Large-Scale Structure of the Deconfined Phase

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Quantum chromodynamics at finite temperature and chemical potential is investigated nonperturbatively at large distance scales in terms of its three-dimensional effective theory, which for the gauge group SU(2) is the Georgi-Glashow-Polyakov model. It is shown that a global SU(2) symmetry breaks if the temperature is high enough and it is argued that this new transition most likely coincides with deconfinement. Some implications for quark-gluon–plasma physics are discussed.

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It is now widely recognized (see, e.g., Svetitsky[1]) that the quark-gluon plasma is nonperturbative over distance scales sensitive to the singular infrared behavior of finite-temperature Yang-Mills theory. That such singularities could cause insurmountable problems for perturbation theory was first pointed out by Linde,[2] who showed that observables could not be perturbatively computed beyond certain characteristic orders, e.g., $O(g^6)$ for the thermodynamic potential. Subsequently, theorists resigned themselves to doing the best they could up to such orders, arguing that at a sufficiently high temperature T, the incalculable higher orders would not be important anyway because of the smallness of the running coupling $g(T)$. However, even this limited calculability has been called into question by recent work on the Debye screening mass.

As is well known, in perturbation theory m_D^2 starts out at the one-loop value $O(g^2)$.[3,4] Gauge-invariant corrections to this value have recently been shown[5,6] to be perturbatively incalculable because of an increase in the effective coupling at distance scales beyond the naive one-loop Debye length. This is consistent with the confining nature of the infrared sector of hot QCD as indicated by the area-law behavior of spacelike Wilson loops at high temperature.[7]

There have been indications[8-10] that despite this confining behavior, the deconfined phase may be characterized by a Higgs-type condensate in the electric sector, which would provide an elegant mechanism for curing its infrared divergences. It was argued[6] that if such a condensate were indeed present, high-temperature perturbative results could be rendered invalid even before the onset of the Linde infrared problem, thus explaining the results of Ref. 5. Since the existence of the condensate is inherently undecidable in perturbation theory, a nonperturbative analysis of the infrared sector of hot QCD is required.[6]

In this Letter, I report on some preliminary investigations in this direction. In order to isolate the infrared dynamics responsible for the nonperturbativity, I have chosen to work at temperatures T that are so high that the running coupling $g(T)$ is small. I can then focus on the infrared sector (momenta $\ll T$) by perturbatively integrating out those modes which do *not* suffer from infrared divergences, namely the so-called nonstatic modes, which include all the fermionic modes. If this integration were also to include the static (zero Matsubara frequency) modes, then, as is well known, all ultraviolet divergences could be removed by essentially the same counterterms as at $T=0$. However, when these counterterms are applied to the nonstatic integrals alone, some divergences survive the renormalization. These residual ultraviolet divergences play the role of counterterms of the effective theory for the static modes, showing up in the guise of its *bare* parameters. They will be cancelled when the static sector is taken into account, reflecting the ultraviolet finiteness of the full theory.

The effective action is found to have the three-dimensional (3D) form[4]

$$S=\int d^3x[(2G^2)^{-1}\mathrm{Tr}F^2(\mathbf{A})+\mathrm{Tr}(\mathcal{D}\phi)^2+m_0^2\,\mathrm{Tr}\phi^2+\tfrac{1}{2}\mu(\mathrm{Tr}\phi^2)^2],$$

where $\mathbf{A}$ represents the magnetostatic potential and ϕ (an adjoint scalar field) the electrostatic potential. The parameters of S can be calculated for the general case of N colors and N_f quark flavors (each having a chemical potential μ_i, $i=1,\ldots,N_f$). A power-counting analysis[4] shows that the (bare) mass parameter m_0^2 needs only be computed to two loops in the nonstatic modes and the induced quartic coupling constant μ to one loop and that couplings higher than the quartic need not be considered. The 3D gauge coupling is $G=g(T,\mu_i)\sqrt{T}$, which does not run as long as T and μ_i are fixed.[4] I shall henceforth suppress the μ_i dependence of g for simplicity. The couplings m_0^2 and μ are given by[4,5,11]

$$m_0^2=m_E^2-\frac{2NG^2}{(2\pi)^3}\left[\int\frac{d^3k}{\mathbf{k}^2}\right]_{\text{regularized}}+(\text{two-loop terms}),\quad \mu=\left[\frac{5N^3-(2N^2-3)N_f}{N(N^2+1)}\right]\frac{G^4}{6\pi^2T},$$

where I have assumed a general 3D ultraviolet regularization, m_E^2 is the (naive) one-loop Debye mass squared,[4] and the two-loop corrections to m_0^2 include logarithmic ultraviolet divergences. (In accordance with the remarks made above, the choice of 3D regularization is arbitrary and need not coincide with that of the 4D theory.) The expression for μ is a new result[11]; details of the calculation will be provided elsewhere. In the special case $N=2$, $N_f=0$, the formula reduces to $\mu=2G^4T/3\pi^2$, a result obtained previously by several authors.[8,9,12]

To economize and simplify still farther, let us choose the gauge group SU(2). Our effective theory is then the 3D Georgi-Glashow model, whose large-scale properties in the Higgs phase are well known as a result of analytical studies initiated by Polyakov.[13] In a classic paper, he showed that its infrared structure is dominated by a Coulomb gas of 3D instantons, which in the present context can be thought of as 4D static 't Hooft–Polyakov monopoles, with long-range interactions which disorder the system. The naively broken global SU(2) symmetry is thereby restored, producing U(1)-type confinement and reconciling the apparently contradictory requirements of confining and Higgs-type behavior. His arguments were later modified when it was discovered that there exists, in addition to the above Coulomb-gas interaction, an always attractive "elastic" force mediated by the Higgs field, which can lead to a monopole condensate if the Higgs-boson mass gets small enough to be comparable to the inverse mean free monopole separation.[14] The observed instability appeared to be characteristic of a first-order transition; however, it should be noted that this dilute-gas approximation used in these calculations breaks down near the critical Higgs-boson mass.

More recently, Dahlem[9] recognized that these results were applicable to finite-temperature SU(2) gauge theory under the assumption of a Higgs condensate. A two-loop perturbative result of Anishetty,[8] which yielded a nontrivial minimum for the Higgs effective potential, was combined with the above dilute-gas approximation to obtain an infrared-finite expression for the thermodynamic potential. It was estimated that the instability of the Higgs phase would occur when the gauge coupling became of order unity, i.e., somewhere around the deconfinement transition. An abrupt change in the monopole density at deconfinement together with the appearance of a Higgs condensate was observed by Polonyi and Wyld[10] in SU(3) Monte Carlo simulations, though for statistical reasons their results can only be regarded as indicative.

The only approximation that can be rigorously justified is that of perturbatively integrating out the nonstatic modes at high temperatures. Having thus arrived at the effective theory, my goal is to establish the Higgs phase without further reliance on perturbative or semiclassical results. I therefore examine the 3D Georgi-Glashow model on the lattice and show that is has a two-phase structure; here I use numerical simulations only in a confirmatory capacity. I describe how the 4D finite-T theory induces renormalization-group flows in the space of bare couplings of the 3D theory, each fixed-T curve providing a possible continuum limit of the model. I show that at sufficiently high T these flows lie in the Higgs phase. Finally I discuss the results.

Let us forget for the moment the connection of the 3D model with finite-temperature physics, and explore its phase structure on the lattice. If we choose unitarity gauge, the partition function after suitable rescaling can be written as

$$Z(\beta,\kappa,\lambda)=\int[\rho^2e^{-\rho^2}d\rho][dU]e^{-S_{\rm eff}[\rho,U]},$$

where

$$-S_{\rm eff}[\rho,U]=\beta\sum_{\rm plaqs}\tfrac{1}{2}\,{\rm Tr}U_{\rm plaq}+\kappa\sum_{\rm links}\rho(n)\rho(n+i)\,\tfrac{1}{2}\,{\rm Tr}[U_i^\dagger(n)\sigma^3U_i(n)\sigma^3]-\lambda\sum_{\rm sites}\rho^4(n).$$

The rescaling relations which connect the bare parameters of the continuum and lattice theories are

$$G^2=4/a\beta,\quad m_0^2=(2/a^2)(\kappa^{-1}-3),$$
$$\mu=8\lambda/a\kappa^2,\quad {\rm Tr}\phi^2=\kappa\rho^2/2a.$$

A semiquantitative picture of the phase diagram can be obtained by examining the model at its boundaries in (β,κ,λ) space; here I shall only give the details of immediate relevance. At $\beta=\infty$, links are frozen to pure gauge values and the plaquette term drops out. The model reduces to a theory of interacting O(3) spins with radial degrees of freedom damped by the λ term, and we can borrow standard results from the theory of critical phenomena (see, e.g., Ma[15]). For $\lambda=0$ we have the Gaussian model which becomes unphysical for $\kappa>\frac{1}{3}$. (In fact, an examination of the functional integral shows that this result holds for *any* value of β. Thus, for $\lambda=0$ there is a line of physicality at $\kappa=\frac{1}{3}$ stretching from $\beta=0$ to $\beta=\infty$.) For $\lambda>0$ we have an SU(2)-breaking second-order transition which moves smoothly away from $\kappa=\frac{1}{3}$ with increasing λ. The position of the critical line can be derived[15] by analysis of the linearized renormalization-group equations near the Gaussian fixed point. If we convert the spherical momentum cutoff of Ref. 15 to our cubic lattice cutoff, the critical line is given in terms of the lattice parameters by

$$\kappa_c=\tfrac{1}{3}+\tfrac{1}{6}[(1+120c_L\lambda)^{1/2}-1],$$

where the constant c_L characterizes the cubic lattice cutoff and is given by

$$c_L \equiv \frac{a}{(2\pi)^3}\left[\int \frac{d^3k}{\mathbf{k}^2}\right]_{\text{lattice}} = \frac{2}{\pi^3}\int_0^{\pi/2}\frac{dx_1\,dx_2\,dx_3}{\sum_i \sin^2 x_i} \cong 0.253.$$

Preliminary Monte Carlo simulations confirm the above analysis, with the transition showing up in the expected places as a steep increase in the thermal average of the link action; details of the completed investigation will be reported elsewhere. The resulting phase diagram of the 3D Georgi-Glashow model consists of a critical surface $\kappa_c(\beta,\lambda)$ which separates a disordered phase characterized by full SU(2)-type confinement (small κ) from an ordered phase characterized by a condensate of the Higgs field as well as by confinement arising from the compact U(1) subgroup (large κ).

To obtain a nontrivial continuum limit of the lattice Georgi-Glashow model, the fixed point ($\beta=\infty$, $\kappa=\frac{1}{3}$, $\lambda=0$) must be approached in a delicate manner dictated by renormalization-group analysis, with some physical mass parameter held fixed. Rather than attempt a general analysis of this three-parameter theory, let us return to the original purpose and consider the Georgi-Glashow-Polyakov model as the infrared limit of SU(2) gauge theory at high temperatures. The mass parameter I shall hold constant is of course the temperature. At fixed T, κ and λ are specific functions of β and accordingly define a curve in parameter space. Thus, the finite-temperature connection provides a one-parameter family of curves in the phase diagram, each curve labeled by a value of T. Let us consider these curves in greater detail.

Converting the bare parameters of the continuum effective theory to their lattice counterparts for $N=2$, we find

$$\beta = 4[aTg^2(T)]^{-1},$$

$$\kappa = [3 - 8c_L/\beta + O(\ln\beta/\beta^2)]^{-1},$$

$$\lambda = [(8-N_f)g^2(T)/24\pi^2](\kappa^2/\beta),$$

where c_L is the constant encountered earlier; the negative sign in front of it will be crucial. The function $g(T)$ is well known from asymptotic freedom: At one loop it is given for SU(2) by $g^2(T) = 12\pi^2/(11-N_f)\ln(T/\Lambda_{\text{QCD}})$.

In the continuum limit $a \to 0$ ($\beta \to \infty$), these curves flow towards the Gaussian fixed point $\kappa=\frac{1}{3}$, $\lambda=0$. Do they approach it from above the critical surface or from below? To answer this question, expand the curves around the fixed point in powers of β^{-1}. Now let the critical surface in the same region be given by $\kappa=\kappa_c(\beta^{-1},\lambda)$. Simple differential geometry then tells us that the curves will lie above the critical surface when

$$\frac{8c_L}{9} > \left(\frac{\partial\kappa_c}{\partial\beta^{-1}}\right)_{\lambda=0} + \frac{1}{9}\frac{(8-N_f)g^2(T)}{24\pi^2}\left(\frac{\partial\kappa_c}{\partial\lambda}\right)_{\beta=\infty}.$$

From our earlier discussion of the form of κ_c near the fixed point, we know that the first term on the right-hand side vanishes, while for the second term we have $\partial\kappa_c/\partial\lambda = 10c_L$. The critical coupling of the finite-temperature theory is then given by

$$(8-N_f)g^2(T_c)/24\pi^2 = \tfrac{4}{5}.$$

(Note that while κ_c depends on c_L, the above relation is independent of the cutoff procedure, as it should be. This is an important check on the calculations.) For $N_f=0$ this gives $g^2(T_c) \simeq 24$, and so the theory is certainly not in the perturbative regime at the transition temperature and the value of T_c it gives can at best be viewed as an order-of-magnitude estimate. On the other hand, we can certainly conclude that if T is sufficiently large a Higgs condensate must be present.

It would not be too surprising if the SU(2)-breaking transition turned out to be the 3D vestige of deconfinement, since a Higgs condensate is necessary for the stability of the monopole gas, while a condensation of the latter could provide the electric "Meissner effect" believed to cause confinement. But since the 3D theory becomes effective only at very high temperatures, such a connection must be made externally. One can argue as follows: T_c cannot be much higher than the deconfinement temperature; otherwise the effective theory would have self-consistently predicted a small value for $g(T_c)$. So the SU(2) breaking occurs either at zero temperature or at deconfinement or at some other not too high temperature. The last possibility is unlikely since numerical simulations have not revealed any structure in that regime. The second is favored over the first because of the Monte Carlo results of Polonyi and Wyld[10] mentioned earlier, which connect the phenomena of deconfinement, which breaks the global $Z(N)$ center symmetry, and electrostatic condensation and/or monopole-gas formation, which breaks global SU(N).

The occurrence of an electrostatic condensate affects the large-scale degrees of freedom of the quark-gluon plasma, drastically altering its collective properties. Let us briefly consider one important aspect viz. how our picture of Debye screening might change. In the absence of symmetry breaking, the naive dimensional-reduction–electrostatic-decoupling scenario (see, e.g., Ref. 4) would hold and perturbation theory would give reliable results for the Debye mass at sufficiently low orders. In the actual case of broken symmetry, only a reduced version of Debye screening due to the residual Higgs field remains, for which perturbative results are inadequate.[6] The results of Polyakov,[13] appropriately translated, suggest magnetic screening masses $O(g^2T)$, a U(1) string tension $O(g^4T^2)$, and a Debye screening mass considerably smaller than the naive perturbative one-loop value. This is a far cry from the conventional picture of the quark-gluon plasma.

While the present approach has permitted an economical focus on the infrared dynamics, it has made connections to the full theory difficult in the transition regime. Therefore I have avoided addressing here such issues as whether the two phases are analytically connected at strong coupling: These are interesting within the context of the 3D model but the answers may be irrelevant to finite-T physics. Nevertheless, the main result, that the vacuum of SU(2) gauge theory at high temperatures is nontrivial over large distance scales, is clearly independent of the use of the effective theory. The latter has, in fact, a decided advantage over the full 4D theory in that dynamical quark fields can be integrated out perturbatively (even at finite chemical potential), avoiding the usual worries of putting them on the lattice. Indeed, dynamical quarks have been insignificant to the results, since I have been concerned with the breaking of gauge as opposed to center symmetry. Global gauge-symmetry breaking might then characterize the plasma transition even for full QCD, though I do not yet know whether it would survive as a true phase transition. This possibility is under investigation.

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