Are There Any Superstrings in Eleven Dimensions?

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Covariant actions are proposed for classical superstrings immersed in eleven space-time dimensions, by construction of simple Chern-Simons terms. Local world-sheet variables are used which are space-time vectors, Majorana spinors, and antisymmetric tensors.

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Strings may be viewed geometrically as immersions of a two-dimensional world sheet $\mathcal{M}_2(z)$ into a D-dimensional ambient space-time manifold $\mathcal{M}_D(x)$. Superstrings[1] require an additional spinor structure θ on $\mathcal{M}_D$, and seem to be the most interesting and consistent models.

Alternatively, strings may be regarded simply as two-dimensional field theories where the local fields $\phi(z)$ transform as specific representations under the symmetries of the ambient space-time manifold. Previous string theories have used ϕ's which were either space-time scalars, spinors, or vectors, the latter variables being the obvious choice from the geometrical point of view. However, from the field-theory viewpoint, there is no obvious reason to avoid ϕ's which are more complicated space-time tensors or spin-tensors. The only immediately apparent problems are the usual ones: Find enough symmetries to eliminate all ghosts in the Minkowski-space formulation of any such models, and/or demonstrate the absence of conformal anomalies in the Euclidean space formulation.

In this rather speculative Letter, I propose some new string models, especially for $D=11$, which use antisymmetric space-time tensors, e.g., $y^{\mu\nu}$, in addition to space-time spinors and vectors, θ and y^μ. These tensors are interpreted as "stringy" generalizations of the well-known local gauge fields.[2] These models were found within a classification of all immersions, $\mathcal{M}_d(z) \subset \mathcal{M}_D(x)$, $d \leq D$, which admit local supersymmetries.[3] Such "superimmersions" are defined by actions which are invariant under transformations whose parameters $\kappa(z)$ are space-time spinors with nontrivial dependence on the submanifold coordinates.

The original example of a superimmersion is the superparticle.[4] Besides superstrings, the next example to be found was that of a three-dimensional object immersed into six space-time dimensions,[5] $\mathcal{M}_4(z) \subset \mathcal{M}_6(x)$, where the world volume swept out by the object is four dimensional. More recently, several other superimmersions have appeared in the literature,[6] including a beautiful example with $\mathcal{M}_3(z) \subset \mathcal{M}_{11}(x)$, where the space-time is allowed to be curved.

The problem of classifying such models constructed only from θ and x variables has apparently now been solved.[7] Classically at least, there exist four fundamental superimmersions: $\mathcal{M}_3(z) \subset \mathcal{M}_4(x)$, $\mathcal{M}_4(z) \subset \mathcal{M}_6(x)$, $\mathcal{M}_6(z) \subset \mathcal{M}_{10}(x)$ and $\mathcal{M}_3(z) \subset \mathcal{M}_{11}(x)$. An elementary discussion of the geometrical and dynamical properties of these fundamental cases may be found in Ref. 3. A number of other examples may be obtained by dimensional reduction,[8] including the well-known superstrings.

On the basis of prior experience with these known superimmersions, invariance under local κ transformations would seem to be a good consistency test for a model which can be checked at the classical level, prior to quantization. In this Letter, only such classical considerations are made, although it is obvious that the critical dimension determined from quantum consistency checks must be reevaluated, and will, in general, change as a result of the presence of the additional antisymmetric tensor variables.

The set of variables to be used for a string immersed in eleven dimensions is suggested by supergravity[9] which involves the *elfbein*, the gravitino, and an independent torsion, $(e^\mu_M, \psi_M, A^{\mu\nu}_M)$, where μ and ν refer to the space-time manifold while M refers to the tangent space. By analogy, I introduce string variables $(y^\mu, \theta, y^{\mu\nu})$, with θ of Majorana type, and define differential forms which are invariant under the constant spinor transformations $\delta\theta=\epsilon$, $\delta y^\mu = i\bar\epsilon\Gamma^\mu\theta$, $\delta y^{\mu\nu}=\bar\epsilon\Gamma^{\mu\nu}\theta$. With $\Gamma^{\mu\nu}\equiv(\Gamma^\mu\Gamma^\nu - \Gamma^\nu\Gamma^\mu)/2$, these differentials are

$$\pi^\mu \equiv dy^\mu - i\bar\theta\Gamma^\mu d\theta, \quad \pi^{\mu\nu} \equiv dy^{\mu\nu} - \bar\theta\Gamma^{\mu\nu} d\theta. \tag{1}$$

Historically, it would be more conventional to consider the supergravity variables $(e^M_\mu, \psi_\mu, A^{MN}_\mu)$, which are obviously related to the previous set. In this form, a connection to the proposed string variables is obtained by contraction with a space-time vector, V^μ, since *a priori*, there is no reason to identify y^μ with the space-time coordinates x^μ. Natural choices for V^μ would involve derivatives of x^μ. For example, one could take the extrinsic curvature $g^{ab}\mathcal{K}^\mu_{ab}$, where $\mathcal{K}^\mu_{ab}$ is the globally supersymmetric generalization of the second fundamental form[10] for the submanifold $\mathcal{M}_2$. Intuitively, this just amounts to an infinitesimal thickening of the world sheet. A finite extension of this choice for V^μ would involve a line integral $\int dx^\mu$ along a curve orthogonal to the surface, which certainly suggests a connection to the supermembrane theory mentioned above [a more technical connection is established below; cf. Eq. (6) and the accompanying remarks]. Alternatively, the string theory proposed here may be related to the limit of infinitesimally narrow membranes. Since membranes are notoriously more difficult to quantize, however, it would seem to be preferable to try to deal with this narrow limit directly.

Therefore, let us define a one-parameter induced metric on the world sheet for the case of a flat ambient space-time,

$$g_{ab} \equiv \pi_a^\mu \pi_b^\mu + c\pi_a^{\mu\nu}\pi_b^{\mu\nu} \tag{2}$$

where $\pi^\mu \equiv \pi_a^\mu dz^a$, $\pi^{\mu\nu} \equiv \pi_a^{\mu\nu} dz^a$. I will fix the parameter c below $(=-1/10)$. The obvious extension of the Nambu area-law action would now be $\int d^2z\sqrt{-g}$, $g=\det(g_{ab})$. However, as in the superstring and supermembrane cases, this action does not behave well under the local κ transformations. I define these to be

$$\begin{gathered}\delta\theta=\kappa(z),\quad \delta y^\mu=-i\bar\kappa\Gamma^\mu\theta,\quad \delta y^{\mu\nu}=-\bar\kappa\Gamma^{\mu\nu}\theta,\\ \delta\pi^\mu=-2i\bar\kappa\Gamma^\mu d\theta,\quad \delta\pi^{\mu\nu}=-2\bar\kappa\Gamma^{\mu\nu}d\theta.\end{gathered} \tag{3}$$

From (2) we then have

$$\delta\sqrt{-g}=-2i\sqrt{-g}\,g^{ab}\bar\kappa(\pi_a^\mu\Gamma^\mu-ic\pi_a^{\mu\nu}\Gamma^{\mu\nu})\partial_b\theta. \tag{4}$$

Now, the set of κ's for which this vanishes seems to be uninteresting since $(\Gamma^\mu\pi_a^\mu-ic\Gamma^{\mu\nu}\pi_a^{\mu\nu})\kappa=0$ is a very strong constraint,[3] just as $\Gamma^\mu\pi_a^\mu\kappa=0$ implies $\kappa=0$ for nonlightlike π^μ's.

As in the usual superstring and supermembrane cases, to find a more interesting local structure on the world sheet, I search for closed, exact three-forms[11] to add Chern-Simons terms to the action. A combination of forms which suffices is immediately found to be

$$f=i\pi^\mu d\bar\theta\Gamma^\mu d\theta-\tfrac{1}{10}\pi^{\mu\nu}d\bar\theta\Gamma^{\mu\nu}d\theta. \tag{5}$$

This is closed and exact (the terms quartic in θ actually cancel) as a consequence of the eleven-dimensional Fierz identity for Majorana spinors,

$$10\Gamma^\mu d\theta(d\bar\theta\Gamma^\mu d\theta)+\Gamma^{\mu\nu}d\theta(d\theta\Gamma^{\mu\nu}d\theta)=0. \tag{6}$$

This identity is in turn an immediate consequence of the identity needed to construct a locally supersymmetric membrane theory in eleven dimensions:

$$\Gamma^{\mu\nu}d\theta(d\bar\theta\Gamma^\mu d\theta)+\Gamma^\mu d\theta(d\theta\Gamma^{\mu\nu}d\theta)=0.$$

The relation in (6) follows by contraction of the latter with Γ_ν, using $\Gamma_\nu\Gamma^{\mu\nu}=-10\Gamma^\mu$. [However, note that in general, a relation like (6) is possible even when the membrane condition fails, e.g., when $D=15$.]

Under the local transformations in (3), using (6), we have

$$\delta f=-2i\,d[\pi^\mu\bar\kappa\Gamma^\mu d\theta+\tfrac{1}{10}i\pi^{\mu\nu}\bar\kappa\Gamma^{\mu\nu}d\theta]. \tag{7}$$

Integration over a three-dimensional slab, $\mathcal{M}_2\otimes[0,1]$, in the usual way then gives

$$\delta f=\int_{\mathcal{M}_2}d^2z[-2i\epsilon^{ab}\bar\kappa(\pi_a^\mu\Gamma^\mu+\tfrac{1}{10}i\pi_a^{\mu\nu}\Gamma^{\mu\nu})\partial_b\theta]. \tag{8}$$

On comparison with (4), the obvious choice for the constant is $c=-\frac{1}{10}$. This choice gives

$$\delta\int_{\mathcal{M}_2}d^2z\sqrt{-g}+\delta\int_{\mathcal{M}_2\otimes[0,1]}f=-4i\int d^2z\sqrt{-g}\,P_+^{ab}\bar\kappa(\pi_a^\mu\Gamma^\mu+\tfrac{1}{10}i\pi_a^{\mu\nu}\Gamma^{\mu\nu})\partial_b\theta, \tag{9}$$

where $P_\pm^{ab}\equiv\frac{1}{2}(g^{ab}\pm\epsilon^{ab}/\sqrt{-g})$ are the usual projection operators. Although the projection operator in (9) is perhaps a bit *ad hoc* at this time, its presence seems most natural upon consideration of previous superimmersions. The point to emphasize here is only that we may fix the relative amounts of π^μ and $\pi^{\mu\nu}$ which appear in the world-sheet metric through a simple *classical* principle. Quantum considerations may override the choice $c=-\frac{1}{10}$ later, but for now it seems to be optimum.

Thus I have obtained my result: With the induced metric,

$$g_{ab}=\pi_a^\mu\pi_b^\mu-\tfrac{1}{10}\pi_a^{\mu\nu}\pi_b^{\mu\nu}, \tag{10}$$

the action,

$$I=\int d^2z\{\sqrt{-g}-i\epsilon^{ab}\bar\theta[\pi_a^\mu\Gamma^\mu+(i/10)\pi_a^{\mu\nu}\Gamma^{\mu\nu}]\partial_b\theta\}, \tag{11}$$

is locally invariant under (3) for all κ satisfying

$$P_-^{ab}(\pi_b^\mu+\tfrac{1}{10}i\pi_b^{\mu\nu}\Gamma^{\mu\nu})\kappa=0. \tag{12}$$

This is the least restrictive condition which I can find for the parameter κ, and appears to yield a nontrivial set (Ref. 3) of κ's through the presence of the projection P_-^{ab}. An equivalent invariant action is obtained through $\epsilon^{ab}\to-\epsilon^{ab}$, $P_+^{ab}\leftrightarrow P_-^{ab}$.

I note in passing that it may be possible to gain some insight into the above by using a coset-space approach, with group elements $U=\exp[i\bar\theta Q+iy^\mu Q_\mu+iy^{\mu\nu}Q_{\mu\nu}]$, where Q, Q_μ, and $Q_{\mu\nu}$ are space-time spinor, vector, and tensor charges, respectively. Perhaps the underlying algebra is a contraction of a subalgebra of the usual orthosymplectic (conformal?) superalgebra, but I will not pursue this here.

I also note some slight generalizations of the above construction. First, it is possible to use two independent Majorana θ's to build an exact three-form, similar to the three-form used for type-II superstring theories in ten space-time dimensions. We replace the forms in (1) by

$$\pi^\mu\equiv dy^\mu-i\bar\theta_1\Gamma^\mu d\theta_1-i\bar\theta_2\Gamma^\mu d\theta_2$$

and

$$\pi^{\mu\nu} \equiv dy^{\mu\nu} - \bar{\theta}_1 \Gamma^{\mu\nu} d\theta_1 - \bar{\theta}_2 \Gamma^{\mu\nu} d\theta_2,$$

and then modify the three-form in (5) by using these new π's as well as by letting

$$d\bar{\theta}\Gamma^\mu d\theta \rightarrow d\bar{\theta}_1 \Gamma^\mu d\theta_1 - d\bar{\theta}_2 \Gamma^\mu d\theta_2$$

and

$$d\bar{\theta}\Gamma^{\mu\nu} d\theta \rightarrow d\bar{\theta}_1 \Gamma^{\mu\nu} d\theta_1 - d\bar{\theta}_2 \Gamma^{\mu\nu} d\theta_2.$$

It may also be possible to find a suitable set of space-time scalars to be used with the set $(y^\mu, \theta, y^{\mu\nu})$ above, to construct a heterotic closed-string theory in eleven dimensions.

Secondly, I note that there is another possible closed and exact three-form involving a single Majorana θ in eleven dimensions. That exact three-form is

$$F = i\pi^\mu d\bar{\theta}\Gamma^\mu d\theta + \frac{i}{720}\pi^{\mu\nu\lambda\rho\sigma} d\theta \Gamma^{\mu\nu\lambda\rho\sigma} d\theta, \qquad (13)$$

where

$$\pi^{\mu\nu\lambda\rho\sigma} \equiv dy^{\mu\nu\lambda\rho\sigma} - i\bar{\theta}\Gamma^{\mu\nu\lambda\rho\sigma} d\theta,$$

and it makes contact with a "failed" supergravity[12] whose local field variables are $(e^\mu_M, \psi_M, A^{\mu\nu\lambda\rho\sigma}_m)$. The exactness of F follows from the Fierz identity

$$720\Gamma^\mu d\theta(d\bar{\theta}\Gamma^\mu d\theta) + \Gamma^{\mu\nu\lambda\rho\sigma} d\theta(d\theta\Gamma^{\mu\nu\lambda\rho\sigma} d\theta) = 0. \qquad (14)$$

By alteration of the other relations above in an obvious way, e.g., making the induced metric bilinear in π^μ_a and $\pi^{\mu\nu\lambda\rho\sigma}_a$, a locally invariant action can be obtained for the variables $(y^\mu, \theta, y^{\mu\nu\lambda\rho\sigma})$.

In general, by taking an arbitrary mixture of f and F, specifically $\eta f + (1-\eta)F$, with a corresponding mixture of bilinears in the definition of g_{ab}, we obtain a theory invariant under those local κ transformations whose parameters satisfy the constraint

$$P^{ab}_{-}\left[\pi^\mu_b \Gamma^\mu + \frac{i\eta}{10}\pi^{\mu\nu}_b \Gamma^{\mu\nu} + \frac{i(1-\eta)}{720}\pi^{\mu\nu\lambda\rho\sigma}\Gamma^{\mu\nu\lambda\rho\sigma}\right]\kappa = 0. \qquad (15)$$

It remains to determine which values of the parameter η are optimum, from either classical or quantum considerations.

Furthermore, by taking the linear combination $72f - F$, it is possible to construct an invariant action which involves only the variables $(y^{\mu\nu}, \theta, y^{\mu\nu\lambda\rho\sigma})$, and does not involve y^μ (a "pointless" superimmersion). It is also amusing to contemplate θ-doubled and/or heterotic mixtures involving f and F.

Finally, we observe that f is also exact for $D=4$. In fact, since

$$\Gamma^\mu d\theta(d\bar{\theta}\Gamma^\mu d\theta) = 0 = \Gamma^{\mu\nu} d\theta(d\bar{\theta}\Gamma^{\mu\nu} d\theta)$$

for a Majorana θ in four dimensions, there is a one-parameter family of theories involving $(y^\mu, \theta, y^{\mu\nu})$ which is locally κ invariant in four-dimensional space-time [i.e., the parameter c in (2) is undetermined for $D=4$].

Perhaps some of the models described above will turn out to be consistent upon quantization. In eleven dimensions, the most likely candidate for a consistent theory is that involving $(y^\mu, \theta, y^{\mu\nu})$, since it requires elimination of the fewest ghosts. Some extension of the Virasora algebra, incorporating the κ symmetry discussed above, may help to show this (for other developments along similar lines, see Zamolodchikov[13] and Bais *et al.*[13]). Work on the quantization is in progress and is really beyond the scope of the present paper.

We can make some simple observations about quantization, however, based on the sum-over-histories approach of Polyakov. If we truncate down to only bosonic variables, since the action is quadratic in them, we may easily work out the world-sheet conformal-anomaly contributions of the antisymmetric tensors in Euclidean space. In this way, on the assumption that the only local invariance is the usual one associated with world-sheet reparameterizations, it is easily established that a theory involving y^μ and $y^{\mu\nu\lambda}$ is anomaly free for $D=6$. It is also clear that models involving $y^{\mu\nu}$ for $D=3$ are equivalent to y^μ models, since in three Euclidean dimensions planes are dual to their normals. It is amusing to note in this latter case that statistical mechanical systems involving local planar structures do arise in nature (e.g., benezene or α-pyridyl molecules distributed on a two-dimensional substrate). However, for the models described above, it remains to establish the effects of the θ degrees of freedom on the conformal properties.

It is also stimulating to consider the possibility of using an *open*-string theory which contains supergravity in eleven dimensions, instead of the more conventional closed-string approach, by choosing a ground state to be a space-time vector, building higher states with Fourier modes of $(y^\mu, \theta, y^{\mu\nu})$ so that the supergravity variables, $(e^\mu_M, \psi_M, A^{\mu\nu}_M)$, appear at the first excited state, and then projectively halving the spectrum so that the supergravity multiplet is lowest in mass squared. Alternatively, the additional space-time vector indices might be attributed to the vector V^μ described above. These issues will have to be discussed further elsewhere.

Whether or not the models described in this paper turn out to be consistent, I believe they represent a useful way of parametrizing the problems associated with supersymmetric string theories in more space-time dimensions. By introducing additional variables, this parametrization has been accomplished at the classical level.

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