Instability of the Sheath-Plasma Resonance

R. L. Stenzel
Department of Physics, University of California, Los Angeles, Los Angeles, California 90024
(Received 29 September 1987)

An electrode with a positive dc bias inserted into a Maxwellian plasma is observed to generate coherent high-frequency oscillations at the sheath-plasma antiresonance frequency ($\omega \lesssim \omega_{pe}$). The phenomenon is explained by a negative resistance associated with the finite electron transit time through the sheath. Harmonics of the fundamental are radiated as electromagnetic waves but are clearly shown *not* to be caused by electron orbital instabilities.

PACS numbers: 52.25.Sw, 52.40.Fd, 52.40.Hf, 52.70.Gw

The sheath-plasma resonance is a well-known phenomenon in the field of rf probes and antennas in plasmas.[1,2] Basically, an electron-depleted sheath forms a vacuum capacitor while a field-free cold plasma behaves inductively below the electron plasma frequency ($K=1-\omega_p^2/\omega^2<0$) such that series and parallel resonances are possible. The exact resonance frequency depends on the electrode geometry and sheath thickness and is typically in the range $0.5<\omega/\omega_p<1$. Sheath-plasma resonances first discovered by Takayama, Ikegami, and Miyazaki[3] have been mainly studied for negatively biased electrodes in space[4,5] and in laboratory plasmas.[6,7] In this Letter I present the behavior of the sheath-plasma resonance for a positively biased electrode drawing electron saturation current through an ion-depleted sheath. It is observed that the series resonance vanishes while the parallel antiresonance becomes spontaneously unstable. This instability, termed the sheath-plasma instability, is though to arise from a negative differential rf resistance of the current-carrying electron-rich sheath. The finite electron transit time through the sheath[8] ($\tau \sim \omega_p^{-1}$) leads to a 180° phase shift between rf current and voltage which drives the resonant system unstable, as is known from the theory of diodes with electron inertia.[9] The sheath-plasma instability is not only interesting as a fundamental process, but is very important when one is interpreting antenna signals from charged space craft, and it may offer an alternative explanation to the proposed Orbitron maser model in earlier works.[10,11]

The experiment is performed in a large-volume (1×2 m^2), cold ($kT_e \lesssim 0.5$ eV), collisionless ($\nu/\omega_p < 10^{-3}$), Maxwellian afterglow plasma ($n_e < 5\times 10^{11}$ cm^{-3}), generated by a pulsed dc discharge in argon ($p<3\times 10^{-4}$ Torr) with a 1-m-diam cathode. An applied axial dc magnetic field of $B_0=5$ G is too small for us to consider the plasma magnetized ($\omega_c^2/\omega_p^2<10^{-4}$). Inserted into the uniform plasma region ($n/\nabla n \simeq 10$ m) are antennas of various shapes (dipoles, single wires, spheres, plates) and a 2-mm-diam diagnostic electron beam (150 eV, <1 mA) which, by weakly exciting the cold beam-plasma instability, gives an accurate ($\Delta f/f \simeq 1\%$) determination of the electron plasma frequency in the range $0.3 \lesssim f_p \lesssim 7$ GHz. The rf electrodes are connected via 50-Ω coaxial lines to a low-noise (noise factor $\simeq 1.5$ dB) tuned receiver ($\Delta f = 3$ MHz) sensitive enough to resolve the thermal fluctuations of the plasma. With a signal generator and and a directional coupler the sheath-plasma resonances are observed in the reflected signal from the antennas. The electrodes are biased ($-200 < V_{dc} < +200$ V) with respect to the plasma potential.

The basic identification of the sheath-plasma resonances is shown in Fig. 1 which displays the reflected signal from a spherical rf probe [radius $R=0.8$ mm $\simeq$ (15-

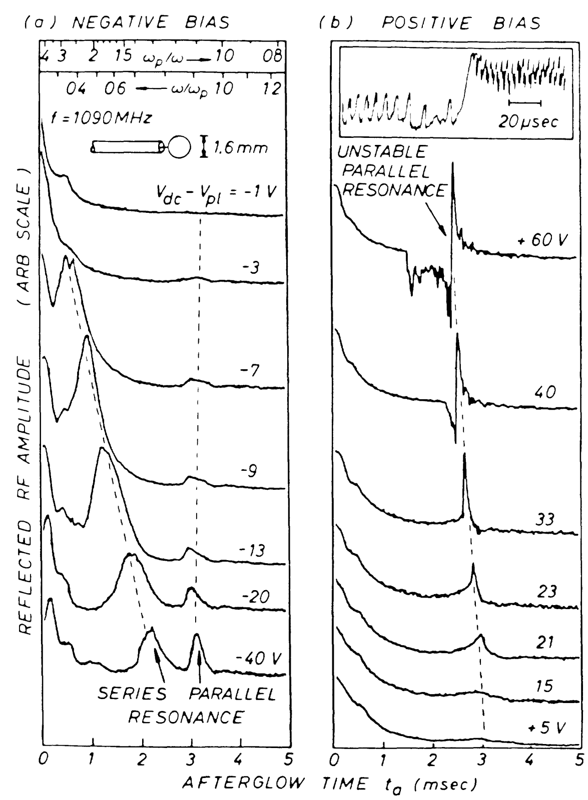

FIG. 1. Identification of the sheath-plasma resonances from the reflected signal on the 50-Ω transmission line. The parallel antiresonance becomes unstable at positive dc bias voltages.

75)λ_D] versus afterglow time at different dc bias voltages. The signal frequency is kept constant while the plasma frequency decreases with increasing time (see ω/ω_p scale). The sheath-plasma (series) resonance depends strongly on the sheath thickness s, i.e., bias voltage V_{dc} [theoretically,[1] $\omega^2/\omega_p^2 \simeq s/(R+s)$ for monopoles] while the (parallel) antiresonance depends little on dc bias ($\omega^2/\omega_p^2 \simeq 1$ for an ideal monopole). At the plasma potential ($V_{pl} \simeq +3$ V) the resonance vanishes ($\omega/\omega_p \rightarrow 0$) and the antiresonance is strongly damped, but for $V_{dc} > V_{pl}$ the antiresonance reemerges as a sharp strong line. Detailed observations (see inset) exhibit an interference structure between two signals with zero beat at the antiresonance. When the rf probe is connected to the sensitive microwave receiver, one observes for negative bias voltages a strong enhancement in the thermal-fluctuation spectrum at the antenna resonances, such as shown for the antiresonance in Fig. 2(b). At the plasma potential these resonant enhancements again vanish. However, for positive bias voltages one observes above a certain threshold ($V_{dc} \gtrsim 30$ V) a strong line emission which rises far above the thermal-fluctuation level and clearly represents an instability. It occurs exactly at the antiresonance frequency identified in the reflection measurements (explaining the beat in the inset of Fig. 1). Just above threshold the emission is highly monochromatic ($\Delta f/f \simeq 0.1\%$), but with increasing dc bias the line broadens and harmonics are generated. Typical curves showing threshold and saturation are shown in Fig. 2(c).

Figure 3 shows the dependence of the emission frequency and its harmonics on afterglow time (i.e., ω_p). At a given receiver frequency f one can observe several lines in the afterglow [Fig. 3(a)]. When one varies f and tracks the lines with afterglow time [Fig. 3(b)] it becomes evident that subsequent lines in time are harmonics of the fundamental sheath-plasma instability. While the fundamental instability is in the evanescent regime for high-frequency waves ($\omega < \omega_p$) the harmonics can excite propagating electrostatic and electromagnetic waves. The latter are readily observed with a horn antenna placed from the outside near a glass port in the vacuum chamber [see Fig. 3(a)]. Electromagnetic waves are thought to be excited by rf currents in the antenna

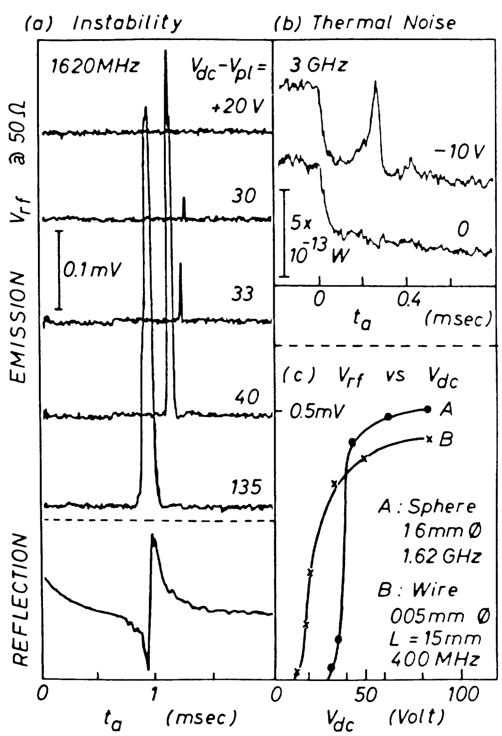

FIG. 2. (a) Emission signal due to the sheath-plasma instability at different dc voltages. The emission occurs exactly at the parallel resonance identified from reflection measurements. (b) For negative dc bias the series resonance causes an enhancement in the detection of thermal noise, but no instabilities. (c) Typical curves of V_{rf} vs V_{dc} showing threshold, growth, and saturation of the fundamental instability. Through line broadening and harmonic generation the total emission continues to grow with V_{dc}.

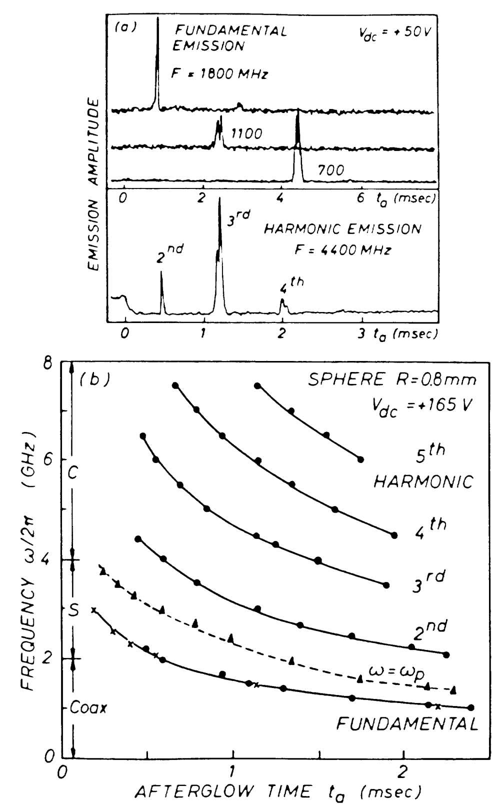

FIG. 3. (a) Variation of emission frequency with density [$\omega_p(t_a)$] and observation of multiple lines due to harmonic generation. (b) Emission frequencies vs afterglow time. Dashed line: electron plasma frequency $\omega_p(t_a)$. Crosses: resonances in reflection. S- and C-band waveguides were used as high-pass filters to prevent harmonic generation in the receiver.

structure rather than by the oscillating electrons in the plasma. It is worth noting that there are no resonances in the reflected signal at the harmonics implying that there are no intrinsic instabilities above $\omega = \omega_p$.

With use of a second movable floating rf probe, the field amplitude has been mapped versus distance from the exciter probe [Fig. 4(a)]. It shows the highly localized field pattern of the nonradiating fundamental mode and a broader pattern of the propagating second harmonic. Electrostatic harmonics are, however, strongly Landau damped. In order to check the influence of the electrode geometry on the instability properties, various electrode structures have been investigated. Single spheres (radius $R = 0.1$–0.8 mm) connected to the end of a semirigid coaxial cable (diameter 0.75–2.1 mm) exhibit with decreasing size a character between monopole and dipole, i.e., the antiresonance is at $\omega/\omega_p < 1$ and depends on sheath thickness [theoretically,[1] $\omega^2/\omega_p^2 = (2g^3+1)/3g^3$ for dipoles, $\omega^2/\omega_p^2 = 1$ for monopoles, where $g = 1 + s/R$]. A symmetric, balun-balanced dipole (length $2L = 3$ cm, diameter 0.05–0.012 mm) exhibits in reflection and noise detection a resonance at $\omega/\omega_p = 0.79$–0.92 for $V_{dc} = -(100$–$1)$ V, and for $V_{dc} > +15$ V it generates emissions at $\omega/\omega_p \simeq 0.7$ and at harmonics. Either side of the dipole is biased separately and it is possible to see two different resonances or emissions. When the dipole is insulated with a 0.1-mm-diam glass tube the dc bias has no effect on the resonance ($\omega/\omega_p \simeq 0.9$) and there are no instabilities. The variation of the length of the wire ($L = 1$–30 mm) does not substantially change the instability frequency or threshold. Thin wires have the lowest threshold voltage ($V_{dc} \simeq +10$ V). Finally, plane probes (1.5-mm square plate, 2-cm diam disc) have been biased positively and are also observed to generate microwave line emissions similarly to wires and spheres. Thus, although the details of the instability depend on geometry it exists universally in the presence of electron-rich sheaths.

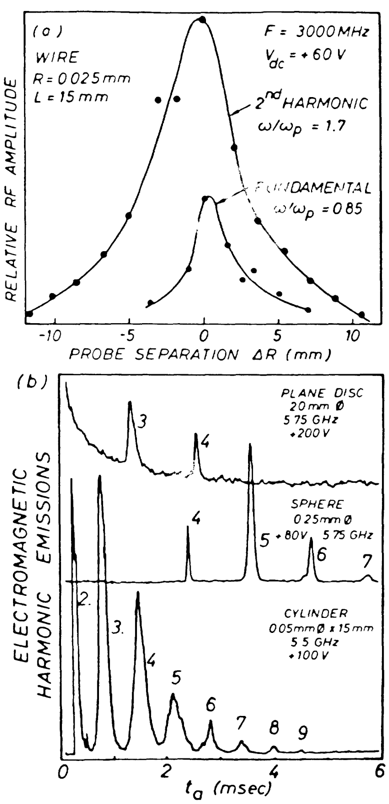

FIG. 4. (a) Radial rf amplitude profile showing a broader half-width for the propagating harmonics than for the evanescent fundamental emission. (b) Comparison of emission spectra for different electrode geometries showing the existence of the sheath-plasma instability for all shapes and ruling out the orbital instability mechanism.

The mechanism of the instability is explained by electron inertia effects in sheaths.[8] It is well known that a diode exhibits a negative differential resistance at high frequencies when the electron transit time τ is comparable to the rf period ($2\pi < \omega\tau < 3\pi$).[9] A very similar situation arises in an ion-depleted, electron-current-carrying sheath at frequencies near the electron plasma frequency ($\omega_p \lambda_D / v_{th} = 1$). Assuming the potential profile of a plane sheath to be given by Child-Langmuir's law, $\phi_{(x)} \simeq V_{dc}(x/s)^{4/3}$, with sheath thickness $s \simeq (eV_{dc}/k \times T_e)^{3/4}\lambda_D$, one finds for the electron transit time

$$\tau = \int_0^s dx/[2e\phi(x)/m]^{1/2} = 3(eV_{dc}/kT_e)^{1/4}\omega_p^{-1};$$

hence, instability ($\omega_p\tau > 2\pi$) requires voltages exceeding $V_{dc} \simeq 20kT_e/e \simeq 10$ V consistent with observations. For cylindrical sheaths the transit time is longer[12]; hence, the threshold voltage is lower the thinner the wire, as is observed. When the negative conductance cancels all losses of the sheath-plasma resonator the system goes spontaneously into oscillations, just like a vacuum-diode cavity oscillator.[9,13] Electron transit-time effects not only change the real part of the ac sheath impedance, but also increase the sheath capacitance with $\omega\tau$ or V_{dc}. This shifts the resonance to higher values of ω_p/ω, i.e., to earlier afterglow times (ω = const), as seen in Figs. 1 and 2. Second-order transit-time effects include harmonic generation (Fig. 3) and dc current changes by rectification (observed, to be shown in a future detailed paper). Radiation losses contribute to the saturation of the instability.

The significance of the sheath-plasma instability is obvious for the field of antennas in plasmas. For example, space-craft structures or antennas can emit high-frequency radiation when charged to a large positive potential, such as during beam injection[14] or the proposed tether experiments.[15] The sheath-plasma oscillation could be mistakenly interpreted as an instability occurring in the plasma volume. Finally, the sheath-plasma instability may offer an alternative explanation for the Orbitron devices[10,11] where microwave emissions are observed from positively biased wires in plasmas and explained by a negative-mass instability of long-lived electrons orbiting the wire. From the present experimental evidence (e.g., emission from plane electrodes and presence of a dc magnetic field[16]) it is clear that our microwave generation is neither due to orbital instabilities[10] nor due to counterstreaming electron beams,[11] but is due to the sheath-plasma instability.

The author acknowledges many useful discussions with Dr. J. M. Urrutia. This work was supported by National Science Foundation Grants No. PHY87-13829 and No. ATM87-02793 and U.S. Office of Naval Research Contract No. N00013-86-K-0611.

[1]G. Bekefi, *Radiation Processes in Plasmas* (Wiley, New York, 1966), Chap. 5.

[2]K. G. Balmain, Radio Sci. **7**, 771 (1972).

[3]K. Takayama, H. Ikegami, and S. Miyazaki, Phys. Rev. Lett. **5**, 238 (1960).

[4]N. Vernet, R. Manning, and J. L. Steinberg, Radio Sci. **10**, 517 (1975).

[5]S. Hoang, Planet. Space Sci. **20**, 1185 (1972).

[6]R. S. Harp and F. W. Crawford, J. Appl. Phys. **35**, 3436 (1964).

[7]T. Dote and T. Ichimiya, J. Appl. Phys. **36**, 1866 (1965).

[8]C. K. Birdsall and W. B. Bridges, *Electron Dynamics of Diode Regions* (Academic, New York, 1966), Chap. 2; R. Rosa, J. Phys. A **4**, 934 (1971); T. M. Antonsen *et al.*, Phys. Fluids **27**, 1257 (1984).

[9]F. B. Llewellyn, *Electron Inertia Effects* (Cambridge Univ. Press, London, 1941), Chap. 5.

[10]I. Alexeff and F. Dyer, Phys. Rev. Lett. **45**, 351 (1980); D. Chernin and Y. Lau, Phys. Fluids **27**, 2319 (1984); I. Alexeff, Phys. Fluids **28**, 1990 (1985).

[11]R. W. Schumacher and R. J. Harvey, Bull. Am. Phys. Soc. **29**, 1179 (1984).

[12]K. R. Spangenberg, *Vacuum Tubes* (McGraw-Hill, New York, 1948), Chap. 8.

[13]J. Müller, Hochfrequenztech. Elektroakust. **43**, 195–199 (1934).

[14]J. Lavergnat, in *Artificial Particle Beams in Space Plasma Studies,* edited by B. Grandal (Plenum, New York, 1982), pp. 87–100; H. B. Garrett, Rev. Geophys. Space Phys. **19**, 577 (1981).

[15]P. Banks, P. Williamson, and K. Oyama, Planet. Space Sci. **29**, 139 (1981).

[16]It has been pointed out by Dyer *et al.*, *International Conference on Plasma Science* (IEEE, New York, 1984), p. 109, that the Orbitron does not oscillate for axial magnetic fields $B \gtrsim 2$ G. However, the sheath-plasma instability is observed for both $B = 0$ and B up to 30 G.