Improved Estimates for Processes $b \to sl^+l^-$, $B \to Kl^+l^-$, and $B \to K^*l^+l^-$

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We present the complete calculation for the short-distance flavor-changing process $b \to sl^+l^-$ and discuss the effect of QCD enhancement, and the resulting e-μ rate difference. We estimate the exclusive rates for $B \to Kl^+l^-$ and $B \to K^*l^+l^-$ using the relativistic constituent-quark model to calculate the different form factors that include recoil effects. The effect of the fourth generation in enhancing the rates is also discussed.

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The importance of our studying flavor-changing one-loop processes as a test of higher-order electroweak theory is obvious. In the light-quark system, however, the presence of large long-distance effects which cannot be calculated reliably makes this study difficult except in the extremely rare process $K \to \pi\nu\bar{\nu}$. The situation is much better in the b-quark system where, on the one hand, the long-distance effects are expected to be smaller and, on the other hand, the loop calculation is sensitive both to m_t, the mass of the top quark, and to the possible quarks belonging to the fourth generation. Recently,[1,2] processes such as $B \to Ke^+e^-$ and $B \to K^*\gamma$ have been studied with this in mind. In this Letter we reexamine the processes $b \to sl^+l^-$ and $B \to Kl^+l^-$ and calculate $B \to K^*l^+l^-$ for the first time. The estimate of flavor-changing one-loop process $b \to sl^+l^-$ now includes, in addition to photon exchange, Z and W^+W^- box diagrams which become important as $m_t > 40$ GeV as shown by Hou, Willey, and Soni.[3] We also include a QCD correction present for real photon exchange,[2] which significantly enhances the decay rate. This same term causes a difference in the rates for $b \to se^+e^-$ and $b \to s\mu^+\mu^-$, and provides a nice test of QCD corrections and the whole scheme of the calculation as well. In calculating the exclusive decays we now include recoil effects by evaluating the form factors at $q^2=0$. These form factors are determined in the two popular constituent-quark models[4,5] currently in vogue and are found to agree quite well with each other. We find that process $B \to K^*e^+e^-$ is 20%, while $B \to Ke^+e^-$ is only a few percent, of the inclusive charmless decay rate for $B \to Ke^+e^- +$anything. We finally calculate the effect of a fourth generation of quarks on this rate. The rate is a function of the mass of the t' quark and the new mixing angles, and can be enhanced by 1-2 orders of magnitude for suitable values of these parameters.

The process $b \to sl^+l^-$ occurs through one-loop diagrams in the standard model, and involves photon, Z, and W^+W^- exchange. The complete calculation can be extracted from Inami and Lim.[6] The effective Lagrangean can be written as

$$L(b \to sl^+l^-) = \frac{G_F}{\sqrt{2}}\left(\frac{\alpha}{4\pi s_W^2}\right)\sum_i V_i(A_i\bar{s}L_\mu b\,\bar{l}L^\mu l + B_i\bar{s}L_\mu b\,\bar{l}R^\mu l + 2m_b s_W^2 F_2^i\bar{s}T_\mu b\,\bar{l}\gamma^\mu l), \tag{1}$$

where we have assumed $m_b \gg m_s$ and

$$L_\mu = \gamma_\mu(1-\gamma_5),\quad R_\mu = \gamma_\mu(1+\gamma_5),\quad T_\mu = -i\sigma_{\mu\nu}q^\nu(1+\gamma_5)/q^2, \tag{2}$$

and $l=e,\mu$; $V_i = U_{is}^* U_{ib}$ (where $i=u,c,t,t',\ldots$, and U is the Kobayashi-Maskawa matrix),

$$A_i = C_i^{\text{box}} + C_i^Z - s_W^2(F_1^i + 2C_i^Z),\quad B_i = -s_W^2(F_1^i + 2C_i^Z). \tag{3}$$

The quantities C_i, F_1^i, etc., are functions of the ith-quark mass. If $x_i = m_i^2/m_W^2$, we have

$$C_i^{\text{box}} = \tfrac{3}{8}(x_i-1)^{-1}[(x_i\ln x_i)/(x_i-1)-1] - \gamma(x_i,\xi), \tag{4a}$$

$$C_i^Z = \tfrac{1}{4}x_i - \tfrac{3}{8}(x_i-1)^{-1} + \tfrac{3}{8}(2x_i^2-x_i)(x_i-1)^{-2}\ln x_i + \gamma(x_i,\xi), \tag{4b}$$

$$F_1^i = \tfrac{4}{9}(\ln x_i)/(x_i-1) - \tfrac{1}{6}x_i(x_i-1)^{-1}[\tfrac{41}{3} + \tfrac{13}{6}(x_i-1)^{-1} - (x_i-1)^{-2}]$$
$$- \tfrac{1}{6}(x_i\ln x_i)(x_i-1)^{-1}[1 - \tfrac{89}{6}(x_i-1)^{-1} - \tfrac{5}{3}(x_i-1)^{-2} + (x_i-1)^{-3}] - 2\gamma(x_i,\xi), \tag{5}$$

$$F_2^i = \tfrac{2}{3}x_i(x_i-1)^{-1}[1 + \tfrac{21}{8}(x_i-1)^{-1} + \tfrac{3}{4}(x_i-1)^{-2} - \tfrac{9}{4}(x_i-\tfrac{2}{3})(x_i\ln x_i)(x_i-1)^{-3}], \tag{6}$$

where gauge-dependent terms $\gamma(x_i,\xi)$ cancel out in the combinations A_i and B_i in Eq. (3).

The inclusive rate for the charmless rate $B \to K l^+ l^-$ + anything can be equated to $\Gamma(b \to s l^+ l^-)$. This rate is[3]

$$\Gamma(b \to s l^+ l^-) = \frac{G_F^2 m_b^5}{192\pi^3} \left(\frac{\alpha}{4\pi s_W^2}\right)^2 \left[|V_i A_i|^2 + |V_i B_i|^2 + 4 s_W^2 V_j (A_j + B_j)(V_i F_2^i) + 16 s_W^4 \left(\ln\frac{m_b}{2m_l} - \frac{2}{3}\right) |V_i F_2^i|^2 \right]. \tag{7}$$

The total B decay width is calculated by use of the QCD-corrected Hamiltonian and phase-space suppression and is

$$\Gamma_{(B)}^{\text{total}} \approx 3 |U_{bc}|^2 [G_F^2 m_b^5 / 192\pi^3]. \tag{8}$$

The dependence of the inclusive rate on top mass and on fourth-generation quark masses and mixing was given in Ref. 3. Our figures include these for comparison. We wish to point out at this stage that there is a QCD correction to F_2 that enhances that term significantly. This was discussed in Ref. 2, where it was pointed out that QCD corrections replace power behavior by logarithmic dependence on x_i. The effect of this correction on the inclusive rate is quite substantial. We ignore QCD corrections to other terms in Eq. (7) because their contribution to the rate is small. The enhancement factor was calculated from a two-loop diagram for $m_t < m_W$ by Shifman, Vainshtein, and Zakharov.[7] A leading-logarithm calculation valid for $m_t \approx m_W$ is now available.[8] The corrected F_2 is given by

$$\tilde{F}_2(m_t) = \left(\frac{\alpha_s(m_W^2)}{\alpha_s(m_b^2)}\right)^{16/23} \left\{ \frac{116}{135}\left[\left(\frac{\alpha_s(m_b^2)}{\alpha_s(m_W^2)}\right)^{10/23} - 1\right] + \frac{58}{189}\left[\left(\frac{\alpha_s(m_b^2)}{\alpha_s(m_W^2)}\right)^{28/23} - 1\right] + F_2(m_t) \right\}. \tag{9}$$

With this value for $\tilde{F}_2$, the rate for $b \to s e^+ e^-$ is shown in Fig. 1. Note the presence of the $\ln(m_b/2m_l)$ term in the rate (7). Since the $\tilde{F}_2$ term now gives substantial contribution, we expect the $b \to s\mu^+\mu^-$ rate to be lower because of $\ln(m_b/2m_\mu)$. The ratio $\Gamma(b \to s e^+ e^-)/\Gamma(b \to s\mu^+\mu^-)$ is found to be 1.35, almost independent of m_t. This is an interesting way of isolating the $\tilde{F}_2$ term experimentally and would provide a nice test for QCD effects in electroweak calculations. We shall show that the $\tilde{F}_2$ term also enhances the exclusive process $B \to K^* e^+ e^-$ but has little effect on $B \to K e^+ e^-$. Thus, $B \to K^* e^+ e^-$ is enhanced compared to $B \to K^* \mu^+ \mu^-$, and we again have an e-μ rate difference.

We first consider the process $B \to K l^+ l^-$. The hadronic matrix elements necessary are

$$\langle K(k) | \bar{s}\gamma_\mu b | B(p) \rangle = (p+k)_\mu f_{KB}^+(q^2) + q_\mu f_{KB}^-(q^2), \tag{10}$$

$$\langle K(k) | \bar{s}\sigma_{\mu\nu} q^\nu b | B(p) \rangle = [q^2 (p+k)_\mu - (m_B^2 - m_K^2) q_\mu] f_T(q^2). \tag{11}$$

Note that the form (11) is dictated by current conservation. Since $m_e, m_\mu \ll m_b$ we can neglect the q_μ terms. We shall assume that the q^2 dependence of the form factors can be approximated by a single pole with mass $\approx m_B$:

$$f^+(q^2) = f^+(0)/[1 - q^2/m_B^2], \quad f_T(q^2) = f_T(0)/[1 - q^2/m_B^2]. \tag{12}$$

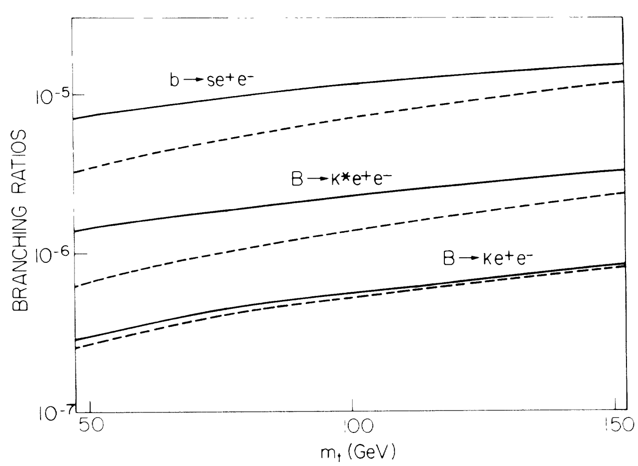

FIG. 1. Branching ratios for $b \to s e^+ e^-$, $B \to K^* e^+ e^-$, and $B \to K e^+ e^-$ as functions of m_t. Dashed lines correspond to the branching ratios calculated without QCD correction, while solid lines represent the QCD-corrected branching ratios.

We have calculated $f^+(0)$ and $f_T(0)$ in a relativistic constituent-quark model[4] (CQM) and find

$$f^+(0) \approx 0.34, \quad f_T(0) \approx f^+(0)/2m_b. \tag{13}$$

We find a similar result from the CQM of Ref. 5. In the calculation for the rate, the q^2 pole in the effective Lagrangean for the quark subprocess in Eqs. (1)-(2) is canceled by q^2 in Eq. (11). Thus there is no logarithmic enhancement of this term unlike the inclusive decay. The full calculation yields

$$\Gamma(B \to K l^+ l^-) = \frac{G_F^2 m_B^5}{192\pi^3} \left(\frac{\alpha}{4\pi s_W^2}\right)^2 \left(\frac{f^+(0)}{2}\right)^2 (H^2 + C^2), \tag{14}$$

$$H = V_i (A_i + B_i + s_W^2 \tilde{F}_2^i), \quad C = V_i (A_i - B_i). \tag{15}$$

We have plotted the rate for $B \to K e^+ e^-$ as a function of m_t in Fig. 1. The result without QCD correction of F_2 is almost the same. The ratio of this exclusive mode to the inclusive process is 4% to 8%, depending on QCD corrections and the top-quark mass. Since the q^2 pole is canceled by q^2 in the numerator in Eq. (11) as required by gauge invariance, we do not have any enhancement for $B \to Ke^+e^-$ over the $B \to K\mu^+\mu^-$ mode.

We shall now evaluate the process $B \to K^* l^+ l^-$. Here we have to calculate the following hadronic matrix elements:

$$\langle K^*(k) | \bar{s} L_\mu b | B(p) \rangle = i\epsilon_{\mu\nu\lambda\sigma}\epsilon^\nu(k)(p+k)^\lambda(p-k)^\sigma V(q^2) + \epsilon_\mu(k)(m_B^2 - m_{K^*}^2)A_1(q^2) - (\epsilon\cdot q)(p+k)_\mu A_2(q^2), \quad (16)$$

$$\langle K^*(k) | \bar{s} T_\mu b | B(p) \rangle = i\epsilon_{\mu\nu\lambda\sigma}\epsilon^\nu(k)(p+k)^\lambda(p-k)^\sigma T_1(q^2) + [\epsilon_\mu(k)(m_B^2 - m_{K^*}^2) - (\epsilon\cdot q)(p+k)_\mu] T_2(q^2). \quad (17)$$

We assume that the q^2 dependence of these form factors is well described by a pole fit:

$$V(q^2) = V/(m_B + m_{K^*})(1 - q^2/m_B^2), \quad (18)$$

$$A_i(q_2) = A_i/(m_B + m_{K^*})(1 - q^2/m_B^2), \quad (19)$$

$$q^2 T_i(q^2) = T_i/(1 - q^2/m_B^2), \quad i = 1,2. \quad (20)$$

We again only quote the results we obtain from using the quark model of Ref. 4:

$$V \approx T_1 \approx 0.37, \quad A_1 \approx A_2 \approx T_2 \approx 0.33. \quad (21)$$

Uncertainties[4] on these quantities are expected to be about 15%. If we had employed the CQM of Ref. 5, we would have obtained $V \approx T_1 \approx 0.43$ and $A_1 \approx A_2 \approx T_2 \approx 0.30$ with similar uncertainties. We shall use the values in Eq. (21). The rate is given by

$$\Gamma(B \to K^* l^+ l^-) = (G_F^2 m_B^5/192\pi^3)(\alpha/4\pi s_W^2)^2 I, \quad (22)$$

where I is an integral over dilepton invariant mass ($z = q^2/m_B^2$),

$$I = \int_{z_{\min}}^{z_{\max}} dz \frac{f(z)}{(1-z)} \left[(|V_i A_i|^2 + |V_i B_i|^2) z + 2 s_W^2 \left(1 + \frac{m_{K^*}}{m_B}\right) V_j (A_j + B_j)(V_i \tilde{F}_2^i) + 2 s_W^4 \left(1 + \frac{m_{K^*}}{m_B}\right)^2 |V_i \tilde{F}_2^i|^2 z^{-1} \right], \quad (23)$$

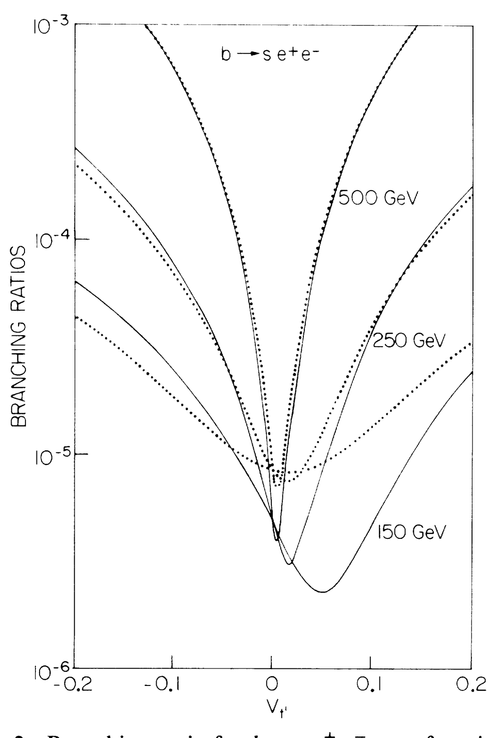

FIG. 2. Branching ratio for $b \to se^+e^-$ as a function of $V_{t'}$ for indicated values of $m_{t'}$. Dotted lines represent QCD-corrected branching ratios, while solid lines correspond to QCD-uncorrected branching ratios.

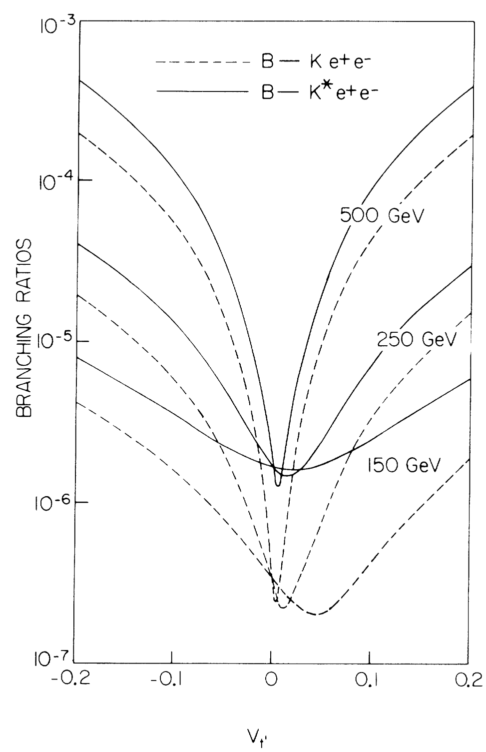

FIG. 3. QCD-corrected branching ratios for $B \to Ke^+e^-$, $K^*e^+e^-$ as functions of $V_{t'}$ for indicated values of $m_{t'}$.

where

$$f(z)=2(1+m_{K^*}/m_B)^{-2}\phi(z)V^2+[3(1-m_{K^*}/m_B)^2+(m_B/2m_{K^*})^2(1+m_{K^*}/m_B)^{-2}(z-5m_{K^*}^2/m_B^2)\phi(z)]A_1^2, \quad (24)$$

$$\phi(z)=(1-z)^2+4z(m_{K^*}/m_B)^2, \quad (25)$$

$$z_{\min}=(2m_l/m_B)^2, \quad z_{\max}=(1-m_{K^*}/m_B)^2, \quad l=e,\mu. \quad (26)$$

The above integral is done numerically for different values of m_t. Note that the presence of z^{-1} in the third term of Eq. (23) enhances the $\tilde{F}_2^i$ contribution relative to other terms as in the case of $b \to sl^+l^-$. We have plotted the rate for $B \to K^*e^+e^-$ with QCD corrections in Fig. 1. This rate is 20% of the rate for $b \to se^+e^-$. If we use the CQM of Ref. 5 the rate agrees within 5% with our quoted results. We expect the inclusive rate to be accurate to about 15% to 20% because of the neglect of QCD corrections to A_i and B_i. The ratio of exclusive to inclusive decay can have uncertainty of about[4,5] 30% from the model dependence of form factors. We again have an e-μ rate difference and find the ratio $\Gamma(B \to K^*e^+e^-)/\Gamma(B \to K^*\mu^+\mu^-)=1.23$, almost independent of m_t.

We have evaluated[9] the rate for $b \to sl^+l^-$ and the exclusive modes $B \to Kl^+l^-$ and $B \to K^*l^+l^-$. The QCD enhancement for real-photon processes like $b \to s\gamma$ and $B \to K^*\gamma$ carries over to the leptonic mode through the phase-space contribution which goes as $\ln(m_b/2m_l)$. This same term leads to the e-μ rate difference. The rate is monotonically and very weakly dependent on m_t in the range 50 GeV $< m_t <$ 150 GeV. The ratio

$$\Gamma(B \to K^*e^+e^-)/\Gamma(b \to se^+e^-)=0.20$$

is independent of QCD corrections and top-quark mass m_t. Any deviation from our prediction could signal new physics. The presence of a fourth generation can lead to a significant enhancement in the rate. We have extended the rate for $b \to se^+e^-$ given in Eq. (10) to the fourth generation. The presence of the t' quark leads to a rate which is a function of $m_{t'}$ and $V_{t'}=U_{t's}^*U_{t'b}$. The unitarity bound requires $|V_{t'}|<0.3$. We plot in Fig. 2 the rate for $b \to se^+e^-$ as a function of $V_{t'}$ for $m_{t'}=150$, 250, and 500 GeV, holding m_t fixed at 65 GeV and $V_c=0.05$. From Fig. 2 it is clear that QCD corrections favor positive small values for $V_{t'}$ over negative $V_{t'}$'s, for $m_{t'}=150$ GeV, and make insignificant changes when $m_{t'}$ becomes large. In Fig. 3 we draw the QCD-corrected branching ratios for $B \to Ke^+e^-$ and $K^*e^+e^-$ as functions of $m_{t'}$ and $V_{t'}$.

We believe that $B \to K^*l^+l^-$ provides a clean test of one-loop electroweak theory and we would urge our experimental colleagues to look for this mode.

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Note added.—Recently Grinstein, Savage, and Wise have calculated QCD corrections to A_i and B_i.[10] The effect of these corrections is to decrease our rate for inclusive decay from 10% to 20% for 50 GeV $< m_t <$ 150 GeV.

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