Taylor-Couette Flow with Periodically Corotated and Counterrotated Cylinders

Thomas J. Walsh and Russell J. Donnelly
Department of Physics, University of Oregon, Eugene, Oregon 97403
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The stability of Taylor-Couette flow is studied as a function of the angular velocities of the inner and outer cylinders, Ω_1 and $\Omega_2=\epsilon\Omega_1\cos(\omega t)$, respectively. Contrary to the theoretical prediction, outer-cylinder modulation stabilizes the flow. The critical Reynolds number, Re_{1c}, is a function of the dimensionless frequency, $\gamma=(\omega d^2/2\nu)^{1/2}$, where d is the gap size and ν is the kinematic viscosity, and a monotonically increasing function of ϵ over the range examined: $0.3 \leqq \epsilon \leqq 2.0$. At larger ϵ stabilization is spectacular, Re_{1c} reaching a value nearly double the value for stationary outer cylinder.

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The hydrodynamic stability of fluids between corotating and counterrotating cylinders has been receiving increasing attention during the past few years.[1] In addition, interest in the problem of external modulation in a variety of geometries and with various driving forces continues to increase. Examples include modulated Taylor-Couette flow,[2-6] modulated Bénard convection,[5,7,8] and transversely modulated cylinders[9,10] as well as general unsteady flows.[11,12]

We report the first study of corotation and counterrotation with modulation in a Taylor-Couette apparatus. We have used an apparatus with the inner cylinder rotating at a constant angular velocity, Ω_1, and the outer cylinder oscillating about zero mean rotation: $\Omega_2=\epsilon\Omega_1 \times\cos(\omega t)$. For consistency with the theory[3,4] of Carmi and Tustaniwskyj we define an inner-cylinder Reynolds number as $\mathrm{Re}_1=(\Omega_1 R_1 d/\nu)(d/R_1)^{1/2}$ where R_1 is the inner cylinder radius, $d=R_2-R_1$ where R_2 is the outer cylinder radius, and ν is the kinematic viscosity.[13] Our results differ qualitatively from the theoretical predictions of Carmi and Tustaniwskyj in that, contrary to their findings,[3,4] modulation of the outer cylinder was found experimentally to delay the onset of secondary flow beyond the inner-cylinder critical Reynolds number for a stationary outer cylinder, $\mathrm{Re}_{1c}^{(0)}$. The critical inner-cylinder Reynolds number for this modulation experiment, Re_{1c}, was found to be a function of both ϵ and the dimensionless frequency of modulation, $\gamma=(\omega d^2/2\nu)^{1/2}$.

A detailed description of the apparatus, fluid sample, detection method, checks on the functioning of the apparatus, experimental protocol, and contributions to error has been published.[6] We describe here a few changes, which were made for this experiment. In the current experiment the inner cylinder was driven by a Compumotor model M57-51E motor and a Compumotor 2100 series indexer. A Hewlett-Packard model 3325A synthesizer/function generator provided a sinusoidal voltage to a voltage-controlled oscillator, which drove a Superior Electric stepping motor that was connected to the outer cylinder. The entire apparatus was kept in a temperature-controlled room. There was a temperature drift in the room of no more than 0.1 °C/h, and runs took from 2 to 5 h. A number of measurements were made over a range of Re_1 from below to above Re_{1c}. Re_1 was slowly increased between measurements; and, after a predetermined value was reached, the measurements were repeated with Re_1 being slowly decreased between measurements. The temperature was checked before every measurement, and the value which was used to determined γ (through its effect on the viscosity) was the value at Re_{1c}. If there was a drift in temperature between Re_{1c} during the up ramp and Re_{1c} during the down ramp, the average was used for our results. This temperature drift resulted in a drift in γ, which was typically about 0.1% and was never more than 1% of γ.

There was a distortion in the modulation waveform, because the response of the voltage-controlled oscillator was not directly proportional to the output of the synthesizer/function generator. This response, ideally a constant in terms of radians per second per volt as a function of voltage input to the voltage-controlled oscillator, was linear over much of the voltage output range of the function generator, having a slope of about 3%. As the output of the function generator approached 0 V, however, this response dropped to zero. The fraction of a modulation period spent in this dropoff region was never more than $\frac{1}{5}$ at Re_{1c}. The amplitude of the outer-cylinder angular velocity, $\epsilon\Omega_1$, was accurate to better than 0.04%; and the angular velocity of the inner cylinder, which was used to determine Re_1, was checked with an optical encoder, the results agreeing with the value set by the computer to within 0.03%. Finally, there was a small drift of the outer cylinder, which was reduced to at most 0.25% of the angular velocity of the inner cylinder before every measurement.

The power limit of the stepping motor, together with the moment of inertia of the outer cylinder, placed limits on ϵ and ω, since the amplitude of outer-cylinder angular acceleration was $\Omega_1\epsilon\omega$. It was, for example, not possible to go to a high enough Re_1 to achieve secondary flow with $\omega=2.20$ rad/sec and $\epsilon=0.5$ for the radius ratio

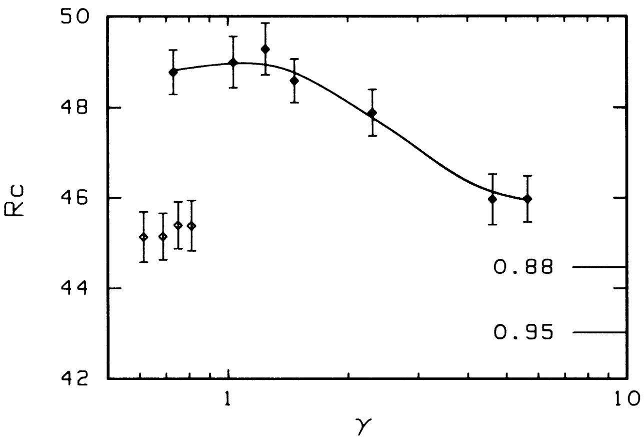

FIG. 1. Critical Reynolds number, Re_{1c}, as a function of the dimensionless modulation frequency, γ, for $\epsilon=0.5$, $\eta=0.88$ (filled lozenges) and $\eta=0.95$ (open lozenges). As a visual aid a curve connects the data for $\eta=0.88$. Horizontal lines on the right-hand side of the graph indicate critical Reynolds numbers for no modulation, $\mathrm{Re}_{1c}^{(0)}$, for each value of η (see text).

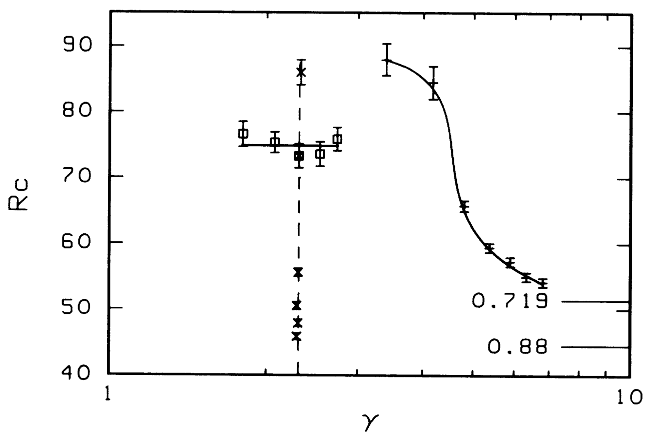

FIG. 2. Re_{1c} as a function of γ for $\eta=0.719$ (plusses) and 0.88 (squares) with $\epsilon=1.5$, and for $\gamma\simeq 2.3$, $\eta=0.88$ with several different ϵ (crosses). As a visual aid solid curves connect the data for $\eta=0.719$ and the data for $\eta=0.88$ with $\epsilon=1.5$. A dashed vertical line connects the data for $\gamma\simeq 2.3$. Horizontal lines on the sides of the graph indicate $\mathrm{Re}_{1c}^{(0)}$ (see text).

$\eta=R_1/R_2=0.95$; hence γ was kept to no more than 0.81 in this case.

Conversely, at low γ the transition region of transient secondary flow broadened, and it became increasingly difficult to determine exactly where secondary flow first appeared. This is because at high γ, modulation has little effect on the bulk of the fluid, the disturbance being confined to a region close to the outer cylinder. As a result Taylor vortices remain after first appearing at some Re_1 just as they do in the absence of modulation. With low γ, however, vortices are transient at and just above Re_{1c}, appearing and disappearing during each modulation cycle while $\Omega_2\simeq 0$. As Re_1 is increased, vortices are present for a greater part of the modulation cycle. In this case the response of our detection system is spread over a range of Re_1. We broaden our error bars accordingly, but because of this phenomenon we report, for example, no data for $\eta=0.719$ with $\gamma<3.39$. The greatest versatility was found with $\eta=0.88$.

Runs were made with three different radius ratios, $\eta=0.719$, 0.88, and 0.95, with $R_2=2.54$ cm. For each radius ratio a run was made with the outer cylinder stationary. For $\eta=0.95$ and $\Gamma=170.87$ ($\Gamma=L/d$ where L is the height of the annulus) runs were made with $\epsilon=0.5$ for a range of γ. For $\eta=0.88$ and $\Gamma=67.91$ runs were made with $\epsilon=0.5$ and 1.5 each with a range of γ, and with $\gamma\simeq 2.30$ for a range of ϵ. For $\eta=0.719$ and $\Gamma=31.93$ runs were made with $\epsilon=1.5$ for a range of γ.

To test the influence of end effects, runs were also made with $\eta=0.88$, $\epsilon=0.5$, and $\gamma\simeq 2.30$ for three aspect ratios ($\Gamma=59.38$, 67.91, and 69.23), and with $\Gamma=66.60$ for various positions of the optical detector. Except for runs testing the influence of end effects by movement of the optical detector, the detector was kept at the middle of the annulus. Re_{1c} was independent of Γ for the aspect ratios that were tested, and end effects were apparent only when the detector was moved more than 75% of the way from the middle to the end of the annulus in which case the detected Re_{1c} lowered as the detector was placed closer to the end cap.

In Figs. 1 and 2 Re_{1c} is plotted as a function of γ. Figure 1 shows our results for $\epsilon=0.5$, and $\eta=0.88$ and 0.95. The solid lines on the right side of the graph give the experimentally measured critical Reynolds numbers for no modulation, $\mathrm{Re}_{1c}^{(0)}(\eta=0.88)=44.46\pm 0.49$ and $\mathrm{Re}_{1c}^{(0)}(\eta=0.95)=43.02\pm 0.51$. We see that for the values of γ for which measurements were taken, the amount of stabilization is roughly independent of η, since the data for a given γ shifts with $\mathrm{Re}_{1c}^{(0)}$. In Fig. 2 we compare the results for $\eta=0.179$ to results for $\eta=0.88$, both with $\epsilon=1.5$. This figure also gives results for $\eta=0.88$, $\omega\simeq\pi$ rad/sec ($\gamma\simeq 2.3$), for a variety of ϵ. These values of ϵ are, in order of increasing Re_{1c}, 0.3, 0.5, 0.7, 0.9, 1.5, and 2.0. The strong dependence of Re_{1c} on ϵ is quite apparent; Re_{1c} increases monotonically with ϵ for the range examined. Note that because of temperature (and, consequently, viscosity) variations from run to run γ varied slightly, $2.29\leqq\gamma\leqq 2.33$, while ω was constant. Again the horizontal lines give $\mathrm{Re}_{1c}^{(0)}$, $\mathrm{Re}_{1c}^{(0)}(\eta=0.719)=51.37\pm 0.66$.

We define the stabilization $\Delta=(\mathrm{Re}_{1c}-\mathrm{Re}_{1c}^{(0)})/\mathrm{Re}_{1c}^{(0)}$ and plot it in Fig. 3 as a function of γ for all data with $\epsilon=0.5$ and 1.5. We see that for a given ϵ the value of Δ is nearly independent of η. When our results were first examined it was expected that this scaling would place all the data for $\eta=0.95$ and 0.88 with $\epsilon=0.5$ on a single line. As this plot demonstrates, however, for the error bars on these data to overlap the uncertainty in Re_{1c}

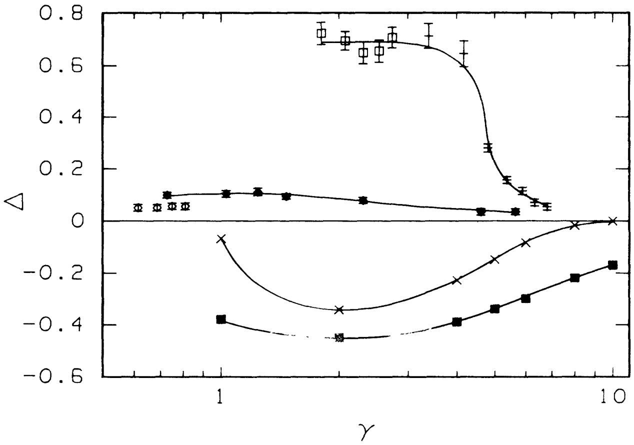

FIG. 3. Scaled experimental data: $\Delta=(\mathrm{Re}_{1c}-\mathrm{Re}_{1c}^{(0)})/\mathrm{Re}_{1c}^{(0)}$ as a function of γ for $\epsilon=0.5$, $\eta=0.88$ (filled lozenges) and 0.95 (open lozenges); and for $\epsilon=1.5$, $\eta=0.719$ (plusses) and 0.88 (open squares). To show the effect of scaling, and as an aid to the eye, a single curve connects all the data for $\epsilon=1.5$. Another curve connects the data for $\epsilon=0.5$ with $\eta=0.88$. Also shown are values of Δ for Carmi and Tustaniwskyj's stability limits with $\epsilon=0.5$ and $\eta=0.693$: linear (crosses) and strong-energy (filled squares). As an aid to the eye both of these sets are connected by a curve.

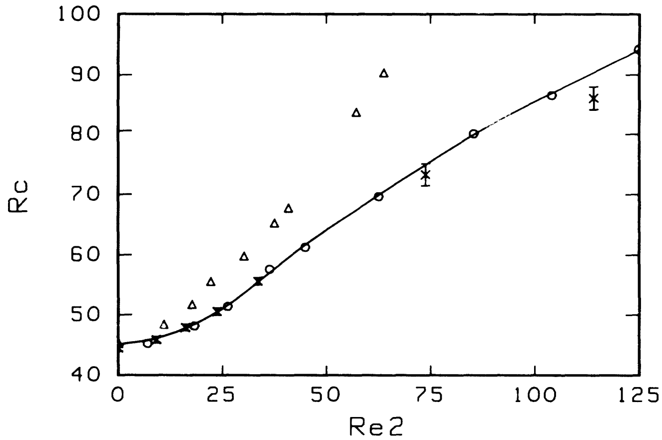

FIG. 4. Taylor's values for constant Ω_1 and Ω_2: corotation (triangles) and counterrotation (circles) (see text), and data for $\eta=0.88$, $\gamma\simeq2.3$ with several ϵ (crosses). Increasing values of Re_2 correspond to increasing values of ϵ. A curve connects Taylor's values for counterrotation.

would have to be about 1% greater. We also include in this figure the values of Δ for Carmi and Tustaniwskyj's strong-energy[3] and linear-stability[4] limits. The experimental Δ's are always positive, implying stabilization, while the theoretical Δ's are always negative, implying destabilization.

One unplotted data point was taken visually at low γ because of the limitations on our system described above. With $\eta=0.88$, $\epsilon=1.5$, $\mathrm{Re}_1=44.46$, and $\gamma=0.66$ transient vortices were found to appear very near $\Omega_2=0$. They were very weak and vanished with corotation and counterrotation. Our results show, therefore, a Δ which is near zero at small γ, increases to a broad peak between $\gamma\simeq1$ and 3.5 and then drops off again to near zero at higher γ. It is interesting that the range of γ over which modulation has its greatest effect, i.e., where $|\Delta|$ is the largest, is roughly the same for both theory and experiment.

We interpret our results in the following way. At low γ transient secondary flow is possible if Ω_2 is close to zero for long enough provided $\mathrm{Re}_1\geqq\mathrm{Re}_{1c}^{(0)}$. The secondary flow, which would appear in unmodulated flow, appears in the nearly steady flow. At this value of γ the viscous wave in the gap has little phase difference across the gap and the pressure and centrifugal fields are nearly the same as those which allow a bifurcation in steady flow.

At intermediate values of γ, where Δ is largest, the pressure and centrifugal fields are quite different from those which exist with constant Ω_1 and $\Omega_2=0$. There is a phase difference in the viscous wave across the gap, and while $\Omega_2\simeq0$ the flow field in much of the gap is still influenced by the previous relative motion (corotation and counterrotation) of the two cylinders. For a range of Re_1 that is greater than $\mathrm{Re}_{1c}^{(0)}$ much of the field resembles conditions in which azimuthal flow is stable, and secondary flow cannot develop.

At high γ the Stokes-layer thickness, $\delta=(2\nu/\omega)^{1/2}$, is much less than d and the effect of modulation is confined to a region near the outer cylinder. The flow field of the bulk of the fluid approaches that for constant Ω_1 with $\Omega_2=0$ as $\gamma\to\infty$. At higher and lower values of γ than we examined, we therefore expect to determine experimentally a Δ which asymptotically approaches zero. We note, however, that in general the stability of modulated hydrodynamic systems in the low-frequency limit remains an unresolved theoretical problem[8,12] which, perhaps, deserves further consideration.

We have observed an interesting scaling relationship holding for the region of optimum stabilization $1<\gamma<3.5$. If we define the outer-cylinder angular velocity $\langle|\Omega_2|\rangle$ as the arithmetic mean of the magnitude of Ω_2 over one modulation cycle, then an outer-cylinder Reynolds number $\mathrm{Re}_2=(\langle|\Omega_2|\rangle R_2d/\nu)(d/R_2)^{1/2}$ can be adopted. Because of the distortion in modulation waveform discussed above, this average must be determined numerically. In Fig. 4 we plot Re_{1c} against Re_2 for our data with $\eta=0.88$ and $\gamma=2.3$. These values are compared to the critical values found experimentally by Taylor[14] with $\eta=0.88$ for corotation and counterrotation (in order to place all data on the same side of the vertical axis we have used the absolute values of Taylor's values of Ω_2 for counterrotation). The data from our experiment falls on the same curve as Taylor's data for counterrotation with the exception of one point. We conclude, therefore, that in the region of optimum stabilization, $1 < \gamma < 3.5$, the effect of the modulation of the outer cylinder is dynamically equivalent to a steady counterrotation of the outer cylinder of magnitude $\langle |\Omega_2| \rangle$.

We note that our system preferentially detects secondary flow near the outer cylinder. It is quite possible that, because of the presence of the viscous wave in the gap, there are at any moment regions in which secondary flow can begin to grow and regions in which it decays when $\mathrm{Re}_1 \simeq \mathrm{Re}_{1c}$. We expect that the results of further theoretical work on this problem may show a stabilization which is somewhat smaller than ours, since theory may report an Re_{1c} at which secondary flow can develop anywhere in the gap and for any length of time, rather than an Re_{1c} at which enough of the flow is unstable that secondary flow can be detected at the outer cylinder. Specifically, since modulation of the outer cylinder stabilizes the flow, the flow near the inner cylinder should become unstable at a lower Re_1 than that at which the flow near the outer cylinder becomes unstable—the amplitude of the viscous wave being greatest near the outer cylinder and zero, because of the no-slip boundary condition, at the inner cylinder. It would be interesting to study the amplitude of local fluctuations with use of laser Doppler velocimetry.

We conclude that the theory underlying our present experiment merits reexamination. Related experiments for which theory exists,[3,4] such as modulation of the outer cylinder about zero mean with the inner cylinder stationary, and mean rotation of the inner cylinder with a superimposed modulation combined with either an in-phase or 180° out-of-phase modulation about zero mean of the outer cylinder, should be tried as a rigorous check on existing and future theoretical work.

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[13]We take this opportunity to correct an error in Ref. 6 in which the Reynolds number was defined as $\overline{\Omega}_1 R_1 d/\nu$ rather than as $(\overline{\Omega}_1 R_1 d/\nu)(d/R_1)^{1/2}$.

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