Observation of Stochastic Resonance in a Ring Laser

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We report the first observation of stochastic resonance in an optical device, the bidirectional ring laser. The experiment exploits a new technique to modulate periodically the asymmetry between the two counter-rotating lasing modes. The measurements verify that the addition of injected noise can lead to an *improved* signal-to-noise ratio (relative to that observed with no externally injected noise).

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The phenomenon of stochastic resonance was introduced by Benzi and co-workers[1] to explain the periodicity of Earth's ice ages. They argued that the small periodic perturbations due to the Earth's wobble could lead to large-scale climatic changes via a nonlinear cooperative effect between periodic and random fluctuations. Several theoretical analyses have appeared in which the main characteristics of this phenomenon were described.[1,2] In the only existing experimental paper on the subject, a discrete two-state electronic Schmitt trigger was investigated by Fauve and Heslot,[3] in which these features were present.

The primary signature of stochastic resonance is that *the addition of random noise can improve the signal-to-noise ratio* of a periodically modulated system, relative to that observed with no externally injected noise. Stochastic resonance is an essentially nonlinear phenomenon, requiring the presence of multiple stable states; the basic ingredients are generic enough that it is expected to occur in a wide variety of physical systems.

In this Letter, we report the observation of stochastic resonance in a bistable optical device, the bidirectional ring dye laser. This is the first instance of this phenomenon in a system with a double-well potential, and large enhancements of the signal-to-noise ratio (up to 11 dB) have been obtained. The bidirectional ring laser (homogeneously broadened) is a bistable optical device[4] in which the depth of the wells of the potential can be controlled by a novel technique that employs an acousto-optic modulator with a tunable-frequency acoustic signal.[5] A change in the acoustic frequency of the modulator may be used to control the direction of lasing of the bistable ring (clockwise or counterclockwise). In addition to the sinusoidal modulation, noise injected into the modulator tuning input causes the laser to switch randomly from one direction to the other. Thus it is possible with this system to investigate carefully the simultaneous action of noise and periodic modulation on the bistable system.

We begin by describing the experimental apparatus, and then present the results of our measurements of stochastic resonance. Finally, we discuss these results in terms of a simple generic model that captures the essential elements leading to stochastic resonance.

The experimental apparatus is shown in Fig. 1. A ring dye laser pumped by an argon-ion laser is operated bidirectionally in a single frequency with intracavity etalons. The laser is vibration isolated; an enclosure protects it from air currents and dust. The longitudinal mode structure of the laser is constantly monitored with a confocal Fabry-Perot interferometer to ensure that the laser operates in a single frequency. One of the output beams of the laser is incident on a photodiode, the output of which is displayed on a digital oscilloscope.

A traveling-wave acousto-optic modulator (AOM) with a variable ultrasonic frequency is introduced into the laser cavity. It was recently discovered that it is possible to control the direction of oscillation of a ring laser with an intracavity AOM.[5] The lasing direction can be electronically controlled by tuning of the frequency of the acoustic waves. If the frequency modulation is periodic, the laser can be switched from clockwise to counterclockwise operation at rates up to 100 kHz by application of the appropriate sinusoidal voltage to the tuning input of the amplifier that drives the AOM.

An operational amplifier is used to sum the periodic modulation signal and the noise voltage. The noise spectrum is flat from dc to 100 kHz. The proportion of the noise and signal in the output of the operational amplifier can be varied over several decades.

The lead molybdate AOM is driven by an amplifier; the output frequency is tuned by the voltage from the operational amplifier over the range from 80 to 110

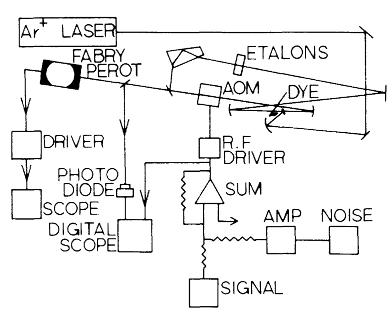

FIG. 1. Experimental apparatus.

MHz. The output of the signal generator contains an adjustable dc bias that determines the central frequency about which the periodic modulation and random excursions occur. The dc bias is set so that the laser spends roughly 50% of its time in each of the two directions. The laser is pumped high enough above threshold so that in the absence of external signal or noise it will spontaneously switch directions only infrequently, i.e., the average dwell time in each direction is several seconds.[4]

The output of the laser in one direction is detected with a fast photodiode, and the signal from it is displayed on a digital oscilloscope. A fast-Fourier transform may be performed on the stored digitized data, and the resulting power spectrum stored in memory. The process is repeated several times, and the resulting averaged power spectrum is displayed. The peak height of the signal and the broad-band noise level at the signal frequency are read off with a cursor to obtain the signal-to-noise ratio.

A sinusoidal modulation signal at 2 kHz was applied to the AOM in addition to the noise voltage. The variations of the output intensity of one of the output beams which turned on and off in response to the input were detected by the photodiode and a digitized time trace was stored. Power spectra obtained from the square of the fast-Fourier transform of the time series are shown in Fig. 2 for increasing amounts of noise. The signal-to-noise ratio (SNR) first increases and then falls off as the input noise is increased, attaining its maximum value in Fig. 2(c). It should be noted that the Lorentzian (broad-band) spectrum resulting from the random switching of the laser output between the off and on states broadens progressively as a function of increasing input noise.

The results obtained from a series of such measurements are shown in Fig. 3, where the stochastic resonance peak displays an enhancement of the SNR by about 11 dB. This is an unambiguous demonstration of the phenomenon of stochastic resonance. These results were obtained over a period of several hours, and unavoidable drifts in temperature caused slow drifts in the laser operating conditions which contributed to the scatter in the data.

Measurements further revealed that the location of the resonance is rather insensitive to the signal frequency: Little if any shift was seen when the signal frequency was changed from 440 Hz to 3 kHz, though the height of the resonance dropped off at the extremes. Further experiments are necessary to delineate the exact nature of the frequency dependence, if it is present, and to probe the limits of the frequency range and its dependence on the laser parameters. These will be described in a future publication.

A simple model of motion in a double-well potential provides physical insight into the mechanism underlying stochastic resonance. The bidirectional ring laser has been shown to obey a Fokker-Planck equation that corresponds to the overdamped motion of a particle in a double-well potential.[4] The on and off states of the beam in a given direction correspond to the particle residing in one or the other of the two wells. The relative depths of the wells are altered periodically by the application of the signal voltage, while the noise voltage is responsible for the transfer of the laser intensity from one well to the other.

At low values of input noise, the transfer from one well to another occurs very seldom; in our experment, the laser is operated high above threshold such that height of

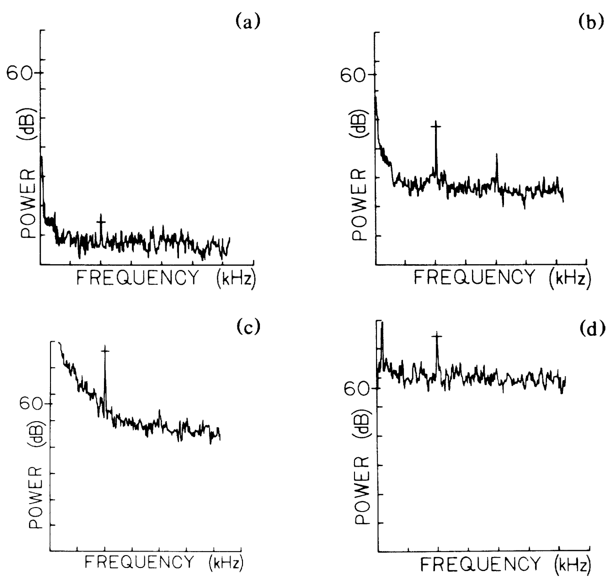

FIG. 2. Power spectrum of laser output vs frequency. The injected noise increases from (a) to (d). The signal frequency (2 kHz) is marked with a cross. The maximum signal-to-noise ratio is seen in (c).

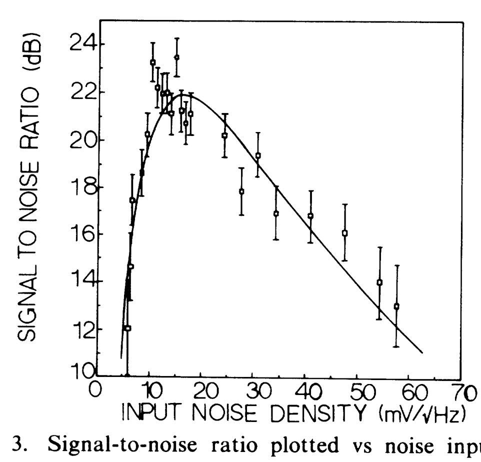

FIG. 3. Signal-to-noise ratio plotted vs noise input. The solid line represents the theoretical result Eq. (2) ($c = 4.4 \times 10^{-4}$, $Q_0 = 6.10 \times 10^{-4}$); an additional horizontal offset of 8.72 mV $Hz^{-1/2}$ had been added to account for the residual noise level.

the barrier between the wells is sufficient to forbid more than an occasional excursion out of the well. Such transitions as do occur are the result of microscopic fluctuations due to spontaneous-emission noise from the dye laser. The modulation signal voltage is small enough that, although the asymmetry of the laser in the clockwise and counterclockwise directions is periodically altered, the difference is insufficient to cause the laser to hop from one well to another. The signal level is thus very small at low levels of input noise: A representative time trace for this regime is shown in Fig. 4(a).

As the noise voltage is increased, the hopping rate between the wells increases, the likelihood of a hop being greatest when the deterministic modulation voltage reaches its extreme values. This is clearly seen in Fig. 4(b), where the switching of the laser between the off and the on state is apparent. At these intermediate values of the noise strength the SNR reaches a maximum. On further increase of the noise voltage, the laser hops between the on and off states so frequently (in response to the noise) that the signal is now buried in the background.

This intuitive explanation of the results is supported by a quantitative analysis of the underlying nonlinear equations.[2,3] The main features of stochastic resonance, however, are robust with respect to many physical details, such as the precise form of the underlying potential, or even its symmetry. In fact, for the limiting regime relevant to the present experiment—namely, when the dwell time is much longer than both the deterministic relaxation time and the noise correlation time—a rather simple rate equation approach captures the essential physics.

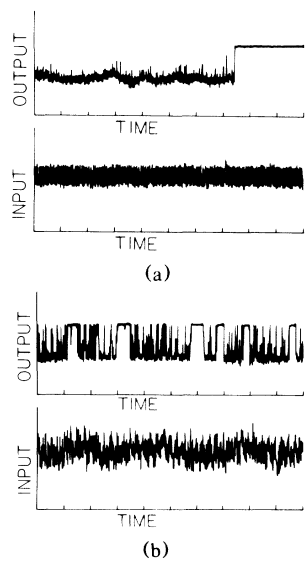

FIG. 4. Laser output and input signal with noise vs time. (a) Low-noise input, showing rare switching of intensity. The time scale is 0.1 s/division. (b) Moderate noise input, showing nearly periodic switching of laser intensity. The time scale is 2 ms/division.

To fix ideas, consider a generic one-dimensional bistable potential with two minima x_+ and x_- separated by a maximum at x_0. The effect of the periodic modulation is to alter the well depths asymmetrically: This causes first x_+, then x_-, to be the globally stable state. The presence of external noise induces occasional transitions across the saddle point x_0. Physically, when the x_+ state is a global minimum, the system preferrentially makes transitions $x_- \rightarrow x_+$, and a time π/ω later, transitions $x_+ \rightarrow x_-$, where ω is the modulation frequency. The probabilities $n_\pm$ of finding the system near the state $x_\pm$ obey the simple rate equations

$$\dot{n}_+ = -W_{\mathrm{down}} n_+ + W_{\mathrm{up}} n_-, \qquad \dot{n}_- = -W_{\mathrm{up}} n_- + W_{\mathrm{down}} n_+, \tag{1}$$

where W_{up} is the transition rate for $x_+ \rightarrow x_-$ and W_{down} is the transition rate for $x_- \rightarrow x_+$, given by the familiar Kramers formula,

$$W_{\mathrm{up,down}} = \mathrm{const}\exp\{-2Q_\pm/D\},$$

where D is the noise variance. Here, we have used the time-periodic potential barriers $Q_\pm = Q_0(1 \pm \epsilon\cos\omega t)$, where ϵ is proportional to the modulation amplitude, and Q_+ (Q_-) is the instantaneous barrier height for the state x_+ (x_-). The overall constant depends on the detailed shape of the potential well, and is of no further interest here.

The linear, time-dependent rate equations (1) can be readily solved[6] for $n_\pm(t)$, and thus for the autocorrelation function and power spectrum. The resulting power spectrum $S(\Omega)$ consists of two pieces: A broad-band part with Lorentzian line shape, and a spike at the modulation frequence ω. As in the experiment, we define the signal-to-noise ratio S/N as the ratio of the strength of the δ function to the value of the spectral density of the broad-band noise at $\Omega = \omega$, with the final result[7]

$$S/N = (c/D^2)\exp\{-2Q_0/D\}. \tag{2}$$

The overall shape of Eq. (2) agrees quite well with the experimental data, as shown in Fig. 3, capturing both the sharply rising edge and the slowly decaying tail. In comparing Eq. (2) with the data, one finds that the value of the maximum value of S/N is a sensitive function of the barrier height Q_0. Since Q_0 is determined by the operating point of the laser, unavoidable drifts in the pump laser power would then explain the pronounced scatter in the experimental data near the peak in Fig. 3.

In fact, one should view this agreement as being somewhat fortuitous, insofar as our use of the Kramers result assumes relatively low input noise, $D \ll Q_0$. The agreement at larger D implies that—qualitatively, at least

—these features are more robust than this simple derivation suggests. However, an understanding of this robustness requires a more detailed theoretical analysis, and will be presented elsewhere.[7] In any event, our intention is not to make detailed comparisons of Eq. (2) with experiment, but rather to emphasize the qualitative understanding achieved by these simple theoretical considerations.

In summary, we have observed the phenomenon of stochastic resonance in an optical bistable system. Nonlinear cooperative effects between periodic and random perturbations can lead to an enhancement of the signal-to-noise ratio as a function of increasing noise input level. The main features agree with theoretical predictions based on a generic model, suggesting that the phenomenon should be observable in a variety of bistable physical systems simultaneously subject to periodic and random perturbations.

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