# Bragg Scattering of Atoms from a Standing Light Wave

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We report the first observation of Bragg scattering of sodium atoms from a standing light wave. We also present a theory which quantitatively predicts the amplitude of the various Bragg orders as a function of the light's detuning and power, and the interaction time. The analog of the *Pendellösung* effect, previously observed in Bragg scattering of neutrons from crystals, is predicted and qualitatively observed for first-order Bragg scattering of atoms from a standing light wave

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Bragg scattering of x rays from crystal planes was demonstrated by W. H. Bragg and his son W. L. Bragg in 1912, in a series of experiments[1] which won them a Nobel prize in 1915. Bragg scattering of neutron de Broglie waves from crystal planes was first observed in 1946, leading to the field of neutron interferometry.[2,3] This Letter presents the first experimental observation of Bragg scattering of atoms from a standing light wave. This observation provides a beautiful example of the complementarity of particles and waves in that we treat the atomic beam as a wave and the intensity maxima of the standing light wave as crystal planes. Our observation also represents a breakthrough in the coherent manipulation of atoms, completing the technology necessary to construct an atomic interferometer (i.e., one which acts by interference of atomic-matter waves).

Momentum transfer to atoms by light in the absence of spontaneous emission results from an interaction of the induced dipole moment of the atom with the field gradient of the standing light wave. Quantum mechanically, the atom trades a photon via absorption and stimulated emission between the counterpropagating traveling waves which compose the standing light wave, thus gaining momentum in discrete units of $2\hbar k$, along the **k** vector of the standing light wave. One can also view this phenomenon as diffraction of an atomic de Broglie wave ($\lambda_{dB}=h/p$) from the *intensity* grating (periodicity $d_{light}=\lambda_{light}/2$) of the standing light wave.[4] Thus, constructive interference occurs at discrete angles given by $\phi=\lambda_{dB}/d_{light}$ which again results in momentum transfer to the atom in discrete units of $2\hbar k$.

The difference between Bragg scattering, in which the atoms scatter mainly into one order (i.e., final momentum state), and the previously observed Kapitza-Dirac scattering,[4-6] in which a large number of momentum states are populated, results from energy-momentum conservation. The absorption and stimulated emission of photon pairs changes the momentum but not the laboratory kinetic energy of the atom. The final momentum vectors must lie on a circle of radius $p_i$ in momentum space as shown in Fig. 1. The focused waist of a Gaussian light beam has a minimum Heisenberg uncertainty

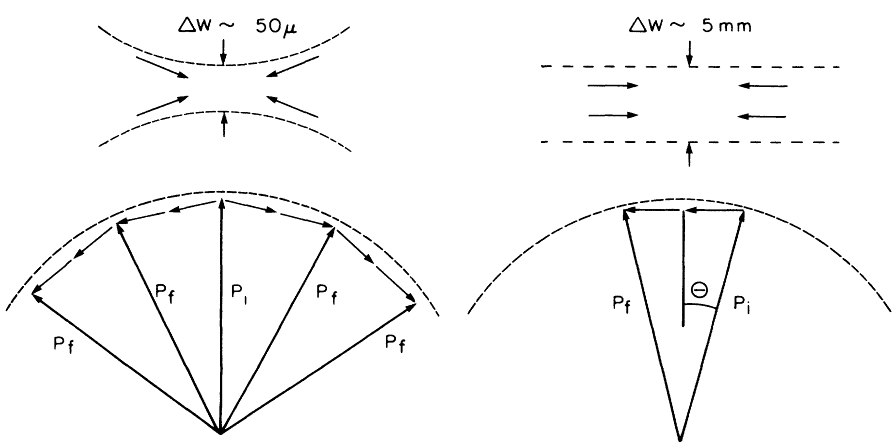


FIG. 1. Comparison of Kapitza-Dirac and Bragg scattering. A tightly focused waist (left) has a large angular uncertainty in the direction of its photons, thus allowing energy conservation for many final momentum states $p_f$—this is the Kapitza-Dirac regime. The observation of Bragg scattering requires a much larger waist for the light where the photons are highly collimated. In this case, the only process which can conserve energy and momentum is Bragg scattering (right).

 

between the rms waist size $\Delta w$ and the rms angular spread $\Delta\phi$ of the **k** vectors of photons traveling through this waist, $\Delta w \Delta p = \Delta w \hbar k \Delta\phi = \hbar/2$. For the previously observed Kapitza-Dirac scattering, the standing light wave was focused very tightly ($\Delta w \simeq 50$ $\mu$m) so that the angular uncertainty in the **k** vector of the light was much greater than the angle $\phi = \lambda_{dB}/d_{light}$ between diffracted orders. Consequently, the atom could scatter into many different orders and still conserve momentum and energy. The diffraction patterns remained symmetric about the initial atomic trajectory even when this trajectory was angled with respect to the nodes of the standing light wave.[6]

To observe Bragg scattering, the angular uncertainty of the photons must be less than the angle $\phi = 60$ $\mu$rad between diffracted orders, implying a minimum waist size for the standing light wave, $\Delta w \simeq 3$ mm. As shown on the right of Fig. 1, the only process which can conserve both momentum and energy is scattering in which the incident angle satisfies the Bragg condition, $m\lambda_{dB} = 2d_{light}\sin\theta$, where $m = 1,2,3,\ldots$ is the order. The Bragg condition allows momentum transfer *only* for discrete initial values of atomic momentum $p_x = m\hbar k$ along the **k** vector of the standing light wave, with transfer *only* to the state $p_x = -m\hbar k$.

Our experiment was done with the high-resolution apparatus used previously to observe Kapitza-Dirac diffraction of atoms by a standing light wave.[4-6] A monoenergetic sodium beam ($\Delta v/v = 11\%$ FWHM) is optically pumped into a pure, two-state system and then collimated with two 10-$\mu$m slits spaced 0.9 m apart. The standing light wave in the interaction region is constructed by retroreflection of a laser beam from a mirror. The angle $\theta$ between the atomic beam and the standing-light-wave nodes can be varied by means of a lead zirconate titanate lever arm on the mirror. The final momentum distributions of the atomic beam are detected 1.2 m downstream from the interaction region by a 25-$\mu$m scanning hot-wire detector. The momentum resolution of this apparatus is $0.9\hbar k$ (FWHM).

Experimental data of first-order Bragg scattering are shown in Figs. 2(a) and 2(b). When the interaction laser is blocked, all atoms are in the undiffracted state $p = 0\hbar k$. When the interaction laser is unblocked and the retroreflecting mirror positioned at an angle of 30 $\mu$rad with respect to the atomic beam, most of the population transfers to the diffracted momentum state $p = -2\hbar k$. For a constant detuning of the light beam, the population in this state first increases and then decreases with increasing laser power, with most of the population returning to the undiffracted state. Higher-order Bragg scattering has been observed for orders $m = 2$, 3, and 4. Figure 2(c) displays data of second-order Bragg scattering.

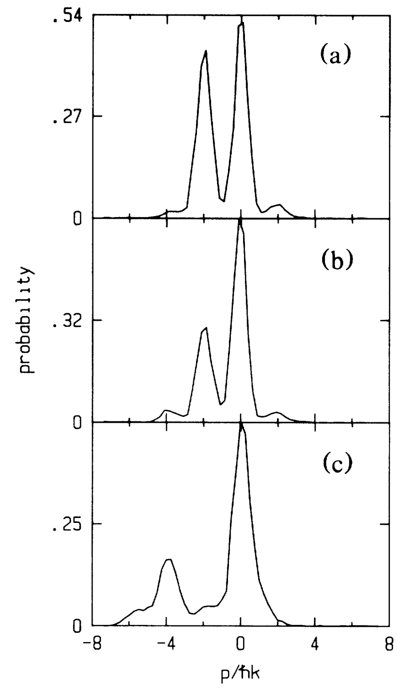


FIG. 2. Experimental data of Bragg scattering. (a) First-order Bragg scattering: $P = 6$ mW; $\Delta = 800$ MHz; $\tau = 6.4$ $\mu$s ($\Delta w = 3.2$ mm). (b) First-order Bragg scattering: $P = 10$ mW; $\Delta = 800$ MHz; $\tau = 6.4$ $\mu$s. (c) Second-order Bragg scattering: $P = 4$ mW; $\Delta = 500$ MHz; $\tau = 3.2$ $\mu$s ($\Delta w = 1.6$ mm). The angle of the standing-light-wave nodes with respect to the atomic beam was 30 $\mu$rad times the order, $m$.

Although there have been many theoretical papers on momentum transfer to atoms by light,[4-11] the theoretical treatment of Bragg scattering has only been treated in detail in Refs. 5 and 11. The theory for Bragg scattering of a two-state atom from a standing light wave with Gaussian field profile $f(t) = \exp[-(t/\tau)^2]$ follows from an extension of the theory given in our previous papers.[4-6] We observe the interaction from the frame of Fig. 1 that sees equal frequencies in the counterpropagating light beams. The Schrödinger equation would appear as in Ref. 7. To account for the initial momentum of the atomic beam, we expand the ground-state probability amplitude as

$$a_1(t) = e^{in_0kx}\sum_n a_{1,n}(t)\,e^{inkx}$$

with initial conditions $n_0 = p_x/\hbar k$ and $a_{1,n}(t=-\infty) = \delta_{n,0}$. In these expressions, $p_x$ is the initial momentum of the atom along the **k** vector of the standing light wave. It is important to note that $n_0$ need not be an integer. In the regime where the detuning $\Delta$ is much greater than the spontaneous linewidth and the peak traveling-wave Rabi frequency $\Omega_0$, the excited-state amplitude is small and may be eliminated yielding

$$i\dot{a}_{1,n} = [\Omega_0^2 f^2(t)/4\Delta](a_{1,n-2} + 2a_{1,n} + a_{1,n+2}) + [\hbar k^2(n+n_0)^2/2M]a_{1,n}, \tag{1}$$

where $a_{1,n}(t)$ is the probability amplitude for the ground-state atom to have momentum $p_x = n\hbar k$. A unitary transformation

$$a_{1,n} = b_{1,n} \exp\left[-in\pi/4 - (i\Omega_0^2/2\Delta)\int_{-\infty}^{t} f^2(t')\,dt'\right]$$

results in a Raman-Nath equation[12,13]

$$\dot{b}_{1,n} = [\Omega_0^2 f^2(t)/4\Delta](b_{1,n-2} - b_{1,n+2}) - [i\hbar k^2(n+n_0)^2/2M]b_{1,n}. \tag{2}$$

The energy levels of the ground-state momentum states indicated by Eq. (2) are shown in Fig. 3; they fall along a parabola.[5] The initial condition, $n_0$, is experimentally controlled by the angle between the atomic beam and the standing-wave nodes [$n_0 = p_x/\hbar k = \theta/(\hbar k/p_i) = \theta/(30\ \mu\text{rad})$]. First-order Bragg scattering ($n_0 = -1$) can be viewed as an absorption and stimulated emission process from the undiffracted ($n + n_0 = -1$) to the diffracted ($n + n_0 = 1$) momentum eigenstate. Second-order Bragg scattering ($n_0 = -2$) can be viewed as two absorption and stimulated emission processes from the $n + n_0 = -2$ to the $n + n_0 = 2$ momentum state via the nearly resonant $n + n_0 = 0$ state. In general, there will be an energy resonance between the initial state $n = 0$ and the state $n = -2|n_0|$. All other states are nonresonant but may be excited in our experiment because of transit-time broadening resulting from the Gaussian profile of the standing light wave ($\Delta\nu = 90$ kHz FWHM for our waist parameter of $\tau = 6.4\ \mu$s).

Although Eq. (2) has no general analytical solution,[13] a few approximations yield an analytical solution for first-order Bragg scattering. If population transfer is limited to the transition $n + n_0 = -1 \rightarrow n + n_0 = 1$ [i.e., the set of $n$ equations (2) is truncated to $n + n_0 = -1$ and $n + n_0 = 1$], then the equations are homomorphic with the rate equations describing a two-level system. Consequently, the probability of our finding the atom in the diffracted momentum state ($n + n_0 = 1$) is

$$P(t=\infty) = |b_{1,2}(t=\infty)|^2 = \sin^2\left[\frac{\Omega_0^2}{4\Delta}\int_{-\infty}^{\infty} f^2(t)\,dt\right] = \sin^2\left[\frac{\Omega_0^2\tau}{4\Delta}\left(\frac{\pi}{2}\right)^{1/2}\right]. \tag{3}$$

To account for the finite momentum resolution of our apparatus as well as the effects of population transfer to the nonresonant neighboring momentum states, the set of equations given by Eq. (2) was numerically solved with the initial condition

$$g(n_0) = \frac{1}{(2\pi)^{1/2}\Delta n_0} \exp\left[-\frac{(n_0+1)^2}{2(\Delta n_0)^2}\right],$$

where $\Delta n_0 = 0.38$ is the momentum resolution of the apparatus. The finite resolution of the apparatus leads to diminished probability of transfer to the diffracted state, although the probability of transfer to the diffracted state still varies sinusoidally with power and maximum momentum transfer still occurs when $(\Omega_0^2\tau/4\Delta)(\frac{1}{2}\pi)^{1/2} = \frac{1}{2}\pi$. The numerical solutions displayed negligible population transfer to the neighboring nonresonant $n + n_0 = -3$ and $n + n_0 = 3$ states for the interaction times used in the experiment.

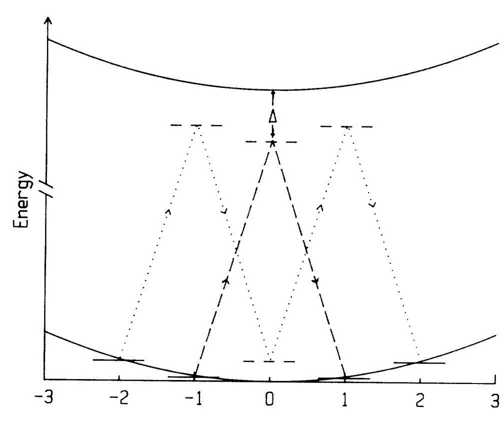


FIG. 3. Energy levels of the momentum eigenstates determined by their respective kinetic energies; $E_n = (n+n_0)^2 \times (25$ kHz). First-order Bragg scattering (dashed lines) can be viewed as a two-photon transition from the undiffracted ($n + n_0 = -1$) to the diffracted ($n + n_0 = 1$) state. Second-order Bragg scattering (dotted lines) can be viewed as two two-photon transitions from the undiffracted ($n + n_0 = -2$) to the diffracted ($n + n_0 = 2$) state.

The population of the diffracted momentum state predicted by this theory is shown in Fig. 4 as a function of laser power ($\propto \Omega_0^2\tau$) for first-order Bragg scattering. Also shown are experimental results of the diffracted-state population for various laser powers. The experiment agrees qualitatively with theory in that the functional form of momentum transfer as a function of laser power (for constant detuning) seems correct. However, imperfections in the experimental apparatus (e.g., polarization imperfections and light-beam aberrations) lead to a reduction in the amount of momentum transfer to the diffracted state and also from the diffracted state back to the undiffracted state. Although spontaneous decays

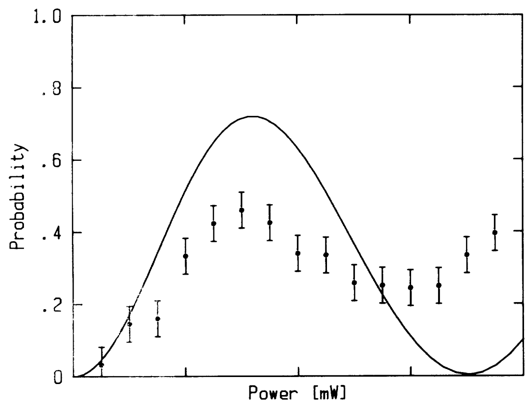


FIG. 4. Plot of probability to be in the diffracted $(n+n_0=1)$ state as a function of laser power, $\Omega_0^2\tau$, for first-order Bragg scattering $(n_0=-1)$: $\Delta=800$ MHz and $\tau=5$ $\mu$s $(\Delta w=2.5$ mm) for these plots. Also shown are experimental data of population in the diffracted state for various laser powers.

were suppressed by detuning far from resonance[4] (the average number of spontaneous decays $\bar{N} \leq 0.1$ for these experiments) the residual spontaneous decay could account for some of the discrepancy between theory and experiment at the highest powers. Measurements of diffracted population versus light power done at different detunings agree qualitatively with theory in the same manner as shown in Fig. 4.

The measurements shown in Fig. 4 display the *Pendellösung* effect, previously observed in Bragg scattering of neutrons from crystals.[14] For Bragg scattering of neutrons from crystals, this effect results in a sinusoidally varying probability between the diffracted and undiffracted waves as the length of the crystal (which has a fixed potential) is increased. For first-order Bragg scattering of atoms from a standing light wave, the *Pendellösung* effect results in a sinusoidally varying probability between the diffracted and undiffracted atomic waves as the power $(\propto \Omega_0^2\tau)$ of the standing light wave is increased as noted above.

The observation and control of Bragg scattering of atomic waves will be useful for the development of an atomic interferometer.[15] Scan (a) of Fig. 2 basically displays an atomic "beamsplitter"—a device which coherently splits an atomic de Broglie wave into two spatially distinct waves. One can reflect these beams and then recombine them with another Bragg-scattering interaction region, thus constructing a Mach-Zender atomic interferometer. This device could be used to measure the ground-state phase shift of an atomic beam, due to electric, magnetic, or gravitational fields, blackbody radiation, the Casimir shift,[16] scattering from atoms (where the real part of the forward scattering cross section could be measured), or rotation of the interferometer (Sagnac effect).

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