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Macroscopic Quantum Tunneling of Vortices in He II

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Experimental evidence is presented showing that the creation of vortices in He II involves macroscopic quantum tunneling through, or thermal activation over, a potential-energy barrier ϵ. The investigations, based on measurements of the rate at which negative ions create vortex rings in the isotopically pure superfluid within the temperature range $50 < T < 500$ mK, have yielded a barrier height of $\epsilon/k_B = 3.1 \pm 0.1$ K, in excellent agreement with the theoretical value predicted by Muirhead, Vinen, and Donnelly.

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Vortices dominate the hydrodynamics and many of the thermal properties of He II. Yet, even now, some thirty years after their existence was first mooted[1] and twenty-six years after the first measurement of the quantum of circulation,[2] the mechanism through which vortices are created in the superfluid is still not properly understood. The remarkable longevity of this long-standing mystery is attributable in large measure to the scarcity of relevant experimental information. Most investigations of vortex generation to date have addressed the quite distinct, but equally interesting, problem of the conditions under which preexisting[3] vorticity expands to form dissipative tangles.[4] In practice, it is this phenomenon that is almost always investigated in flow and thermal counterflow experiments, though possible exceptions are to be found in the measurements of Hess[5] and also, in particular, in the recent studies by Beeken and Zimmermann[6] and by Avenel and Varoquaux[7] of flow through microscopic orifices. The latter results, however, are the subject of continuing debate[8,9] as to their correct interpretation. The present Letter describes a completely different experimental approach to the problem of vortex creation, based on the use of negative ions[10] which are small enough (radii $\simeq 1$ nm) to be negligibly influenced by remanent vorticity.

The rate ν at which vortex rings are created by negative ions moving through the liquid was measured by means of the electric-induction technique[11,12] described previously, with use of an isotopically purified[13] sample of ^{4}He in order to eliminate the complications[14] that would otherwise have arisen from the ^{3}He isotopic impurities found in naturally occurring helium. An extension of the earlier work[11,12] to considerably lower pressures (12 bars) and temperatures (50 mK) was accomplished by the exploitation of techniques developed for the determination of the Landau critical velocity.[15] The 1.5-l cell developed for the latter measurement was also used for the present work, but with its electrode structure modified to incorporate an 11-mm induction space, thus enabling ν to be measured. The cell was mounted in a simple dilution refrigerator that allowed its temperature to be stabilized within the range $50 < T < 500$ mK. The measured variation of ν with reciprocal temperature at three fixed electric fields E is shown in Fig. 1 for a pressure P of 12 bars. The variation of ν with E in the temperature-independent regime below 100 mK is shown in Fig. 2, also at $P = 12$ bars.

Superficially, the experimental results look rather similar to those obtained previously[11,12] at higher pressures, but there are important differences. In particular, the rapid rise in ν with increasing T (Fig. 1) now takes place, not at $\simeq 0.5$ K as previously, but at $\simeq 0.2$ K where the density of thermal rotons in the liquid is negligible. One can be confident, therefore, that the temperature dependence of ν is not associated with the roton-driven

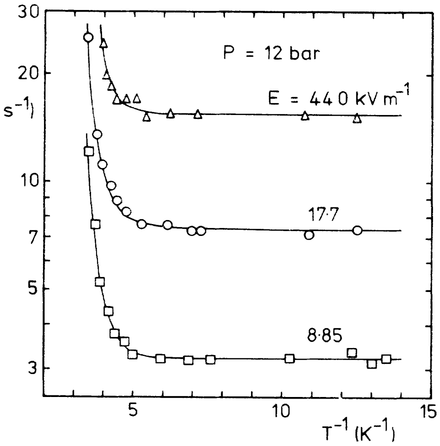

FIG. 1. Vortex nucleation rate ν for negative ions in isotopically pure He II plotted as a function of reciprocal temperature T^{-1} for three electric fields E. The curves represent fits of (1) to the experimental data (points).

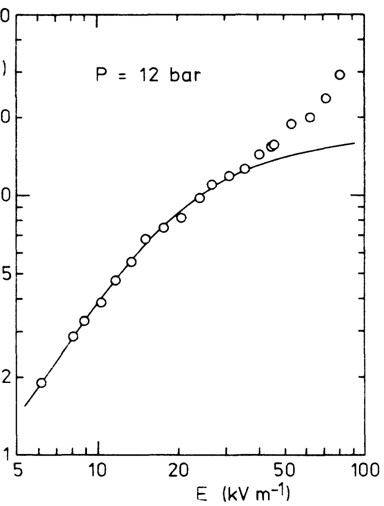

FIG. 2. Vortex nucleation rate ν plotted as a function of electric field E for the temperature-independent regime below 100 mK. The curve represents a fit of the theory of Ref. 16 to the experimental data (points) for $E < 35$ kV m^{-1}.

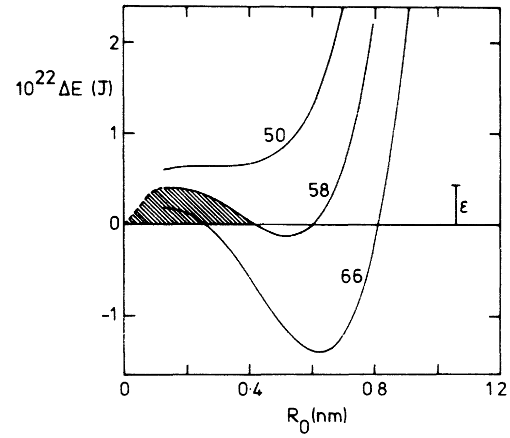

FIG. 3. Change in energy ΔE at constant impulse that occurs when a vortex loop of radius R_0 is formed in the equatorial plane by a negative ion, for the three different ionic velocities (meters per second) shown by the numbers adjacent to the curves (after MVD, Ref. 26). The energy barrier for the case of an ion that slightly exceeds the critical velocity (middle curve) is the hatched area. The experimental barrier height ϵ deduced from the data of Fig. 1 is indicated by the bar.

vortex nucleation process discovered in the earlier experiments.[12] It is found that $\nu(T)$ can be fitted accurately (lines in Fig. 1) by

$$\nu = \nu(0) + A \exp(-\epsilon/k_B T), \qquad (1)$$

where $\nu(0)$ represents the temperature-independent value of ν for $T < 0.2$ K, and A and ϵ are constants. The fitted value of ϵ is independent of E within experimental error: The weighted mean obtained from fitting the $\nu(T)$ data for several values of E was $\epsilon/k_B = 3.1 \pm 0.1$ K which, we may note, is considerably smaller than the roton energy gap (cf. the higher-pressure data of Ref. 12).

Behavior of the kind represented by (1) is, of course, normally associated with the existence of an energy barrier, such that the temperature-independent term $\nu(0)$ represents quantum-mechanical tunneling through the barrier and the Arrhenius term represents thermal activation over it. The data of Fig. 1 can thus be construed as confirmation of the long-standing suggestion of Vinen[17] that there must be a potential-energy barrier impeding vortex creation in He II.

On the basis of this idea, and of ideas contributed over the years by a number of other workers,[18–25] Muirhead, Vinen, and Donnelly[26] (MVD) have recently proposed that vortex creation by ions moving through He II at low T occurs through a form of macroscopic quantum tunneling (MQT). Reference should be made to their paper[26] for details of the argument and essential caveats and qualifications, but, in essence, MVD calculate the change in total energy ΔE at constant impulse that occurs when a vortex loop of radius R_0 is formed. Some results for the energetically most favorable configuration are shown in Fig. 3. Because the MVD calculations are hydrodynamic in nature, they cannot be carried down to very small R_0, but it seems reasonable to suppose that $\Delta E \rightarrow 0$ as $R_0 \rightarrow 0$ as sketched by the dashed line for the case where the ionic velocity v is only slightly larger than its critical value v_c (middle curve). The resultant potential-energy barrier is the hatched area. For comparison, the barrier height ϵ deduced from the data of Fig. 1 is indicated by the bar on the right-hand side of the figure: The agreement is excellent (although probably coincidentally so, given that the theory has a quantitative uncertainty[27] of up to a factor of $\simeq 2$). The curves actually refer to 17 bars, rather than to 12 bars, but the size of the barrier at threshold is not expected by MVD to be strongly influenced by pressure. The measured value of ϵ supports the picture of nucleation from a vortex loop,[19,20,26] rather than from an encircling ring[16,22] for which the corresponding energy barrier is an order of magnitude larger, at least within the thermal-activation regime.

It is of considerable interest to try to estimate the first critical velocity[11,12] v_c from the data of Fig. 2, but, for reasons discussed in detail elsewhere,[12,24] the calculation is not straightforward. The complications arise mainly from the time dependence of v: Each ion accelerates in the superfluid under the applied electric field; shortly after passing the Landau critical velocity for roton creation v_L, it loses momentum through the emission of a pair of rotons and suffers a corresponding discontinuous decrement in v; it then accelerated once more. The cycle repeats at a frequency ($\simeq 10^{10}$ s^{-1}) which depends on E. The resultant $v(t)$ has a mean value which is slightly greater than v_L and which increases with E. Superimposed on this is an approximately sawtooth-shaped velocity variation with an amplitude of $\simeq \pm 4 \text{ ms}^{-1}$. There is a finite, but relatively small, probability that the acceleration of the ion may terminate, not in roton emission, but in the vortex creation process discussed in this Letter. As E is increased, the whole sawtooth rises, the maxima of $v(t)$ become larger, on average, and the ion has a correspondingly enhanced probability of reaching v_c. Consequently ν increased with E as observed in the experiments (Fig. 2).

In order to extract a value of v_c from the $\nu(E)$ data, it is clearly essential to take explicit account of the distribution of ionic velocities; it is, of course, the ions in the high-velocity tail of the (approximately rectangular) distribution function that are likely to exceed v_c and create vortices. Bowley[24] showed that by solution of the appropriate Boltzmann equation, it is possible to calculate ν in terms of v_c and of the instantaneous nucleation rate N_0 at the threshold: that is, the nucleation rate for an ion that moves at a constant velocity marginally in excess of v_c. In doing so he found it necessary to make an assumption about the way in which the nucleation rate varies with v. He proposed, as the simplest plausible assumption, that

$$\nu = N_0 \theta(v - v_c), \tag{2}$$

where θ is the unit step function. Bowley assumed N_0 to be a constant. Although the situation is now known to be more complicated[12] than this, and the MVD theory (in which N_0 corresponds to the tunneling rate) also suggests that N_0 should increase rapidly with v, it is nonetheless reasonable to suppose that (2) with N_0 constant will be a reasonable approximation in relatively weak E, close to threshold, where $v - v_c$ is small. By fitting Bowley's theory[24] to the lower-field data of Fig. 2, with v_c and N_0 as adjustable parameters, we obtain the curve shown in Fig. 2, for which $v_c = 59.5 \pm 0.3 \text{ m s}^{-1}$ and $N_0 = (2.5 \pm 0.3) \times 10^4 \text{ s}^{-1}$. The derived critical velocity is within 10% of that calculated by MVD and hence, like the barrier height ϵ, it may be regarded as being in remarkably good agreement with their theory.

The MVD model appears to have been strongly supported by the experimental results of Figs. 1 and 2. There are, however, a number of observations that should be made. First, as already noted, the MVD calculations are inherently incapable of describing the shape of the potential at small R_0; a quantum-mechanical calculation of the tunneling rate is now urgently required. The calculation will need to take proper account of the acceleration of the ion, which implies that the tunneling process always takes place through a barrier that is shrinking (see Fig. 3).

Second, the data of Fig. 1, described by (1) with ϵ equal to the barrier height, appear at first sight to differ markedly from results[12] at higher P where the temperature-dependent part of ν was found to be proportional to the thermal roton density. This apparent discrepancy can be resolved, however, if we infer (which was not predicted by MVD) that the barrier height increases with P. Thus, the temperature at which the rapid rise occurs in ν will also increase with P until it has entered the temperature range ($T \gtrsim 0.5$ K) where rotons are the dominant thermal excitations.

Third, it should be mentioned that, for $P \gtrsim 15$ bars, an additional phenomenon has been observed[28] in which ν at first decreases slightly with increasing T. The reason is not yet understood, but it appears[29] that the effect is not due simply to a small reduction in the ionic drift velocity caused by excitation scattering.

Fourth, we would comment that the tunneling process envisaged by MVD, and apparently confirmed by the experiments, refers to a *boson* system. It also seems to differ in other important respects from the kind of MQT phenomena most intensively studied to date,[30] where the properties of SQUIDs have been particularly in mind. For example, the extent to which the MVD potential-energy curve (Fig. 3) can play a role analogous to a classical potential remains unclear; furthermore, R_0 cannot become negative. (There is also room for discussion of whether in fact the MVD quantum tunneling mechanism is best described as macroscopic: Although the tunneling process certainly involves correlated changes of state for a macroscopic number of atoms, the ion itself and the created vortex loop are only semimacroscopic, both being on the scale of nanometers.)

Fifth, we wish to emphasize that the excellent agreement obtained between experiment and the MVD "loops" theory relates to the processes in relatively weak electric fields ($E \lesssim 35 \text{ kV m}^{-1}$ at $P = 12$ bars) that are associated with the first critical velocity.[12] The departure of the data from the theoretical curve[24] (Fig. 2) in stronger electric fields (also observed in earlier work at higher pressures[11,12]) can be interpreted either as evidence for a complicated velocity dependence of N_0 or, alternatively, as an indication that additional nucleation mechanisms come into play when the ionic velocity is large enough. Thus, the present data do not *necessarily* preclude the occurrence of, for example, axially symmetric nucleation processes,[16,22] even though the latter have associated critical velocities and energy barriers that are higher[26] than those for vortex loop creation.

Finally, although the particular probe used in these experiments has been the negative ion, we would expect essentially the same physics to apply quite generally to any object moving through the superfluid, or to superflow relative to a fixed tube or orifice. The absolute magnitudes of the critical velocity and barrier height will naturally depend on the particular system under study and, in the case of flow experiments,[6,7] are likely to be strongly influenced by microscopic excrescences (protuberances) on the walls.

In conclusion, we have confirmed experimentally that vortex creation in He II is impeded by a small potential-energy barrier. Vortices can be produced either by macroscopic quantum tunneling through the barrier or, at high enough temperatures, by thermal activation over it.

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[1] H. London, *Low Temperatures: A Report on the International Conference at Cambridge* (Physical Society, London, 1947), p. 48; see, also, F. London, *Superfluids* (Wiley, New York, 1954), p. 151.

[2] W. F. Vinen, Proc. Roy. Soc. London A **260**, 218 (1961).

[3] D. Awschalom and K. W. Schwarz, Phys. Rev. Lett. **52**, 49 (1984).

[4] J. T. Tough, in *Progress in Low Temperature Physics,* edited by D. F. Brewer (North-Holland, Amsterdam, 1982), Vol. 8, Chap. 3.

[5] G. B. Hess, Phys. Rev. Lett. **27**, 977 (1971).

[6] B. P. Beeken and W. Zimmermann, Phys. Rev. B **35**, 1630 (1987).

[7] O. Avenel and E. Varoquaux, Phys. Rev. Lett **55**, 2704 (1985); E. Varoquaux, M. W. Meisel, and O. Avenel, Phys. Rev. Lett. **57**, 2291 (1986).

[8] K. W. Schwarz, Phys. Rev. Lett. **57**, 1448 (1986), and **59**, 1167 (1987).

[9] O. Avenel and E. Varoquaux, Phys. Rev. Lett. **59**, 1168 (1987).

[10] A. L. Fetter, in *The Physics of Liquid and Solid Helium,* edited by K. H. Benneman and J. B. Ketterson (Wiley-Interscience, New York, 1976), Pt. 1, Chap. 3.

[11] R. M. Bowley, P. V. E. McClintock, F. E. Moss, and P. C. E. Stamp, Phys. Rev. Lett. **44**, 161 (1980).

[12] R. M. Bowley, P. V. E. McCintock, F. E. Moss, G. G. Nancolas, and P. C. E. Stamp, Philos. Trans. Roy. Soc. London A **307**, 201 (1982).

[13] P. C. Hendry and P. V. E. McClintock, Cryogenics **27**, 131 (1987).

[14] R. M. Bowley, G. G. Nancolas, and P. V. E. McClintock, Phys. Rev. Lett. **52**, 659 (1984), and Philos. Trans. Roy. Soc. London A **313**, 537 (1985).

[15] T. Ellis and P. V. E. McClintock, Philos. Trans. Roy. Soc. London A **315**, 259 (1985).

[16] R. M. Bowley, J. Phys. C **17**, 595 (1984).

[17] W. F. Vinen, in *Liquid Helium,* International School of Physics "Enrico Fermi," Course 21 (Academic, New York, 1963), p. 336.

[18] G. W. Rayfield and F. Reif, Phys. Rev. **136**, A1194 (1964).

[19] G. W. Rayfield, Phys. Rev. Lett. **19**, 1371 (1967).

[20] R. J. Donnelly and P. H. Roberts, Philos. Trans. Roy. Soc. London A **271**, 41 (1971).

[21] G. E. Volovik, Pis'ma Zh. Eksp. Teor. Fiz. **15**, 116 (1972) [JETP Lett. **15**, 81 (1972)].

[22] K. W. Schwarz and P. S. Jang, Phys. Rev. A **8**, 3199 (1973).

[23] E. B. Sonin, Zh. Eksp. Teor. Fiz **64**, 970 (1973) [Sov. Phys. JETP **37**, 494 (1973)].

[24] R. M. Bowley, J. Phys. C **9**, L367 (1976).

[25] M. Ichiyanagi, J. Low Temp. Phys. **23**, 599 (1976).

[26] C. M. Muirhead, W. F. Vinen, and R. J. Donnelly, Philos. Trans. Roy. Soc. London A **311**, 433 (1984).

[27] W. F. Vinen, private communication.

[28] P. C. Hendry, N. S. Lawson, C. D. H. Williams, P. V. E. McClintock, and R. M. Bowley, Jpn. J. Appl. Phys. Suppl. **26**, No. 3, 73 (1987).

[29] P. C. Hendry, N. S. Lawson, C. D. H. Williams, P. V. E. McClintock, and R. M. Bowley, in "Elementary Excitations in Quantum Fluids," edited by K. Ohbayashi and M. Watabe (Springer-Verlag, Berlin, to be published).

[30] A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. **46**, 211 (1981), and Ann. Phys. (N.Y.) **149**, 374 (1983).