Doering *et al.* Reply: Hänggi, Jung, and Talkner raise an interesting point in their Comment[1] on our Letter.[2] Specifically, they point out the difference between the mean first-passage time (MFPT) from one well of a bistable potential to either (a) the unstable point of the state variable (x in our notation) or (b) the separatrix in the phase space of the state variable and the colored-noise variable (the x,z plane). We agree that there is a difference between these two first-passage times, and that alternative (b) may be more appropriate for quantitative comparison with the low-lying spectrum of the Fokker-Planck operator. We stand by our analysis, however, because we explicitly treated case (a) above.

Since the work reported in Ref. 2 was completed, we have performed numerical simulations of the colored-noise–driven process to measure the MFPT considered in our analysis. Our simulations were carried out according to the first-order technique described by Sancho *et al.*[3] for the two-variable Markov process (x,z). Figure 1 is a plot of the result of the simulations for the increase factor of the MFPT over its white-noise value (T/T_0) versus correlation time (ϵ^2) for two values of the noise amplitude ($\sigma^2=0.10$ and $\sigma^2=0.08$). The MFPT's were normalized by the low-amplitude, white-noise value $T_0=(\pi/\sqrt{2})\exp(1/4\sigma^2)$. The solid curve in Fig. 1 is the theoretical prediction from our perturbative analysis [Eq. (23) in Ref. 2], *with no free parameters.* The agreement is quite satisfactory in light of both the inevitable statistical errors in the simulation (5000 samples of the first-passage time were measured for each point in Fig. 1), and the fact that the analytic form is based on the $\sigma^2\rightarrow 0$ limit of the ϵ-expansion coefficients (for which we expect approximately 10% errors at these values of σ^2). It is impractical to perform simulations at significantly smaller noise amplitudes because of the exponential growth of the MFPT. We stress that our prediction of the leading correction of order ϵ is unique among the recent theoretical treatments (see Refs. 9 and 10 in Ref. 2).

The boundary-layer contribution, i.e., the lowest-order increase in the prefactor for the MFPT proportional to ϵ, is apparent in Fig. 1. It is worthwhile noting that although we have not computed the MFPT to the separatrix for the bistable system, the methods of Ref. 2 are easily applied to the transition from one well of the bistable potential to the other. The result is that the boundary-layer contribution is exponentially suppressed in this case—its coefficient in the ϵ expansion of the increase factor for the MFPT over its white-noise value is proportional to $\exp(-1/4\sigma^2)$ in the $\sigma^2\rightarrow 0$ limit. This fact, as well as the magnitude of the ϵ^2 contribution, is in quantitative agreement with the small-ϵ spectral analysis of Hänggi, Jung, and Talkner. This is also an indication of the invalidity, for non-Markovian processes, of the usual identification (for symmetric potentials) of the MFPT from one well to the unstable fixed point with either (a) half the MFPT from one well to the other or (b) half of the inverse of the first nonvanishing eigenvalue of the Fokker-Planck operator. Presumably, as Hänggi, Jung, and Talkner point out, the MFPT to the separatrix in the phase space is more appropriate for comparison with these time scales.

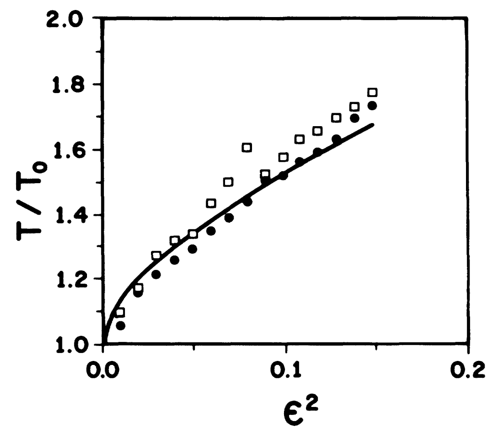

FIG. 1. Plot of the increase in the MFPT over its white-noise value (T/T_0) vs dimensionless correlation time (ϵ^2) for two values of the noise amplitude: $\sigma^2=0.10$ (squares) and $\sigma^2=0.08$ (circles). The time step in the simulations was $\Delta t=0.01$ in the dimensionless units of Ref. 2. The solid line is the theoretical result of Ref. 2.

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