Interacting Superstrings at Finite Temperature

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A real-time finite-temperature formalism for interacting superstrings is described. It is used to compute the tree amplitude for open superstrings at finite temperature.

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Superstring theory[1] is the only known example of a renormalizable quantum theory of gravity. Therefore understanding its role in the early Universe is crucial. An important aspect in this problem is to understand the finite-temperature effects of superstrings.

Recently a real-time finite-temperature formalism for the bosonic string has been constructed.[2,3] Using it, the finite-temperature Veneziano amplitude and the one-loop string amplitude have been calculated.

The basic idea of the real-time finite-temperature formalism [thermo field dynamics (TFD)] is to construct a thermal vacuum $|0(\beta)\rangle$ such that the statistical average $\langle A\rangle$ of a dynamical variable A can be expressed in the form

$$\langle A\rangle \equiv tr(Ae^{-\beta H})/\mathrm{tr}(e^{-\beta H}) = \langle 0(\beta)|A|0(\beta)\rangle. \tag{1}$$

To construct $|0(\beta)\rangle$ it has been shown[4] that one has to double the physical degrees of freedom. It is common to denote the unphysical variables by a tilde.

The basic axioms of TFD can be summarized[2] in the following equations:

$$[A(t),\tilde{B}(t)]=0 \ \forall A,\tilde{B}, \tag{2}$$

$$(AB)^{\sim}=\tilde{A}\tilde{B}, \tag{3}$$

$$(c_1A_1+c_2A_2)^{\sim}=c_1^*\tilde{A}_1+c_2^*\tilde{A}_2, \tag{4}$$

$$(\tilde{A})^{\dagger}=(A^{\dagger})^{\sim}, \tag{5}$$

$$|0(\beta)\rangle^{\sim}=|0(\beta)\rangle, \tag{6}$$

$$A(t,\mathbf{x})|0(\beta)\rangle=\sigma\tilde{A}^{\dagger}(t-i\tfrac{1}{2}\beta,\mathbf{x})|0(\beta)\rangle, \tag{7}$$

$$\langle 0(\beta)|A(t,\mathbf{x})=\langle 0(\beta)|\tilde{A}^{\dagger}(t+i\tfrac{1}{2}\beta,\mathbf{x})\sigma^*, \tag{8}$$

$$(\tilde{A})^{\sim}=\sigma A, \tag{9}$$

where c_1 and c_2 are complex numbers and $|\sigma|=1$. Sometimes one uses the thermal doublet notation

$$A^{\alpha}=\begin{cases}A, & \alpha=1,\\ \tilde{A}^{\dagger}, & \alpha=2.\end{cases}$$

In Ref. 2, TFD has been applied to string field theory. The finite-temperature string propagator has been given by

$$\Delta^{\alpha\beta}(p)=\left(U_B(|p_0|)\frac{\tau}{L_0-1-i\alpha'\tau\delta}U_B|p_0|\right)^{\alpha\beta}, \tag{10}$$

where

$$U_B(\omega)=(e^{\beta\omega}-1)^{-1/2}\begin{pmatrix}e^{\beta\omega/2} & 1\\ 1 & e^{\beta\omega/2}\end{pmatrix},$$

$$\tau=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \tag{11}$$

$$L_0=\tfrac{1}{2}p^2+\sum_{n=1}^{\infty}[\alpha^{\mu}_{-n}\alpha^{\mu}_{n}+n(c_{-n}b_n+b_{-n}c_n)],$$

α_n^{μ} are the oscillator modes and c_n (b_n) the ghost (antighost) mode of the string. The vertices are

$$V^{\alpha\beta\gamma}(p)=\begin{cases}v, & \alpha=\beta=\gamma=1,\\ -v, & \alpha=\beta=\gamma=2,\\ 0, & \text{otherwise.}\end{cases} \tag{12}$$

Although the results (10) and (12) have been derived with the covariant formulation of string field theory, correspondence with the first quantized theory makes one believe that (10) and (12) are correct even in noncovariant gauge. Therefore we use (10) and (12) to define the propagators and the vertices in the light-cone gauge where

$$L_0=\tfrac{1}{2}p^2+\sum_{n=1}^{\infty}\alpha^{i}_{-n}\alpha^{i}_{n}. \tag{13}$$

Now using the light-cone gauge to calculate the Veneziano amplitude at finite temperature I obtain

$$A_u(s,t)=g^2\left(\frac{e^{\beta|k_{01}+k_{02}|}}{e^{\beta|k_{01}+k_{02}|}-1}B(-\tfrac{1}{2}s-i\tfrac{1}{2}\delta,1-\tfrac{1}{2}t)-(e^{\beta|k_{01}+k_{02}|}-1)^{-1}B(-\tfrac{1}{2}s+i\tfrac{1}{2}\delta,1-\tfrac{1}{2}t)\right), \tag{14}$$

where s and t are the Mandelstam variables

$$s=-(k_1+k_2)^2,\quad t=-(k_2+k_3)^2, \tag{15}$$

and $B(x,y)$ is Euler's beta function. The result (14) is identical to that of Ref. 1. Similarly the results of the one-loop

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amplitudes are identical. Hence from now on the light-cone gauge will be used with the rules (10) and (12) with the modification (13) for the bosonic string.

Now our attention is turned to superstrings. For simplicity, only open superstrings are considered. The rule for the finite-temperature vertices is still (12) since it is a consequence of the doubling-up rule. However, since the superstring has both bosonic and fermionic modes its propagator can have one of the following forms:

$$\Delta_B^{\alpha\beta}(p)=\left[\frac{U_B(|p_0|)\tau U_B(|p_0|)}{L-i\tau\alpha'\delta}\right]^{\alpha\beta}, \tag{16}$$

$$\Delta_F^{\alpha\beta}(p)=\left[\frac{U_F(|p_0|)IU_F(|p_0|)}{L-i\tau\alpha'\delta}\right]^{\alpha\beta}, \tag{17}$$

where

$$U_F(\omega)=(e^{\beta\omega}+1)^{-1/2}\begin{bmatrix} e^{\beta\omega/2} & 1 \\ -1 & e^{\beta\omega/2}\end{bmatrix}, \tag{18}$$

$$L=\tfrac{1}{2}p^2+\sum_{n=1}^{\infty}(\alpha_{-n}^i\alpha_n^i+\tfrac{1}{2}n\bar{S}_{-n}\Gamma^-S_n), \quad I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \tag{19}$$

S_n^a are the Majorana-Weyl fermionic oscillators of the superstring, and

$$\Gamma^-=\frac{1}{\sqrt{2}}(\Gamma^0+\Gamma^9).$$

Γ^μ, $\mu=0,1,2,\ldots,9$, are the ten-dimensional Dirac matrices.

Which one of Δ_B and Δ_F will be the $T\neq 0$ superstring propagator? The answer I give is the following. It will depend on the statistics of the emitted particles at the same vertex. If they have the same statistics, the superstring propagator will be Δ_B. If they have opposite statistics, e.g., a spinor and a gauge vector, the superstring propagator is Δ_F. This is a direct analogy with field theory. Furthermore, it is applicable to all tree and one-loop calculations for which light-cone gauge can be used.

Let me calculate the tree amplitude of four particles at finite temperature. I will specify two particles at different vertices to be the massless vectors. The other two can be either two vectors or two spinors. I am using the notation of Schwarz.[5] The amplitude is given by

$$\begin{aligned}
\langle k_1|V_B(\zeta_{21}k_2)\Delta V_B(\zeta_3,k_3)|k_4\rangle_{T\neq 0}
&=\tfrac{1}{2}g^2\zeta_2\cdot\zeta_3(1+\tfrac{1}{2}t)[B(1-\tfrac{1}{2}s-i\tfrac{1}{2}\delta,-1-\tfrac{1}{2}t)(1\pm F_\mp)\mp B(1-\tfrac{1}{2}s+i\tfrac{1}{2}\delta,-1-\tfrac{1}{2}t)F_\mp]\\
&\quad+\tfrac{1}{2}g^2[-(\zeta_2\cdot k_1)(\zeta_3\cdot k_2)+R_0^{ij}(\zeta_2^ik_2^j\zeta_3\cdot k_4-\zeta_3^ik_3^j\zeta_2\cdot k_1)+R_0^{ij}R_0^{kl}\zeta_2^ik_2^j\zeta_3^kk_3^l]\\
&\quad\times[B(-\tfrac{1}{2}s-i\tfrac{1}{2}\delta,1-\tfrac{1}{2}t)(1\pm F_\mp)\mp B(-\tfrac{1}{2}s+i\tfrac{1}{2}\delta,1-\tfrac{1}{2}t)F_\mp]\\
&\quad+\tfrac{1}{2}g^2[\zeta_2\cdot k_3\zeta_3\cdot k_4+\zeta_2\cdot k_1\zeta_3\cdot k_2+\zeta_2\cdot k_3\zeta_3\cdot k_2+\zeta_2\cdot\zeta_3k_2\cdot k_3\\
&\quad+R_0^{ij}(-\zeta_2^ik_2^j\zeta_3\cdot k_2+\zeta_3^ik_3^j\zeta_2\cdot k_3+\zeta_2\cdot\zeta_3k_2^ik_3^j+\zeta_2^i\zeta_3^jk_2\cdot k_3-\zeta_2\cdot k_3k_2^i\zeta_3^j-\zeta_2^ik_3^j\zeta_3\cdot k_2)]\\
&\quad\times[\mp B(1-\tfrac{1}{2}s+i\tfrac{1}{2}\delta,-\tfrac{1}{2}t)F_\mp+B(1-\tfrac{1}{2}s-i\tfrac{1}{2}\delta,-\tfrac{1}{2}t)(1\pm F_\mp)],
\end{aligned} \tag{20}$$

where

$$F_\mp\equiv(e^{\beta|k_{01}+k_{02}|}\mp 1)^{-1}, \tag{21}$$

and the upper (lower) sign is used if the other two particles are vectors (spinors). I expect the calculation of other tree and one-loop amplitudes to be straightforward.

The formulation of Ref. 2 has been generalized to superstrings. The four-particle tree amplitude has been calculated at finite temperature. It is clear that as $T\to 0$ the known $T=0$ amplitude[5] is regained. I anticipate that the finite-temperature superstring amplitudes will be important in superstring cosmology.

[1]See, for example, M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory* (Cambridge Univ. Press, Cambridge, England, 1987).

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[5]J. H. Schwarz, Phys. Rep. C **89**, 223 (1982).