Gauge Model of Generation Nonuniversality Revisited

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Six years ago, a gauge model of generation nonuniversality was proposed in which the τ lifetime was predicted to be longer, and B^0-$\bar{B}^0$ mixing to be greater, than in the standard model. Since current experimental data appear to favor both of these interpretations, an updated and improved analysis is here presented. In particular, a plausible branching fraction of the order 10^{-4} is predicted for $b \rightarrow sl^+l^-$, just below the present experimental limit and well above the standard-model expectation of 10^{-6}.

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Electron-muon universality is a well-established phenomenon. With the discovery of the τ lepton, it is generally assumed that it also interacts in exactly the same way as the e and μ. This means that given the $\tau \rightarrow e\bar{\nu}_e\nu_\tau$ branching fraction, one can predict the τ lifetime. Of course, both quantities are measured experimentally and the most recently reported[1] world average of the τ lifetime is

$$\tau_\tau^{\text{expt}} = (3.02 \pm 0.08) \times 10^{-13}\ \text{s}, \quad (1)$$

which is about 2.5 standard deviations away from the theoretical prediction of

$$\tau_\tau^{\text{th}} = (2.79 \pm 0.06) \times 10^{-13}\ \text{s}. \quad (2)$$

This discrepancy, if confirmed by more data, would signal the breakdown of generation universality and require a change in our understanding of the physics involved. Actually, all this was already anticipated six years ago, when a gauge model of generation nonuniversality was proposed,[2] where e-μ universality is the result of a mass-scale inequality, $v_{03} \ll v_{12}$, in much the same way as strong isospin is the result of $m_u, m_d \ll 1$ GeV. However, e-μ-τ universality is not mandatory and some deviation is to be expected.

The model is based on the group $\mathrm{U}(1) \otimes \mathrm{SU}(2)_1 \otimes \mathrm{SU}(2)_2 \otimes \mathrm{SU}(2)_3$ with a gauge coupling g_0, g_1, g_2, g_3. The Higgs boson are doublets under $\mathrm{U}(1) \otimes \mathrm{SU}(2)_i$ and self-dual quartets under $\mathrm{SU}(2)_j \otimes \mathrm{SU}(2)_k$ with vacuum expectation values v_{0i} and v_{jk}. The left-handed fermions are doublets under $\mathrm{U}(1) \otimes \mathrm{SU}(2)_i$, with each generation coupling to a separate SU(2). The right-handed fermions are singlets coupling only to U(1). Mixing among quarks is given by two 3×3 unitary matrices U and D:

$$(u', c', t')^{\mathrm{T}} = U(u, c, t)^{\mathrm{T}}, \quad (3)$$

and

$$(d', s', b')^{\mathrm{T}} = D(d, s, b)^{\mathrm{T}}, \quad (4)$$

where the primed (unprimed) states are interaction (mass) eigenstates. The lepton states are assumed to be unmixed. The electromagnetic coupling is given by

$$e^{-2} = g_0^{-2} + g_1^{-2} + g_2^{-2} + g_3^{-2}; \quad (5)$$

the Fermi weak coupling is generalized to a matrix

$$\left(\frac{4G_{\mathrm{F}}}{\sqrt{2}}\right)_{ij} = \begin{cases} \dfrac{1}{v_{03}^2}, & i=3 \text{ or } j=3 \text{ or both}, \\ \dfrac{1}{v_{03}^2} + \dfrac{1}{v_{13}^2 + v_{23}^2}, & \text{otherwise}, \end{cases} \quad (6)$$

so that any weak interaction involving the third generation has its effective strength reduced by

$$\xi^{-1} = (v_{13}^2 + v_{23}^2)/(v_{03}^2 + v_{13}^2 + v_{23}^2), \quad (7)$$

and

$$\sin^2\theta_W = (1 - e^2/g_0^2) - (e^2/g_3^2)(1 - \xi^{-1}). \quad (8)$$

One additional parameter,

$$C = (e^4/g_3^4)(1 - \xi^{-1})\xi^{-1}, \quad (9)$$

is present in this model. With use of Eqs. (5), (8), and (9), it can be seen that

$$0 < C < \sin^4\theta_W(\xi - 1). \quad (10)$$

Experimentally, C can be constrained by $e^+e^- \rightarrow \mu^+\mu^-$ cross-section data because it appears in the effective interaction

$$(4G_F/\sqrt{2})[(j^{(3)}-\sin^2\theta_W j^{em})^2+C(j^{em})^2]. \quad (11)$$

Using $\xi^2=\tau_\tau^{expt}/\tau_\tau^{th}$ and Eqs. (1) and (2), we find

$$1-\xi^{-1}=0.039\pm0.016, \quad (12)$$

and

$$0<C<0.003\ (0.004), \quad (13)$$

where $\sin^2\theta_W=0.23$ has been used and 1σ (2σ) limits are indicated. The constraint on C from $e^+e^-\to\mu^+\mu^-$ data at present is not as restrictive as Eq. (13), and so in the following discussion, C will be allowed to vary according to Eq. (10).

Recently, a nonzero and somewhat surprisingly large mixing in the B^0-$\bar{B}^0$ system was reported.[3] The parameter

$$r=(x^2+y^2)/(2+x^2-y^2), \quad (14)$$

where $x=\Delta m_B/\Gamma_B$ and $y=\Delta\Gamma_B/2\Gamma_B$, was measured as

$$r=0.21\pm0.08. \quad (15)$$

This goes against the long-held expectation in the standard model that r should be unobservably small, although many authors are quick to point out after the fact that if all the uncertainties, both experimental and theoretical, are stretched to their limits, this result is still allowed as long as m_t is greater than 50 GeV or so. On the other hand, an observable B^0-$\bar{B}^0$ mixing was definitely predicted in Ref. 2. In addition to the usual second-order charged-current contribution to Δm_B, there is now a first-order neutral-current contribution given by

$$\frac{\eta_B\Delta m_B}{m_B}=\frac{8G_F}{3\sqrt{2}}|D_{bd}^*D_{bb}|^2(1-\xi^{-1})f_B^2B_B, \quad (16)$$

where f_B is the B^0 decay constant and B_B the so-called bag factor. Taking the position that the standard-model contribution to Δm_B is as expected, i.e., small, we assume in the following that the fraction η_B is in fact near 1. In Eq. (14), y^2 is expected to be negligible; hence x^2 can be evaluated from Eq. (15). Using the most recently reported[4] world average of the B lifetime,

$$\tau_B=(1.18\pm0.14)\times10^{-12}\ \mathrm{s}, \quad (17)$$

we then find

$$\Delta m_B=(4.1\pm1.1)\times10^{-13}\ \mathrm{GeV}, \quad (18)$$

and

$$|D_{bd}^*D_{bb}|^2=(4.0\pm2.0)\times10^{-6}(\eta_B/1.0)[(150\ \mathrm{MeV})/f_B\sqrt{B_B}]^2. \quad (19)$$

Now $|D_{bb}|$ should be near 1, and so $|D_{bd}^*|$ is about 2×10^{-3} in this model.

Next we consider $K_L\to\mu^+\mu^-$. The total rate has a well-known absorptive part coming from $K_L\to\gamma\gamma\to\mu^+\mu^-$. The remainder,

$$\Gamma_{disp}=\Gamma(K_L\to\mu^+\mu^-)-1.2\times10^{-5}\Gamma(K_L\to\gamma\gamma), \quad (20)$$

has contributions from the dispersive $\gamma\gamma$ amplitude and the usual second-order weak interaction as well as the first-order neutral-current interaction in this model given by

$$(\eta_K')^2\Gamma_{disp}=(G_F^2f_K^2m_Km_\mu^2/16\pi)(1-4m_\mu^2/m_K^2)^{1/2}(1-\xi^{-1})^2|D_{bd}^*D_{bs}|^2. \quad (21)$$

Using[5] the K_L lifetime of 5.183×10^{-8} s and branching fractions of $(9.1\pm1.9)\times10^{-9}$ and $(4.9\pm0.4)\times10^{-4}$, respectively, for $K_L\to\mu^+\mu^-$ and $K_L\to\gamma\gamma$, we find

$$|D_{bd}^*D_{bs}|^2=1.94\times10^{-8}[(1.0\pm0.6)/(1.0\pm0.4)^2](\eta_K'/0.5)^2, \quad (22)$$

where $f_K=159$ MeV has been used and the large errors in Γ_{disp} and $(1-\xi^{-1})^2$ are indicated separately. The fraction η_K' is arbitrarily estimated at 0.5 in the belief that it should not be negligible, as otherwise $|D_{bs}|$ would be too small and not in keeping with the natural hierarchy[2]

$$|D_{bd}|\ll|D_{bs}|\ll|D_{bb}|\simeq1. \quad (23)$$

Comparing Eq. (22) against Eq. (19), we then obtain

$$|D_{bs}|^2=4.85\times10^{-3}[(1.0\pm0.6)/(1.0\pm0.5)](\eta_K'/0.5)^2(1.0/\eta_B)[f_B\sqrt{B_B}/(150\ \mathrm{MeV})]^2, \quad (24)$$

which shows that $|D_{bs}|$ is about 0.07 in this model. The errors in Eq. (24) are less severe than in Eq. (22) because one factor of $1-\xi^{-1}$ drops out when we take the ratio. Because of unitarity, $|D_{bs}|$ should be very nearly equal to $|D_{sb}|$. Now the $b\to c$ transition is characterized in the standard model by the matrix element V_{cb}, whose magnitude is evaluated[4] at 0.046 ± 0.006 by use of the B lifetime. In this model,

$$|V_{cb}|=|U_{uc}^*D_{db}+U_{cc}^*D_{sb}+U_{tc}^*D_{bb}\xi^{-1}|\simeq|D_{sb}+U_{tc}^*\xi^{-1}|. \quad (25)$$

Hence a value of the order 0.07 for $|D_{bs}|$ as given by Eq. (24) is not unexpected. If a number much greater or smaller than $|V_{cb}|$ is found, that would be much less convincing. In the above, if the preliminary value $(6.05\pm0.04\pm0.08)\times10^{-4}$ for the $K_L^0\to\gamma\gamma$ branching fraction by the NA31 Collaboration is used instead, we only need to change η'_K from $\frac{1}{2}$ to $\frac{2}{3}$ and all other estimates will remain the same. Also, B_s^0-$\bar{B}_s^0$ mixing should be maximal in this model.

We now come to our major result, namely the process $b\to sl^+l^-$. The effective interaction is

$$(G_F/2\sqrt{2})(1-\xi^{-1})D^*_{bs}D_{bb}\bar{s}\gamma_\mu(1-\gamma_5)b\bar{l}\gamma^\mu(a+\gamma_5)l, \tag{26}$$

where $a=-1+4\{\sin^2\theta_W-[C/(\xi-1)]^{1/2}\}$. Hence

$$\Gamma(b\to sl^+l^-)/\Gamma(b\to c\bar{\nu}l^-)=\tfrac{1}{4}(1+a^2)(1-\xi^{-1})^2|D^*_{bs}D_{bb}/V_{cb}|^2, \tag{27}$$

where a relative phase-space factor of 2 has been included. If we take $|D_{bs}|^2=4.85\times10^{-3}$, $|D_{bb}|=1$, $|V_{cb}|=0.046$, $1-\xi^{-1}=0.039$, $0.08<|a|<1$, and a branching fraction[5] of 0.12 for $b\to c\bar{\nu}\bar{l}$, then

$$B(b\to sl^+l^-)=(1\text{ to }2)\times10^{-4}. \tag{28}$$

Of course, the large errors in $|D_{bs}|^2$ and $1-\xi^{-1}$ have not been included, but we believe the above estimate to be a plausible one, provided that there is indeed a discrepancy between the measured τ lifetime and the standard-model prediction. Experimentally, the exclusive decay $B\to Kl^+l^-$ is known[6] to have an upper limit on its branching fraction of the order 10^{-4}. If it is a significant component of the inclusive decay, then it should be observable in the near future at electron-positron storage rings such as the Cornell Electron Storage Ring and DORIS at DESY. In contrast, the standard-model prediction[7] for $B(B\to Kl^+l^-)$ is of the order 10^{-6}, and it is not expected to be enhanced by quantum chromodynamics or by the existence of supersymmetric particles. However, if m_t is as large as 200 GeV, a branching fraction of the order 10^{-5} is possible.[8] Existence of a fourth generation may also make it go up.[9] Of course, there will be a corresponding rise in the $b\to s\gamma$ rate for these cases, and $B\to K^*\gamma$ may be observable at the 10^{-3} level instead of 10^{-4}, as predicted[10] in the standard three-generation model with modest values of m_t. On the other hand, there can be no first-order neutral-current contribution to $b\to s\gamma$, so that $B\to K^*\gamma$ is not expected to be enhanced beyond 10^{-4} in the generation-nonuniversality model. Going back to Δm_B, we note that if m_t is indeed large, then η_B will be smaller than 1. Keeping ξ fixed will then lead to a smaller $|D_{bd}|$ and consequently a larger $|D_{bs}|$ if η'_K is unchanged. This means that $B(b\to sl^+l^-)$ will be greater than 10^{-4}, in potential conflict with data. Other rare decays such as $K^+\to\pi^+\nu\bar{\nu}$ are not enhanced in this model.

Since we assume that there is no need to revise the standard-model parameters which predicted a small Δm_B, we must also check that there is very little new contribution to Δm_K. The analog of Eq. (16) is

$$\eta_K\Delta m_K/m_K = (8G_F/3\sqrt{2})|D^*_{bd}D_{bs}|^2(1-\xi^{-1})f_K^2B_K. \tag{29}$$

Comparing this with Eq. (21) for $K_L\to\mu^+\mu^-$, we find

$$\eta_K=\frac{128\pi\Gamma_{\text{disp}}(1-4m_\mu^2/m_K^2)^{-1/2}B_K(\eta'_K)^2}{3\sqrt{2}G_Fm_\mu^2\Delta m_K(1-\xi^{-1})^2} = 0.02\frac{(1\pm0.6)}{(1\pm0.4)}\left(\frac{B_K}{0.4}\right)\left(\frac{\eta'_K}{0.5}\right)^2, \tag{30}$$

which shows clearly that the first-order neutral-current contribution to Δm_K is indeed small. Hence there is also no need to revise the standard-model prediction for ϵ'/ϵ, which is in agreement with the recently reported[11] preliminary experimental result

$$\epsilon'/\epsilon=(3.5\pm0.7\pm0.4\pm1.2)\times10^{-3}. \tag{31}$$

As for other measurable effects of this model, both $e^+e^-\to\tau^+\tau^-$ and $e^+e^-\to b^-b$ forward-backward asymmetries should be reduced by the factor $\xi^{-1}=0.961\pm0.016$. Current experimental results[12] normalized to the standard-model predictions are

$$A_\tau=\begin{cases}0.84\pm0.16\text{ (SLAC PEP)},\\0.91\pm0.11\text{ (DESY PETRA)},\end{cases} \tag{32}$$

and

$$A_b=1.08\pm0.29, \tag{33}$$

where corrections due to B^0-$\bar{B}^0$ mixing have been included. Obviously, the errors at present are too big to be decisive tests of this model.

The observed W and Z bosons are to be identified with the first set of weak gauge bosons in this model, with masses M_{W_1} and M_{Z_1}. Let M_W and M_Z be the standard-model predictions; then

$$M^2_{W_1}/M^2_W\simeq1+\sqrt{C}[s^2(\epsilon-1)^{1/2}-\sqrt{C}]/\xi s^4\le1+\tfrac{1}{4}(1-\xi^{-1}), \tag{34}$$

$$M^2_{Z_1}/M^2_Z\simeq1+\sqrt{C}[s^2(\xi-1)^{1/2}-(1-s^2\xi)\sqrt{C}]/\xi s^4(1-s^2)\le1+[4(1-s^2)]^{-1}(1-\xi^{-1})/(1-s^2\xi), \tag{35}$$

and

$$M_{Z_2}^2 \simeq M_{W_2}^2 \simeq \xi s^4 M_W^2/\sqrt{C}\,[s^2(\xi-1)^{1/2}-\sqrt{C}\,] \geq 4M_W^2/(1-\xi^{-1}), \quad (36)$$

where $s^2=\sin^2\theta_W$. Using $\xi^2=1.082\pm0.037$ and $s^2=0.23$, we then find the following 1σ (2σ) bounds:

$$M_{W_1}/M_W - 1 < 0.007\ (0.009), \quad (37)$$

$$M_{Z_1}/M_Z - 1 < 0.012\ (0.015), \quad (38)$$

$$M_{Z_1}\cos\theta_W/M_{W_1} - 1 < 0.009\ (0.011), \quad (39)$$

$$M_{Z_2} \simeq M_{W_2} > 680\ (600)\ \text{GeV}, \quad (40)$$

where $M_W=80$ GeV is assumed in the estimation of M_{W_2} and M_{Z_2}. The deviations from the standard model in the W and Z masses are thus at most of the order 1 GeV, well within experimental errors at present. Precision measurements of M_{Z_1}, at e^+e^- colliders in the near future as well as those of $M_{Z_1}-M_{W_1}$ at hadron colliders will be important tests of this model.

Finally we mention that lepton mixing is certainly also allowed in this model. Hence exotic processes such as $\mu\rightarrow eee$ are possible, and present experimental limits can be used to bound the mixing angles in this sector. In addition, there could be intergenerational fermions[13] which transform nontrivially under two different SU(2) factors. They may even be considered the supersymmetric partners of the Higgs-boson quartets which are already present in this model. In the near future, better measurements of the τ lifetime as well as improvements in B-decay statistics will be crucial tests of the idea of generation nonuniversality as presented in this paper.

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