemblance to it, aside from sharing the same global symmetries. In fact, our numerical data suggest that there is no photon in the renormalized theory. Instead, we suggest below that multifermion operators, as in the Nambu-Jona-Lasinio model of chiral-symmetry breaking,[1] become renormalizable at the new fixed point and lead to an interacting theory. A multidimensional coupling-constant space which includes the bare charge and couplings of four-fermion interaction terms will be essential in the study of the renormalization-group flows of this theory.

Consider noncompact quantum electrodynamics formulated on a four-dimensional hypercubic lattice with four species of light fermions,

$$S = S_0 + \sum_{x,y} \bar{\psi}_x M_{x,y} \psi_y, \tag{1}$$

where M is the Dirac operator defined as

$$M_{x,y} = \tfrac{1}{2} \sum_\mu \eta_{x,\mu} (U_{x,\mu}\delta_{y,x+\mu} - U^*_{y,\mu}\delta_{y,x-\mu}) + m\delta_{x,y} \tag{2}$$

and

$$S_0 = -\tfrac{1}{2}\beta \sum_p \theta_p^2, \tag{3}$$

where, x, μ, and p denote sites, directions, and plaquettes of a four-dimensional hypercubic lattice and $U_{x,\mu} = \exp(i\theta_{x,\mu})$. $\theta_{x,\mu}$ are variables proportional to the gauge fields living on links, while $\bar{\psi}_x, \psi_x$ are Grassmann variables on sites (we use the staggered formulation of lattice fermions and $\eta_{x,\mu}$ is the appropriate phase). θ_p is the sum of the four $\theta_{x,\mu}$ taken around a plaquette and $\beta = 1/e^2$. The rest of the notation is standard. The action is locally gauge invariant and because the gauge variables are noncompact, $-\infty < \theta_{x,\mu} < +\infty$, and S_0 is a quadratic form, the pure gauge sector of the theory is free of topological excitations. The noncompact character of the theory does not complicate the lattice formulation or the numerical methods as long as we consider only gauge-invariant quantities.

The quenched theory, Eq. (3), was studied by simulation methods several years ago and chiral-symmetry breaking was discovered at strong coupling.[2] The quenched theory has also been studied analytically in continuum space-time by use of the Schwinger-Dyson equation to sum the planar-photon-exchange graphs.[3-5] It was found that the theory has a line of fixed points emanating from the origin $e^2 = 0$ and terminating at $e_c^2 = \pi/3$, where the theory breaks chiral symmetry dynamically and has an interesting continuum limit.[4] This critical point is nonperturbative and interacting because the pion is made out of quarks and antiquarks with the π-q-$\bar{q}$ coupling being nonzero as a consequence of Goldstone's theorem.[4] Of course, the quenched theory has some trivial features. For example, the numerical simulation showed that it lacks real temperature dependence[2] because the underlying dynamics is free and scale invariant. In addition, it lacks the fermion contribution to coupling-constant renormalization, so it does not address the crucial question of whether nonasymptotically free field theories with light fermions exist.

Using the formulation of Eqs. (1)-(3), we simulated the theory with four species of light fermions by use of the hybrid stochastic algorithm. The fermions were included in the system by the use of stochastic complex fields as discussed in the work of Gottlieb *et al.*[6] and applied in the work of Dagotto and Kogut[7] to compact quantum electrodynamics. The continuous evolution of the system in "molecular-dynamics time" was replaced by discrete time steps of size $dt = 0.02$ with the analysis of Ref. 6 indicating that systematic errors in expectation values are $O((dt)^2)$. The bare fermion mass was taken to be 0.0125 in lattice units which is small enough to study the chiral properties of the theory. A thorough discussion of our simulation methods and error analysis, including systematic effects and statistical errors accounting for correlation times, will be given elsewhere.[8] There we will also discuss the influence of the number of flavors (N_f) on our results. We have already observed that e_c^2 moves toward stronger couplings as N_f increases from 2 to 12.

We have done extensive measurements of $\langle\bar\psi\psi\rangle$, the Polyakov-line correlation functions, and Wilson loops from size 1×1 through 4×4 on 6^4, 8^4, 2×8^3, 4×8^3, and 6×12^3 lattices. Tables of results and fits must be deferred to a lengthier publication.[8]

In Fig. 1 we show the chiral condensate $\langle\bar\psi\psi\rangle$ computed on both 6^4 and 8^4 lattices. Each data point on the 6^4 lattice consisted of at least 100000 sweeps of the algorithm and the error bars on the data account for correlations between sweeps. The curve falls to zero rapidly as $\beta = 1/e^2$ increases from $\beta = 0.16$ to 0.20, but we found no sign of metastability so the transition to a chiral-symmetry-broken phase appears to be continuous. The 8^4 data are less extensive (10000-20000 sweeps per point) but also support this view. Recall that the compact theory has a first-order chiral transition and does not pass this test.[7] It is important to note that the 8^4 data points are shifted toward *weaker* coupling as compared with the 6^4 points. Such considerable finite-size effects are not expected in a theory which suffers from a zero-charge problem (the Landau ghost) and is free. It is interesting to make this point quantitative. According to traditional wisdom, weak-coupling quantum electrodynamics suffers from Landau's ghost, which means that its renormalized coupling vanishes no matter how large the bare coupling is chosen. This is interpreted in perturbation theory as complete screening—the strength of the fermion-photon coupling is screened to zero. If the theory has a cutoff, then a theory with a zero-charge problem is expected to have nonnegligible interactions

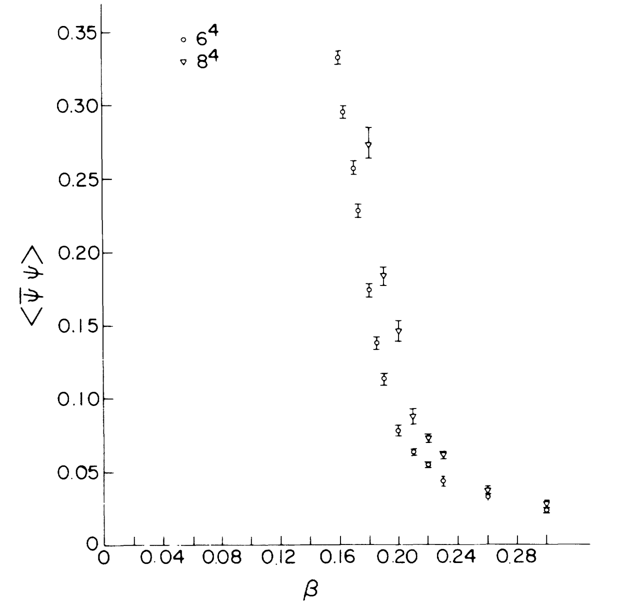

FIG. 1. Chiral condensate $\langle\bar\psi\psi\rangle$ calculated on 6^4 and 8^4 lattices in noncompact quantum electrodynamics with four species of fermions. The mass of the fermion is 0.0125 in lattice units.

only over distances comparable to the space-time cutoff. This smearing effect can make cutoff theories interacting although they become free in the continuum limit. Can the data of Fig. 1 be interpreted in this way? We answer this question by simulating a zero-charge theory whose short-distance features match those of the full theory. Consider a free massive vector meson,

$$S_m = -\tfrac{1}{2}\beta\sum_p \theta_p^2 - \tfrac{1}{2}m^2\sum_l \theta_{x,\mu}^2, \tag{4}$$

where the screening length m^{-1} is comparable to the cutoff a. The heavy-quark potential implied by Eq. (4) is $V(r) = \beta^{-1}e^{-mr}/r$, so its effective charge squared $\beta^{-1}e^{-mr}$ is exponentially small for any physical distance. To determine m^2 we match the average plaquette $\langle\theta_p^2\rangle$ of this model to the full model of Eqs. (1)-(3) and allow m^2 to have β dependence. We find that for β between 0.16 and 0.30, $m(\beta)$ is always close to 0.500, ranging from 0.445 at $\beta = 0.16$ to 0.530 at $\beta = 0.300$. Given $m(\beta)$ we then calculated $\langle\bar\psi\psi\rangle$ by simulating the massive vector meson theory on 6^4 and 8^4 lattice and Fig. 2 resulted. We found small finite-size effects with the *opposite* sign compared to Fig. 1. We anticipated this result—the range of the attractive interaction caused by Eq. (4) is only a few lattice spacings, so the symmetry breaking on the larger system (8^4) is *less* than that on the smaller (6^4) system. Presumably, $\langle\bar\psi\psi\rangle$ measurements on larger lattices will approach from above a limiting mean-field curve $(\beta-\beta_c)^{0.5}$ characteristic of a free field. In conclusion, our zero-charge model which is fitted to the short-distance data fails to explain the sym-

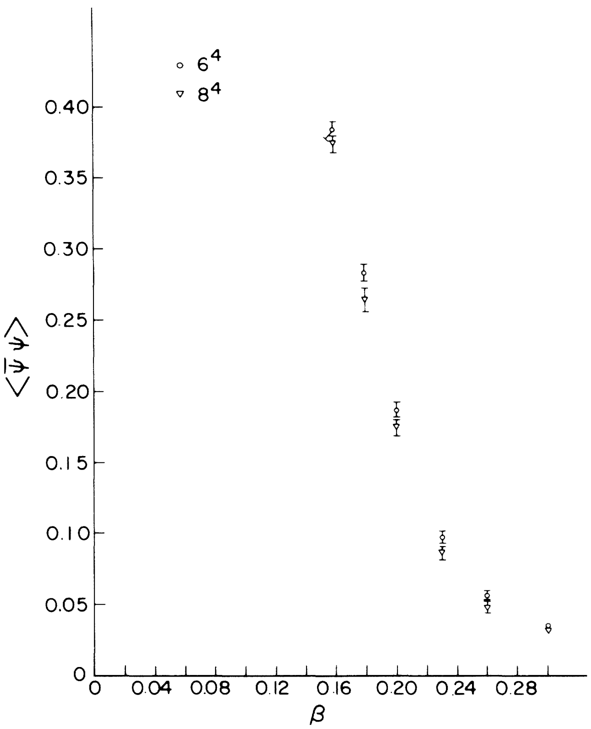

FIG. 2. Chiral condensate calculated on 6^4 and 8^4 lattices in the cutoff, zero-charge model.

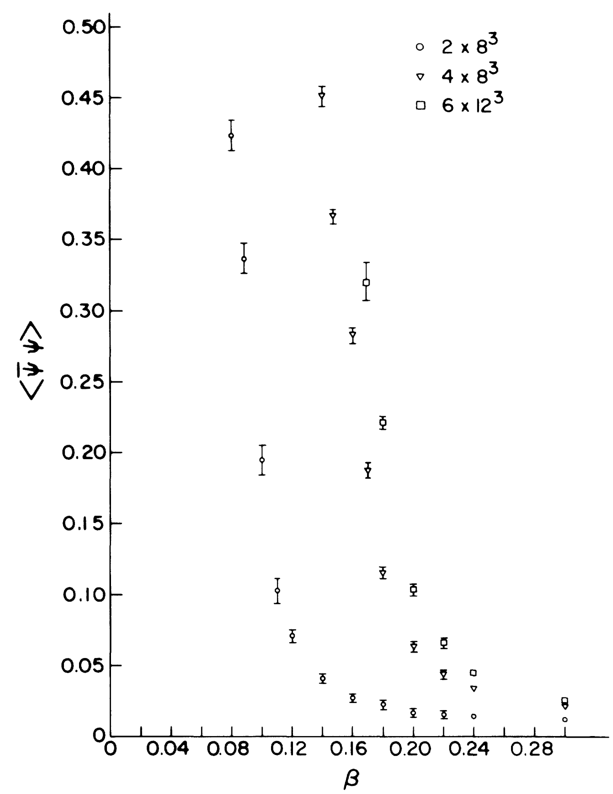

FIG. 3. Symmetry restoration at finite temperature. Lattice simulations of $\langle\bar{\psi}\psi\rangle$ on 2×8^3, 4×8^3, and 6×12^3 lattices in the full theory.

metry breaking in the full theory. The chiral, light-fermion properties of the full theory appear to be nontrivial.

Next we studied the finite-temperature features of the full theory Eqs. (1)-(3). Figure 3 shows the chiral condensate calculated on 2×8^3, 4×8^3, and 6×12^3 lattices. As the number of lattice spacings in the temporal direction (N_τ) increases, the temperature varies ($aT=1/N_\tau$) and the critical coupling shifts to smaller values (Fig. 3). It is interesting to contrast this result to the quenched quantum electrodynamics simulations which showed no shift as a function of coupling.[2] Figure 3 suggests that the full theory dynamically generates physical correlation lengths which diverge as e approaches e_c. The systematics resemble lattice simulations of quantum chromodynamics where the correlation lengths diverge at zero coupling rather than at $e_c \approx \sqrt{\beta_c}^{-1} \approx 2.0$ as observed here.

Finally, we studied the heavy-quark potential through Wilson-loop correlation functions and Polyakov lines on all the lattices simulated. We found a simple and puzzling result—a screened Coulomb potential as described by Eq. (4) with a screening length of approximately two lattice spacings fit the Wilson loops and Polyakov lines even at spatial separations of 4, 5, and 6 lattice spacings. There was no indication of a Coulomb potential associated with a massless photon or even of a short-range potential associated with a massive photon whose mass could be held finite as the cutoff is removed. Tabulated data and plots of heavy quark potentials will be presented in our lengthier article.[8] These results suggest that the theory does suffer from a zero-charge problem and that there is no vector field in the renormalized Lagrangean! How can such a theory be nontrivial? An interesting possibility is that there are *renormalizable multifermion terms* in the renormalized Lagrangean at e_c. We have in mind Nambu-Jona-Lasinio models in which the four-fermion interaction term is induced by the basic bare Lagrangean and the four-fermion term becomes renormalizable as e is increased to e_c where that term causes chiral-symmetry breaking. This mechanism occurs in the quenched planar approximation to the model[5]—at the end of that theory's fixed line of critical couplings the chiral invariant operator $(\bar{\psi}\psi)^2+(\bar{\psi}i\gamma_5\psi)^2$ has scale dimension *four*. This hypothesis can be tested in the full theory by the simulation of multipoint correlation functions in the scaling region of the lattice theory. This will not be easy. If this speculation is true, then the Nambu-Jona-Lasinio model[1] when viewed from a modern renormalization-group perspective will have unforeseen relevance.

The result that chiral-symmetric products of four-fermion fields become renormalizable at the critical point in the quenched, planar model can be understood intuitively. That theory's critical point has been visualized as a collapse mechanism,[4] similar to the relativistic hydrogen atom with a bare proton charge of $Z=137$. At this coupling the bound-state fermion-antifermion wave function collapses under the stress of attraction and the fermions collide with probability one. This effect is, therefore, equivalent to a four-point coupling which causes $O(1)$ effects, i.e., a *renormalizable* four-point interaction. Our computer data suggest that this mechanism persists when dynamical fermions are included in the dynamics. In that case, strong coupling creates the screening pairs which collapse and generate the pointlike, multifermion couplings at the critical point.

It would be useful to have a generalization of the Schwinger-Dyson equations to the full theory (the four-point–interaction terms must be included in the self-consistent coupled equations) where the screening and collapse mechanisms occur together. Are these phenomena consistent and can they generate a nontrivial fixed point? The computer simulations suggest a positive answer, but analytic insights are needed. We plan to probe the scaling region of the lattice theory next and measure f_π, etc. We will also study the fermion propagator since there are quenched planar calculations which state that the fermion is confined at e_c.[3] It would be particularly interesting if this theory illustrated "ultraviolet confinement" where strong forces at short distances screen the photon completely and collapse of the wave function eliminates low-momentum poles and cuts in the fermion propagator.[3]

If our results and their interpretations are correct, they will be useful in grand unification schemes. Ultraviolet-stable fixed-point theories are needed in Technicolor[9] and preon dynamics. One can also speculate that this new phase of quantum electrodynamics is relevant to the properties of ordinary, weakly coupled electrodynamics in strong external fields and it may play a role in the interpretation of the narrow peaks in electron and positron spectra observed in heavy-ion collisions.[10] We are investigating this possibility through additional simulations.

The work of two of us (E.D. and J.B.K.) is supported in part by the National Science Foundation, Grant No. NSF-PHY-82-01948, and another (A.K.) is supported by the Department of Energy, Contract No. DE/FG02/85ER40213. Our computer simulations were done at the National Center for Supercomputing Applications, University of Illinois. One of us (E.D.) thanks P. Hasenfratz, M. Lüscher, and A. B. Migdal for useful discussions.

[1]Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).

[2]J. Bartholomew, S. H. Shenker, J. Sloan, J. Kogut, M. Stone, H. W. Wyld, J. Shigemitsu, and D. K. Sinclair, Nucl. Phys. **B230 [FS10]**, 222 (1984).

[3]R. Fukuda and T. Kugo, Nucl. Phys. **B117**, 250 (1976).

[4]P. Fomin, V. Gusynin, V. Miransky, and Yu. Sitenko, Riv. Nuovo Cimento **6**, 1 (1983); V. Miransky, Nuovo Cimento **90A**, 149 (1985); A. J. G. Hey, D. Horn, and J. E. Mandula, Phys. Lett. **80B**, 90 (1978).

[5]C. N. Leung, S. T. Love, and W. A. Bardeen, Nucl. Phys. **B273**, 649 (1986).

[6]S. A. Gottlieb, W. Liu, D. Toussaint, R. L. Renken, and R. L. Sugar, Phys. Rev. D **35**, 2531 (1987).

[7]E. Dagotto and J. B. Kogut, Phys. Rev. Lett. **59**, 617 (1987); E. Dagotto and J. B. Kogut, University of Illinois Report No. ILL-(TH)-87-45, 1987 (to be published).

[8]E. Dagotto, A. Kocic, and J. B. Kogut, to be published.

[9]M. Bando, T. Morozumi, H. So, and K. Yamawaki, Phys. Rev. Lett. **59**, 389 (1987).

[10]T. Cowan *et al.*, Phys. Rev. Lett. **56**, 444 (1986); L. S. Celenza, V. K. Mishra, C. M. Shakin, and K. F. Liu, Phys. Rev. Lett. **57**, 55 1986; Y. J. Ng and Y. Kikuchi, Phys. Rev. D **36**, 2880 (1987); D. G. Caldi and A. Chodos, Phys. Rev. D **36**, 2876 (1987).