Experimental Observation of Nonsymmetrical $N=2$ Solitons in a Femtosecond Laser

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We report here the first experimental observation of nonsymmetrical $N=2$ solitonlike pulses. These solitons are produced by a passively mode-locked dye laser. The evolution of the pulse spectrum has been recorded and compared with theoretical predictions of the nonlinear Schrödinger equation.

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Since the first production of subpicosecond pulses by use of passive mode-locking of dye lasers,[1] great progress has been achieved. With the introduction of systems to control the intracavity group velocity dispersion, pulse durations as short as 30 fs have been obtained.[2] It is now well established that soliton mechanisms are responsible for the formation and shortening of pulses in these passively mode-locked and disperison-controlled lasers.[3,4] Femtosecond pulses have also been produced with either fiber Raman amplified soliton lasers working in the 1.5-μm range,[5] or soliton narrowing in optical fibers.[6,7] In all works dealing with temporal soliton propagation,[4,7] spatial soliton self-guided propagation,[8] or theoretical studies on soliton lasers,[9] only symmetrical shapes were considered. This leads to the misconception that solitons could only have symmetrical shapes. In this Letter, we report the first (as far as we know) observation of the time evolution of nonsymmetrical $N=2$ solitons. These pulses are directly produced by a colliding pulse mode-locked (CPM) laser. Moreover, this experimental observation gives a new proof of the soliton character of the pulses obtained from CPM lasers.

With use of the slowly varying-envelope approximation, pulse propagation in a nonlinear medium can be described with the so-called nonlinear Schrödinger equation (NLSE). The pulse envelope amplitude $u(z,t)$ satisfies the following equation:

$$i\,\partial u/\partial z+\tfrac{1}{2}\,\partial^2 u/\partial t^2+|u|^2u=0, \qquad (1)$$

where t is the pulse local time and z is the normalized distance in the propagation medium.

Zakharov and Shabat[10] have shown that the NLSE can be solved by the inverse scattering method. The stable solutions for propagation are called soliton bound states. Each solution is characterized by a set of complex constants $\{\xi_j,C_j\}_{j=1\cdots N}$. The complex ξ_j's are the poles of the soliton and the C_j's are its residues. Zakharov and Shabat showed that bound states are obtained only if all the poles ξ_j's are on a line parallel to the imaginary axis. If they are not, the solution separates into different pulses. With no loss of generality we will then assume that the N poles ξ_j's characterizing a N-order soliton lie on the imaginary axis. Therefore we write $\xi_j=i\eta_j$.

Using Zakharov and Shabat's results, one can obtain the analytical expression for an N-order soliton. The $N=2$ soliton shape $u(z,t)$ can be written down explicitly[11]:

$$u(z,t)=2N(z,t)/D(z,t), \qquad (2)$$

where

$$N(z,t)=C_1[1+A_2]\exp(-2\eta_1 t+4i\eta_1^2 z)+C_2[1+A_1]\exp(-2\eta_2 t+4i\eta_2^2 z) \qquad (3)$$

and

$$D(z,t)=1+(C_1^2/4\eta_1^2)\exp(-4\eta_1 t)+(C_2^2/4\eta_2^2)\exp(-4\eta_2 t)+A_1A_2 +2[C_1C_2/(\eta_1+\eta_2)^2]\exp[-2(\eta_1+\eta_2)t]\cos[4(\eta_1^2-\eta_2^2)z], \qquad (4)$$

with

$$A_j=C_j^2[(\eta_1-\eta_2)/(\eta_1+\eta_2)]^2/4\eta_j^2\exp(-4\eta_j t),\quad j=1,2. \qquad (5)$$

In the expression (2) an $N=2$ soliton is uniquely determined by four constants: two imaginary poles $i\eta_1$ and $i\eta_2$, and two residues C_1 and C_2. In fact, Haus and Islam[9] have shown that the four degrees of freedom of the complex residues C_j's affect the soliton in restricted ways. They found that the imaginary parts of the residues only introduce a global soliton phase shift which can be absorbed in a shift of the origin of t. So we can say that an $N=2$ soliton is uniquely described by a set of four real constants $\{\eta_1,\eta_2,C_1,C_2\}$. Symmetrical solitons are obtained when residues are related by[9]

$$C_j=\prod_{k=1}^{N}(\eta_j+\eta_k)\left(\prod_{\substack{k=1\\k\neq j}}^{N}|\eta_k-\eta_j|\right)^{-1}. \qquad (6)$$

Note that even for the restricted case of symmetrical solitons, one can find an infinity of $N=2$ solitons. The usual solution characterized by $u_0(z=0,t)=2\,\mathrm{sech}\,t$ corresponds to poles $\eta_1=\frac{1}{2}$ and $\eta_2=\frac{3}{2}$, and to residues $C_1=2$ and $C_2=6$. Other sets of poles with residues following the expression (6) give symmetrical solitons as those sketched in Ref. 9. When Eq. (6) is not verified, the $N=2$ soliton temporal shapes generally present a double humped structure with one of the peaks much higher than the second one.[1] All these $N=2$ solitons present a periodical evolution of their shape. The period T_p only depends on the pole values[9] $T_p=\pi/(\eta_2^2-\eta_1^2)$. As an example, evolutions of an $N=2$ soliton temporal shape and spectrum during a period T_p are sketched in Fig. 1, for $\eta_1=0.7$, $\eta_2=1.3$, $C_1=0.7$, and $C_2=1.1$ [the sum of the poles is chosen to give $\eta_1+\eta_2=2$, in order to have the same energy as the solution given by $u_0(z,t)$].

Such a nonsymmetrical soliton is not easily experimentally observed because it is very difficult to generate the exact $u(z=0,t)$ in amplitude and phase before launching the pulse in a nonlinear medium. In the experiment described here, the generating and propagation media are not separated. We have used a CPM dye laser containing a sequence of four prisms which allows a precise adjustment of the group velocity dispersion inside the cavi-

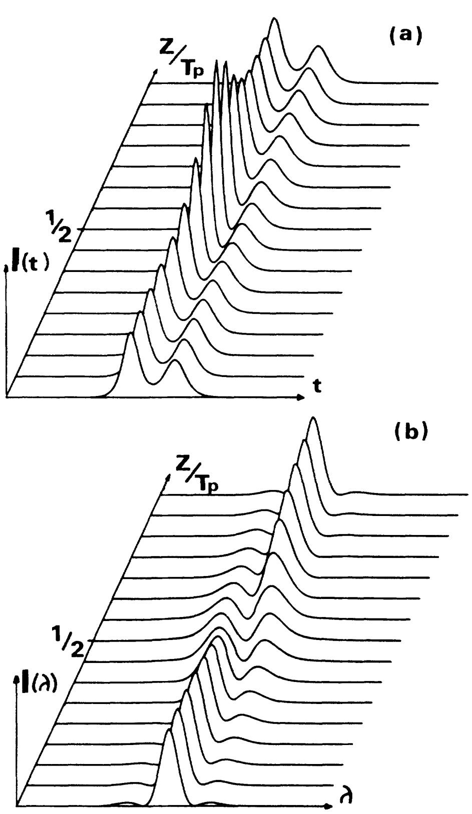

FIG. 1. Theoretical evolution of a nonsymmetrical $N=2$ soliton: (a) temporal pulse shape; (b) pulse spectrum.

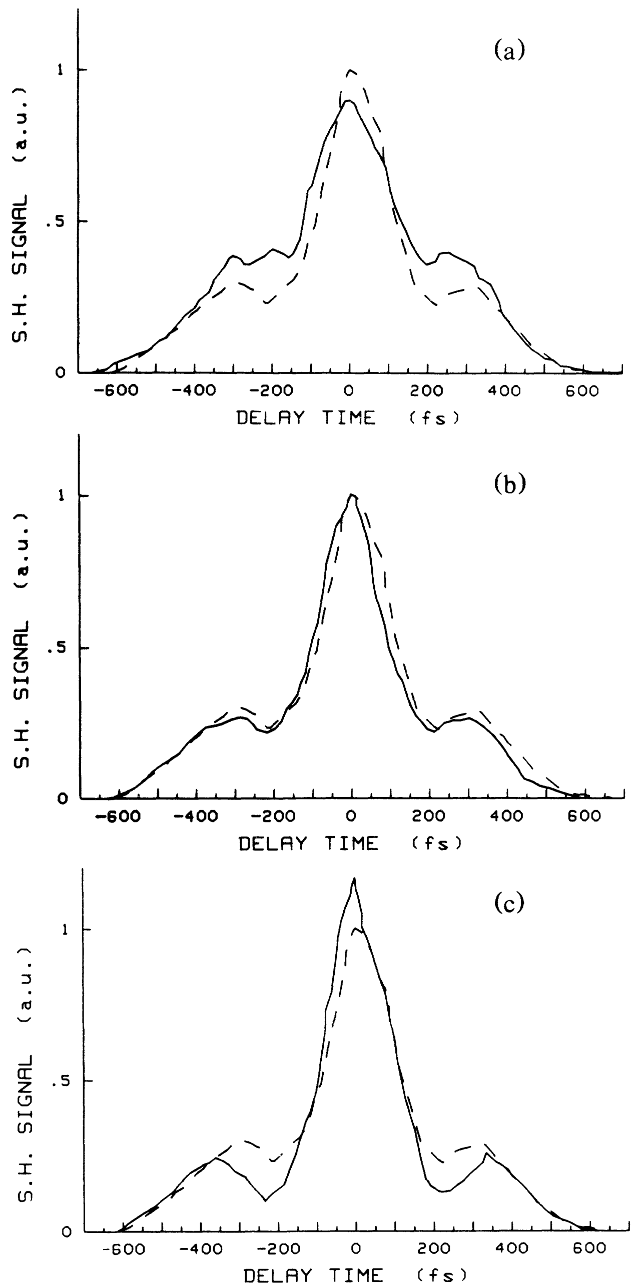

FIG. 2. Autocorrelation function recorded (a) at the beginning of the soliton period ($t=0$), (b) after 275 cavity round trips ($t=T_p/4$), and (c) after 550 cavity round trips ($t=T_p/2$). As a comparison the autocorrelation function averaged over the entire period is given in dashed lines.

ty.[2] In its usual working regime, our laser produces stable pulses with duration as short as 40 fs at 620 nm. These pulses correspond to $N=1$ solitons.[4] If we introduce in the cavity less negative dispersion than the value corresponding to the minimum pulse width, the laser wavelength shifts toward the red. By focusing or defocusing the laser-beam spot in the diethyloxadicarbocyanine iodide jet, and thus varying the incident optical power density, we are able to obtain $N=1$, $N=2$, and $N=3$ solitons at 622 nm. These pulses are characterized by a modulated pulse-train envelope with the $(N-1)$ characteristic frequencies of a N soliton. In a recent paper[4] we have studied the evolution of the pulse temporal shape in $N=3$ soliton regime.

In order to study $N=2$ solitons, the beam spot in the diethyloxadicarbocyanine iodide jet was defocused until only one frequency (near 80 kHz) was observed in the pulse-train envelope modulation. The pulse autocorrelation function at different points of this period was then recorded with the experimental technique described in Ref. 4. The result is displayed in Fig. 2, together with the autocorrelation function averaged on the whole of the soliton period. One can only see a small variation of the amplitude of the wings and a small increase of the autocorrelation maximum between the beginning [Fig. 2(a)] and the middle [Fig. 2(c)] of the period. The fact that the pulse wings never disappear during the period seems to indicate that this $N=2$ soliton is not characterized by a sech^2 intensity temporal shape at the beginning of the period.

In order to obtain more information on the exact profile of this pulse, we have recorded the evolution of its spectrum along the soliton period. We have used an optical multichannel analyzer triggered synchronously with the pulse-train envelope modulation. This modulation was used to produce a very short high-voltage pulse which gated the microchannel plate intensifier. The gate width was about 10 ns which is less than the laser cavity round-trip time (12 ns). We can then obtain the spectrum of single pulses located at different points of the soliton period. Figure 3 shows the experimental recordings of the pulse spectrum evolution along one soliton period T_p. This figure is clearly consistent with the spectrum evolution of a nonsymmetrical $N=2$ soliton.

We have tried different combinations of poles and residues values in Eq. (3) in order to obtain a general evolution of the soliton autocorrelation and spectrum close to the experimental results sketched in Figs. 2 and 3. In all cases, the pulses are composed of a high peak followed by a small one. These two pulses exchange a part of their energy during their evolution. The time delay between the two pulses can be obtained from the autocorrelation trace and is about 250 fs. Such a pulse-shape evolution explains why we never obtained autocorrelation traces without wings. As exhibited in Fig. 1(b), the set of values $\eta_1=0.7$, $\eta_2=1.3$, $C_1=0.7$, $C_2=1.1$ gives a reasonable fit with experimental data for the spectrum. The agreement is not so good for the autocorrelation traces: The ratio between the peaks of Figs. 2(c) and 2(a) is clearly smaller than expected from the theoretical plot of Fig. 1(a). Several effects can explain this discrepancy. First, time jitter in the triggering of the autocorrelator[4] amounts to a decrease in the time resolution of the measurement, and therefore increases the

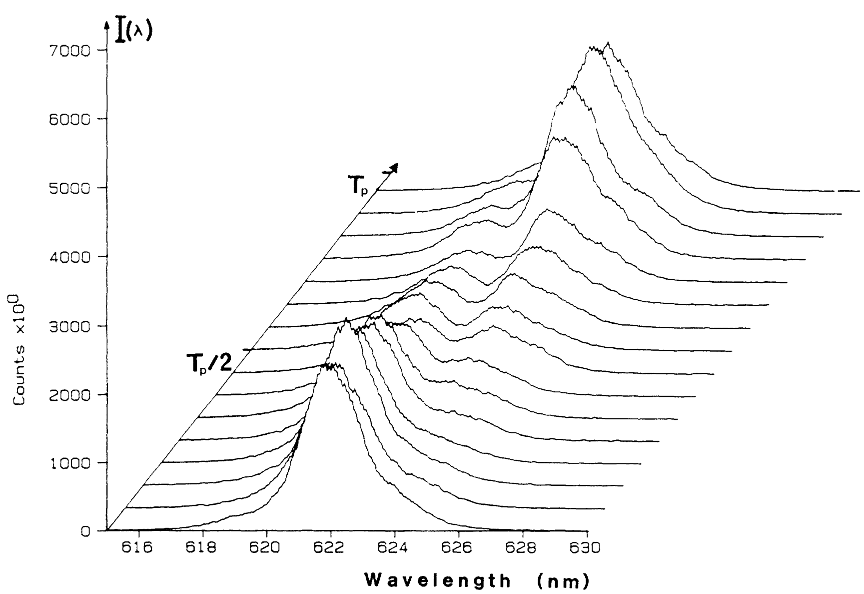

FIG. 3. Experimental recording of the pulse spectrum evolution during a soliton period. Note that the shift between each curve corresponds to about 70 cavity round trips. It can be deduced from the comparison between these results and Fig. 1 that the large intensity peak occurs before the small one.

width while decreasing the maximum of the peak. Second, the energy of the pulse actually changes during the soliton period, contrary to the hypothesis of the NLSE. This energy modulation is observed on the pulse-train envelope and corresponds to a minimum energy for the maximum predicted peak power ($t = T_p/2$). Both of these effects can account for a 40% decrease of the peak-value ratio, which gives fair agreement with experimental data.

It can therefore be concluded that the CPM laser produces pulses with changing temporal shapes, which can be described by theoretical results for a $N=2$ soliton, at least as a first approximation. This result suggests the use of a perturbation technique on the NLSE[5] in order to determine why and how the laser selects a particular type of soliton. Such an approach could include saturable gain and loss in order to explain the observed energy modulation as a periodic motion of the poles.[9,11] Moreover, some mechanisms nonsymmetrical relative to the pulse local time, such as self-phase modulation,[12] could explain the production of nonsymmetrical solitons.

In conclusion, we present here the first experimental observation of a nonsymmetrical soliton. We have performed a spectrum analysis of its evolution along the soliton period. This result indicates that, in this particular regime, a CPM laser produces double-peaked pulses with a shape that evolves with a period of 1100 cavity round trips. The remarkable consistency of these experimental results with the nonlinear Schrödinger equation (even if the nonlinear properties of the laser cavity are much more complex than those supposed in this equation) suggests a new approach for the theoretical description of CPM lasers.

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