Disordered Systems Which Escape the Bound $\nu \geq 2/d$

Recently Chayes, Chayes, Fisher, and Spencer[1] (CCFS) have derived the inequality $\nu \geq 2/d$ for the correlation-length exponent ν of a generic disordered system in d dimensions. Among the stated conditions are that the disorder should be uncorrelated on long length scales and that there should be a continuous transition with a diverging correlation length, ξ. This result raises the following question: When should a system be considered truly disordered? Indeed, one can often represent pure, uniform systems in a form that *appears* random or disordered. Does this imply that the uniform system should also satisfy $\nu \geq 2/d$?

To answer these questions and gain insight into the CCFS theorem consider the nearest-neighbor site-random Ising model with

$$-\mathcal{H} = J\sum_{[i,j]} \rho_i \rho_j s_i s_j \quad (s_i = \pm 1), \tag{1}$$

the ρ_i being independently distributed according to

$$P(\rho;\lambda,\tilde{\lambda}) = \lambda\delta(\rho-\tilde{\lambda}) + (1-\lambda)\delta(\rho - c\tilde{\lambda}); \tag{2}$$

here λ and $\tilde{\lambda}$ are variable *disorder parameters.*[1] The case $c=0$, $\tilde{\lambda}=1$ is the usual dilute Ising model, but consider, instead, $c=-1$: This is a form of the Mattis model. The gauge transformation $s_i' = \rho_i s_i/\tilde{\lambda}$ (all i) shows that (1) is equivalent to a pure Ising ferromagnet with $J' = \tilde{\lambda}^2 J$. Hence this disordered model has the pure exponents. A recent estimate for $d=3$ is[2] $\nu = 0.632 \pm 0.001$ which certainly violates $\nu \geq 2/d$!

Now one must note that CCFS address *only transitions which can be undergone by the variation of a disorder parameter.* Accordingly, let us fix $\tilde{\lambda}=1$ and consider the (T,λ) phase diagram. The gauge transformation is valid for all λ so that the critical locus, $T_c(\lambda)$, is independent of λ; i.e., the critical line parallels the λ axis and, hence, cannot be crossed by our varying λ! Thus the model fails the CCFS conditions and, whether regarded as disordered or not, can escape the inequality $\nu \geq 2/d$.

Conversely, fix λ ($\neq 0,1$) and vary $\tilde{\lambda}$: This is equivalent to the variation of T and does carry the system through the transition. What has failed? The answer is that varying $\tilde{\lambda}$ produces *singular changes* in the overall probability distribution, $\Pr_{\tilde{\lambda}}[\Omega \equiv \{\rho_i\}]$.[1] To see this, replace $\delta(x)$ by

$$\delta_\kappa(x) = (\kappa/\sqrt{2}\pi)\exp(-\tfrac{1}{2}\kappa^2 x^2);$$

then $\kappa \to \infty$ reproduces (2). Now follow the CCFS analysis: Their Eqs. (5)–(8) remain valid if a is replaced by κ and $n(\Omega)$ by $\sum_i \rho_i \tanh(\kappa^2\tilde{\lambda}\rho_i)$. For $\kappa\tilde{\lambda} \gg 1$ the Cauchy-Schwartz inequality[1] still yields a bound $\kappa|\Lambda|^{1/2}$ on $|d\Pr_{\tilde{\lambda}}[Y]/d\tilde{\lambda}|$. Then $\nu \geq 2/d$ follows[1] *provided* that $a=\kappa<\infty$. However, the model escapes the inequality in the Mattis limit $\kappa \to \infty$. It thus appears that an appropriate disorder parameter, λ, should yield *singularities in* $d\Pr_\lambda/d\lambda$ *no worse than* δ *functions.*

As to further applications it is clear that a critical locus parallel to the λ axis is not generic. If $(dT_c/d\lambda)_T \neq 0$ the transition can be undergone by the changing of λ and then $\xi \sim |\lambda - \lambda_c|^{-\nu_\lambda}$ with $\nu_\lambda \geq 2/d$. Typically, the transition will be universal along $T_c(\lambda)$ and the λ axis is not special: Varying T at fixed λ then yields $\xi \sim |T-T_c|^{-\nu_T}$ with $\nu_T = \nu_\lambda \geq 2/d$. This should apply to the usual spin-glass models.[3] When $(dT_c/d\lambda)_T = 0$ the exponent ν_T need not meet the bound (but may well do so).

At a *multicritical* point, (T_m,λ_m), with disorder new features arise. Near (T_m,λ_m) one expects scaling to hold with two distinct scaling axes and two correlation exponents, ν_1 and ν_2, one for an approach along each axis.[4] Generically, neither scaling axis is parallel to the λ axis: Then *both* ν_1 *and* ν_2 must obey the inequality. If the second scaling axis is normal to the λ axis one still has $\nu_1 \geq 2/d$; however, the second exponent need satisfy only $\nu_2 \geq 2/md$, where the full nonlinear scaling field[4] at $T=T_m$ varies as $|\lambda-\lambda_m|^m$ ($m=2,3,\ldots$). At the $d=3$ Ising spin-glass/ferro/para multicritical point[3] we find $\gamma_1 \simeq 3.0$ from series extrapolation[5] along the Nishimori line: Provided $\eta \geq -2.3$, which is hard to doubt, this implies $\nu_2 > 2/d$ as required. On the other hand, Nishimori estimates $\nu_T \simeq 0.51 \pm 0.06$ (and $2-\eta \simeq 2.0$) by the Monte Carlo renormalization-group method.[3] This value for ν_T is permitted provided the second scaling axis is *parallel to the T axis*[3]; otherwise it must be discounted.

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