Josephson Effect and Quantum Phase Slippage in Superfluids

O. Avenel

Service de Physique du Solide et de Résonance Magnétique, Centre d'Etudes Nucléaires de Saclay, 91191 Gif-sur-Yvette, Cedex, France

and

E. Varoquaux

Laboratoire de Physique des Solides, Université Paris-Sud, 91405 Orsay, France

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We describe the operation of a hydromechanical resonator immersed in ^{4}He and ^{3}He-B superfluids and opened on the main bath by two holes, a micro-orifice acting as a weak link and a long parallel channel. Transitions between adjacent quantum states differing by one hydrodynamic circulation quantum can be induced coherently, yielding a succession of steps arranged in a staircase pattern. In ^{3}He at about 0.8 mK and 0 bar, a nearly ideal, nondissipative hydrodynamic Josephson effect can be observed.

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Josephson suggested in 1962[1] that supercurrents would develop between two superconductors weakly connected via a tunnel junction. At zero applied voltage, a quantum-mechanical phase difference $\delta\phi$ would generate a dc current, given, in the case of *ideal* superconductive tunneling, by

$$J = J_c \sin\delta\phi, \tag{1}$$

J_c being the critical current in the weak link. Also, an electrochemical potential difference across the tunnel junction, $\delta\mu$, would lead to the appearance of an ac current the frequency and magnitude of which are governed by Eq. (1) with

$$\hbar\,\delta\dot{\phi} = -\delta\mu. \tag{2}$$

These suggestions were quickly confirmed by experiments which showed furthermore that suitable weak links are not limited to pure tunnel junctions but can be made in a large variety of ways—point contacts, microbridges, etc.—with geometrical dimensions which may be comparable in size to, or larger than, the superconductive coherence length ξ.[2] In these latter cases, the current-phase relation is not purely sinusoidal, not even necessarily single valued, but it does retain a periodicity of 2π in the phase. Such a J-$\delta\phi$ relation can be described in a phenomenological way[2,3] with the help of a nonideality parameter α by

$$J_w = J_c \sin\zeta, \quad \delta\phi = \zeta + \alpha\sin\zeta. \tag{3}$$

For $\alpha = 0$, the ideal case of Eq. (1) is recovered. For $\alpha \gg 1$, the J-$\delta\phi$ characteristics are indistinguishable from parallel slanting straight segments. Equation (3) provides an interpolation scheme between these two limits and describes in a convenient, if not necessarily very accurate, way both the *hysteretic* case ($\alpha > 1$), where the J-$\delta\phi$ curve folds back on itself and where dissipation occurs, and the purely *dispersive* case ($\alpha < 1$).

The discovery of this new class of effects in superconductors led Richards and Anderson,[4] on the basis of the deep-rooted analogy between superconductivity and superfluidity,[5,6] to get started a long-lasting search for the corresponding phenomena in superfluid ^{4}He and, later, in superfluid ^{3}He-B in which the theoretical picture[7,8] for Josephson-type effects is more satisfactory than in ^{4}He. Anderson introduced the unifying concept of *phase slippage*[6] to account for the motion of the phase resulting from Eq. (2) both for mere ideal fluid acceleration and when vortices move across the potential flow lines. This concept also applies to the ideal case discussed by Josephson and to phase slips produced by depairing effects.[2] The features common to these different cases, included in (3), are the requirement of phase coherence throughout the superfluid and the periodicity by 2π of the basic properties of the weak link.

This experimental search was largely inconclusive in both ^{4}He[9] and ^{3}He[10] until individual phase slips could be observed in ^{4}He.[11] These last experiments were carried out in a hydromechanical resonator whose dynamical behavior was governed by a single micro-orifice.[11-13] They showed that, at the critical velocity threshold, the weak-link energy E_w (which, in the case of ^{4}He, is simply the flow kinetic energy) is reduced, when a phase slip takes place, by the quantity $\Delta E_w = \kappa_0 J_c$, κ_0 being the quantum of circulation. These experiments brought about the first observation of individual phase-slip events in a superfluid.

Here we report on observations made in a double-hole hydrodynamic resonator which bridge the gap between these quantized dissipative events and a nearly ideal case described by Eq. (3) with $\alpha < 1$. This double-hole cell is a miniature chamber closed by a flexible membrane, immersed in the main superfluid bath and connected to it by two different paths, a weak link (of $0.3 \times 5\ \mu m^2$ bored in a 0.2-μm-thick nickel foil) and a long channel (in the shape of a 0.25-mm-i.d., 5-mm-long hole partially filled with a 0.22-mm-o.d. wire). The size of the weak link, is large with respect to the coherence length in ^{4}He ($\xi \approx 1.5$ Å) but of the same order as that of ^{3}He ($\xi \approx \hbar v_F / 2\pi k_B T_c \approx 700$ Å at zero pressure and temperature).

We monitor the membrane displacement d with the help of a highly sensitive superconducting gauge. The superfluid current created by an overall rotation of the cell would also be of interest; since we cannot perform such an operation, we have no direct handle on this physical quantity. When the resonator is excited by our applying an external drive to the membrane, the phase difference between the two baths, $\delta\phi$, as measured along the long channel, increases according to the ac Josephson equation (2) with $\delta\mu = v\delta P$, v being the atomic volume for ^{4}He and twice the atomic volume for ^{3}He, and δP the hydrostatic pressure difference. The critical velocity regime is reached first in the weak link, whereas the flow in the long channel remains ideal: $J_l = \rho_s s_l v_l$, where ρ_s is the superfluid density, s_l the area of the long channel, and v_l the superfluid velocity. This velocity can be written, by definition of the effective (or hydraulic) length c_l, as $v_l = \kappa_0 \delta\phi / 2\pi c_l$. The fluid in this Helmholtz-type system has a natural resonance at a frequency Ω_H which varies as $[\sigma(L_l^{-1} + L_w^{-1})]^{1/2}$, σ being the elastic constant of the membrane and L_l, L_w the inertances of the two holes ($L_i = \rho c_i / \rho_s s_i$). In this cell, Ω_H is mainly determined by the long hole (i.e., $L_l \ll L_w$). The total fluid mass displaced by the flexible membrane, of area S and moving with velocity $\dot{d}$, is equal (if we neglect compressibility and assume normal fluid clamped) to the sum of the superfluid mass flow through both holes:

$$\rho S \dot{d} = J_w + \rho \kappa_0 \delta\phi / 2\pi L_l. \tag{4}$$

As J_w, given by Eq. (3), is periodic in $\delta\phi$, $\dot{d}$ exhibits in terms of $\delta\phi$ a constant mean slope to which is superposed a (weak) periodic modulation. The pitch of this modulation is given, letting $\delta\phi \to \delta\phi + 2\pi$ in Eq. (4), by $\Delta\dot{d} = \kappa_0 / L_l S$. Since the system is resonant with a high quality factor, d ($\simeq \dot{d}/\omega$) will also show, above the critical amplitude threshold, a superstructure with regular stepwise increases the height of which is given by

$$\Delta d = \kappa_0 / L_l S \omega. \tag{5}$$

This periodic superstructure in the response of resonant systems perturbed by a weak quantum link results in the well-known "staircase pattern" as first discussed by Zimmerman, Thiene, and Harding[14] for electrodynamic rf-SQUID's, and by Zimmermann and co-workers[9] in superfluid resonators.

Such staircase patterns, as observed for the first time in both ^{4}He at 10 mK[15] and ^{3}He-B in the vicinity of $0.8T_c$ at zero pressure in the cell described above, are shown in Figs. 1–3. These patterns are obtained in the following way. The resonator is driven at frequency ω close to Ω_H by our applying to the membrane a voltage which is the sum of a dc bias V_{dc} and a small ac component V_{ac}. The power fed into the resonator is proportional to $V_{dc} V_{ac} \dot{d}$ (and therefore increases more rapidly than V_{ac}). The output of the displacement gauge is read with a fast digital voltmeter which yields, after processing, a full-wave peak rectification of the response. This peak absolute amplitude of the response at successive half-cycles is plotted in the figures versus the drive level V_{ac} which is slowly ramped up with time.

Let us first examine staircase patterns in ^{4}He, shown in Fig. 1 at $T = 10$ mK. Above the critical velocity threshold, corresponding to a critical phase difference $\delta\phi_c$ of about $26 \times 2\pi$, steps develop with an exact regularity in amplitude which corresponds to Eq. (5) within the accuracy of the displacement gauge calibration [$\simeq 30\%$ (Ref. 11)]. The apparent "scatter" on the plateaus is not random noise but is mainly due to phase slip events taking place in the weak link as shown in the insets in Fig. 1. These slips, which are the isolated dissipation

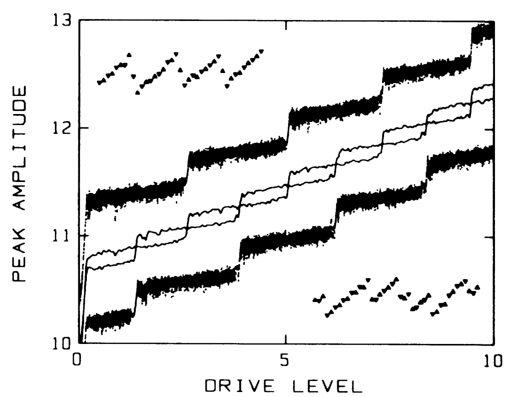

FIG. 1. Absolute peak amplitude of the resonator oscillation at each half-cycle vs drive level in ^{4}He at 10 mK, zero pressure, and $\Omega_H = 6.4$ Hz. The scale units are arbitrary but consistent between this figure and Figs. 2 and 3. The scale of the peak amplitude, as labeled in the figure, applies to the interwoven staircase pattern constructed by superposition of the two patterns shown above and below, shifted vertically by ± 0.5 scale unit, obtained for two values of the phase differences at rest close to 0 (top) and π (bottom). For clarity, the superposed patterns have been filtered digitally. The single, shifted patterns display the scatter on the plateaus as recorded. This apparent scatter results in part from the occurrence of elementary phase slips (Ref. 11) as shown on magnified portions of the first plateaus in the upper left and lower right corners for 0 and π biasing phases, respectively. (Scales have been multiplied by 100 along the abcissas and 3 along the ordinates.) The triangles represent the signed (for up and down directions) peak absolute amplitude at successive half-cycles while the drive level is slowly ramped up in time. Jumps occur in pairs of opposite direction at successive half-cycles for 0 bias as predicted in Ref. 9, one at a time but with alternating directions for bias of π.

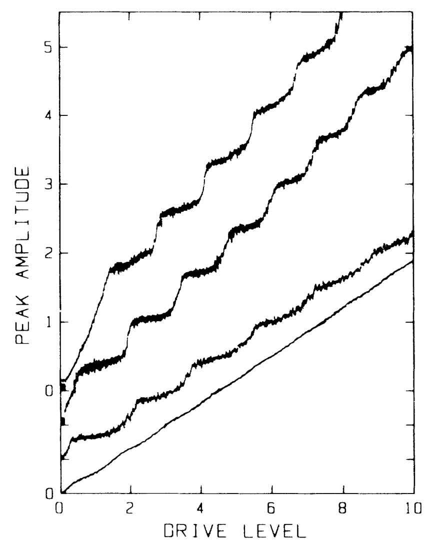

FIG. 2. Peak amplitude vs drive level in ^{3}He-B at various temperatures. From bottom to top, 2.825 Hz, $0.89T_c$; 3.60 Hz, $0.83T_c$; 4.43 Hz, $0.77T_c$; and 5.40 Hz, $0.69T_c$ for zero bias. Frequencies are shifted from resonance (see Fig. 3) by comparable amounts at the various tempertures.

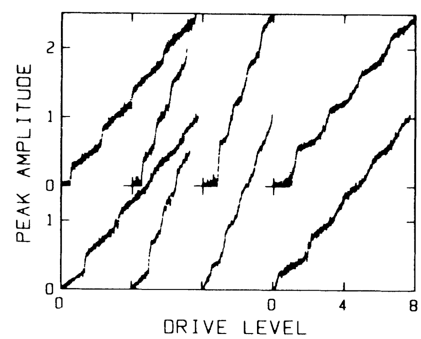

FIG. 3. Peak amplitude vs drive level in ^{3}He-B at $0.83T_c$ and varying frequency for $\delta\phi_x=0$ (bottom) and $\delta\phi_x=\pi$ (top) at 3.3, 3.4, 3.5, and 3.6 Hz from left to right. The patterns have been shifted and truncated for clarity.

events studied in Ref. 11, correspond to the discontinuous jumps from one determination to an adjacent one when Eq. (3) is multivalued (i.e., when $\alpha > 1$). As the drive level increases, the phase in the weak link slips more and more frequently until a pair of $+2\pi$ and -2π slips occurs at *each* half-cycle, exhausting the possibilities of slips with overall phase excursions of $\pm\delta\phi_c$. At this drive level, the total excursion of $\delta\phi$ (defined along the long channel) increases by 2π above $\delta\phi_c$, giving rise to a step in the response with a height given by Eq. (5).[9,14] Since many well-defined steps can be tracked (more than fifty) and since the rest state of the system (see below) remains unaltered when the drive is ramped up and down, the phase slips have to be identical to one another and they have to preserve accurately the phase coherence in the superfluid system: Phase slips in the weak link correspond to transitions between macroscopic quantum states differing by one quantum of circulation and have to be by 2π exactly, in accordance with Anderson's concept of phase slippage.[6]

The phase coherence discussed above may, however, be broken by our applying drive levels 3 orders of magnitude larger than the drive at the critical threshold. Severe overdriving causes incomplete (i.e., non-2π) phase slips to occur, possibly by leaving a vortex pinned part of the way across the flow path. Any value of the residual phase difference when the diaphragm is brought back to rest, $\delta\phi_x$, which arises from the remnant bias current trapped in the loop formed by the two holes and which simulates an applied rotation, may be selected by trial and error. Figure 1 displays two situations in which $\delta\phi_x$ is close to 0 and to π. The superposition of these two limiting cases yields an unmistakable interwoven pattern. The trapped bias current shows no decay in time. On most instances, the system is found in a state of zero bias upon cooling down through the λ transition.

To study superfluid ^{3}He-B, we increased the sensitivity of the displacement gauge by a factor of 10. We also had to reduce the background level of mechanical noise even further. The temperature, obtained by nuclear demagnetization, is measured with a cerium-doped lanthanum magnesium nitrate thermometer calibrated at 12 mK and at the superfluid transition temperature T_c. The transition is signaled by a rapid readjustment of the membrane to hydrostatic equilibrium when ^{3}He becomes superfluid in the long hole. The Helmholtz frequency, which varies as $\rho_s^{1/2}$, is at first very low, a fraction of a hertz, and increases when the temperature decreases. Patterns with the periodic structure of Eq. (5) are shown in Fig. 2 at various temperatures, starting from about $0.9T_c$ where they can first be distinguished. At that temperature, the patterns are smooth, mostly free of scatter, and the critical phase difference $\delta\phi_c$ is a fraction of 2π only. When the temperature is lowered, $\delta\phi_c$ increases while plateaus with marked edges and apparent "scatter" on the flat parts develop, indicating the onset of *dissipative* energy jumps. The staircase patterns in ^{3}He become similar to those in ^{4}He and so do the phase slips, which can also be resolved experimentally. These qualitative features alone indicate clearly that the system evolves from a purely *dispersive* regime ($\alpha < 1$) at high temperature to a dissipative regime at low temperature closer to the hard hysteretic case found in ^{4}He.

This conclusion is further borne out by the observed frequency dependence of the shape of the patterns, presented in Fig. 3 at $T \sim 0.83T_c$ for values of the phase difference at rest $\delta\phi_x$, again obtained by overdriving, close to 0 and π. The pattern shape (but not the step size) is seen to be dependent on the value of the frequency shift from resonance. Such a frequency dependence is not present in the hard hysteretic regime of ^{4}He with dissipative phase slips (apart from the trivial dependence of the average slope). This dispersion effect reflects the change of the resonance frequency of the hydrodynamical system when α in Eq. (3) is not large with respect to 1.

A particularly striking situation is met at this temperature ($0.83T_c$) when the weak link is biased at a phase difference of π. When the resonator is excited from its idle state with low drive levels, the membrane registers no net flow of fluid at first but only a chaotic motion of small amplitude: The ideal potential flow generated in the long channel by the applied pressure difference is nearly exactly balanced by the *counterflow* taking place in the weak link. This cancellation effect, which is close to complete at this particular temperature, shows clearly that classical hydrodynamics is no longer at work.

All these observations can be analyzed quantitatively in terms of a model based on Eqs. (2)–(4) to which is added the equation accounting for the balance of force on the membrane. Numerical simulations, which we hope to describe elsewhere, yield best fitting values for the nonideality parameter α (a fully undetermined parameter here since J_c and L_l can be evaluated separately). In the situation of Fig. 2 at $0.89T_c$, where the temperature-dependent coherent length is of the order of 2000 Å, α is found to be about 0.5, confirming that the coupling through the weak link is purely dispersive. These observations establish experimentally the existence in a superfluid of the direct analogs of the electrodynamic Josephson effects.

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