Evidence for a Scaling Solution in Cosmic-String Evolution

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We study, by means of numerical simulations, the most fundamental issue of cosmic-string evolution: the existence of a scaling solution. We find strong evidence that a scaling solution does indeed exist. This justifies the main assumption on which the cosmic-string theories of galaxy formation are based. Our main conclusion coincides with that of Albrecht and Turok in previous work, but our results are not consistent with theirs. In fact, our results indicate that the details of string evolution are very different from the standard dogma.

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The idea that cosmic strings might be the seeds for galaxy formation has been around for many years,[1] and it has recently attracted a fair amount of attention because of its apparent ability to account for aspects of large-scale structure.[2] More recently Ostriker, Thompson, and Witten[3] have proposed that cosmic strings of the superconducting variety might be responsible for explosive galaxy formation. Despite this interest in cosmic strings, the fundamental issue of how a string network evolves in the early universe has not yet been resolved. An implicit assumption in most of the work on cosmic strings to date is that the energy density of the strings scales like radiation during the radiation-dominated era, but this assumption has not yet been confirmed.

Strings (if they exist) should have formed when the Universe was very young, but they are expected to be relevant at much later times because the majority of the string length at the time of formation is in the form of infinitely long strings. These infinitely long strings cannot radiate away into gravitational radiation, and so they are expected to survive indefinitely. This is the reason that cosmic strings might be relevant for galaxy formation, but it also poses a potential problem for the cosmic-string scenario.

Although cosmic strings are complicated configurations of Higgs and gauge fields, their behavior in a cosmological context can be almost completely described by equations of motion derived from the Nambu action which describes fundamental strings. The only time when the details of the field theory play an important role is when two strings cross. When this happens they can either pass through each other or they can "intercommute" (break and reconnect the other way). Numerical calculations by Shellard[4] and Matzner[5] for the simplest field theories with strings indicate that intercommutation occurs in almost every case, and this is what is assumed in this paper. Without intercommutations, it can be shown that the energy density of the infinite strings will scale roughly like *nonrelativistic* matter. Thus, if strings do not intercommute, then the cosmic strings would become the dominant form of matter in the Universe, bringing the radiation-dominated era to a premature end. With intercommutation, it has been postulated that such a "disaster" can be avoided by the process by which the infinitely long strings cross themselves and break off small loops which can decay into gravitational radiation and thereby transfer their energy density into gravity waves. This scenario has been studied analytically by Kibble[6] and Bennett,[7] who showed that either loop production is not sufficient to avoid a string-dominated universe, or the strings will settle down to a scaling solution in which the number of strings crossing a given horizon volume is fixed and the energy density in strings scales as t^{-2} like radiation. It has not been possible to decide between these two possibilities analytically, so that this question must be resolved through numerical simulations.

The first numerical simulation of cosmic-string evolution was done by Albrecht and Turok[8] (hereafter referred to as AT). Although this was important pioneering work, their program was fairly crude, and their published results have been challenged on the basis of analytic work and have been shown to be inconsistent with the "standard" model for loop production.[7]

In this Letter we present the first results of a large numerical study of the evolution of a cosmic-string network. First, we outline our numerical method, drawing attention to where it differs from the AT method. Then, we present our results and discuss some of the numerical difficulties that we have faced. Our conclusion is that a scaling solution does indeed exist, but many of its properties are very different from previous assumptions and from the AT results.

The basic strategy for our simulations is as follows: First, we generate our initial conditions following the general procedure introduced by Vachaspati and Vilenkin.[9] The one improvement that we have made is to replace the generated sharp corners by arcs of circles (this helps to minimize the number of discontinuous derivatives that the program must deal with). Then, we solve the partial differential equation describing the string motion using a finite-differencing scheme. Finally, intercommutation is included by our checking for crossings between different string segments during the time step, and reconnecting the string segments the opposite way, when necessary.

It is important to note that our initial conditions are not supposed to be a particularly good representation of the string spectrum at the time when the strings begin to move freely (when the friction from the surrounding medium becomes negligible[10]). If a scaling solution exists, then it should be reached from a large variety of initial conditions. We have chosen these initial conditions because their large-scale structure is similar to what we would expect for real strings. This should minimize the relaxation time to the scaling solution if it exists.

In order to evolve the generated configuration, we solve the partial differential equation derived from the Nambu action in an expanding universe with metric $ds^2=a^2(-d\tau^2+d\mathbf{x}^2)$. The equation is[11]

$$\ddot{\mathbf{x}}+2\left(\frac{\dot{a}}{a}\right)\dot{\mathbf{x}}(1-\dot{\mathbf{x}}^2)=\left(\frac{1}{\epsilon}\right)\left(\frac{\mathbf{x}'}{\epsilon}\right)', \qquad (1)$$

in the gauge where $\dot{\mathbf{x}}\cdot\mathbf{x}'=0$ (i.e., the velocity is perpendicular to the string). Dots denote derivatives with respect to conformal time τ, primes denote partial derivatives with respect to the string length parameter σ, and $\epsilon=[\mathbf{x}'^2/(1-\dot{\mathbf{x}}^2)]^{1/2}$. ($\mu a\int\epsilon\, d\sigma$ is the strong energy, where μ is the string tension.) Spatial derivatives at midpoints are obtained by finite differences, while we use a modified "leapfrog" scheme for the time integration. We evolve ϵ semi-implicitly according to $\dot{\epsilon}=-2(\dot{a}/a)\epsilon\dot{\mathbf{x}}^2$ which can be derived from Eq. (1). Each loop carries its own time step satisfying the Courant condition. The overall evolution scheme is second-order accurate.

A major numerical difficulty is that the strings have physical discontinuities in $\dot{\mathbf{x}}$ and $\mathbf{x}'$, which result from intercommutations. These "kinks" have a long lifetime and are likely to have important implications for the spectrum of loops produced. To avoid the development of short-wavelength instabilities near the kinks, we introduce some numerical diffusion that we try to keep at a minimal level. This is accomplished by an averaging of the velocities over neighboring points, but *only* when an instability starts to develop, i.e., whenever the quantity $\dot{\mathbf{x}}^2+(\mathbf{x}'/\epsilon)^2>1$ by a few percent. We have tested this procedure on loops with kinks in flat space and found that the numerical diffusion was only invoked at the kinks, and that the numerical loop trajectory stayed much closer to its analytic value than with either a scheme without diffusion or one that is always diffusive, like the one used by AT.

In order to determine if two string segments crossed during the time step, we check the volume of the tetrahedron spanned by the four points on the two segments. If it changed sign during the step, the configuration is checked at the time that the volume is zero, to see if a crossing did really occur (the positions of the points are extrapolated linearly between time steps). This procedure is almost exact, the one exception being when a crossing occurs as the tetrahedron volume changes sign twice. (This was was explicitly checked for in one run and found to be very rare.) Our procedure was tested with a two-parameter family of flat-space loop trajectories for which the self-intersections can be found algebraically, and the scheme did not miss a real crossing or find a spurious one in hundreds of tests. In contrast to our technique, AT divided their box into 80^3 cells, and performed intercommutations whenever two string segments were in the same cell at the same time. Obviously, this procedure yields some spurious crossings.

Finally, when two segments have been determined to cross, we interchange partners and locally "resample" the string to help reduce the amount of diffusion invoked in the subsequent evolution. More details of our numerical techniques will appear elsewhere.[12]

We have performed two runs with boxes of size $36\xi_0$, 14 runs with boxes of size $28\xi_0$, and more than 40 runs with smaller boxes. (ξ_0 is the correlation length of the strings in the initial configuration.) A typical run on a $28\xi_0$ box with ten sampling points per correlation length would use 160000 points and run for a time interval of $16\xi_0$ (about 100 CPU hours on a Cray-2 computer). Our largest run had twice as many points and ran for a time interval of $\Delta\tau=30.4\xi_0/c$. In contrast, AT ran most of their runs on $16\xi_0$ boxes with five points per correlation length[13] and about 18000 points. Their typical time interval was $5\xi_0/c$.

Figure 1 shows a cube of side ct that has been "cut" out of the final configuration of a typical $28\xi_0$ run after 500 time steps and expansion by a factor of 2.25. (This shows about 1/10 of the total volume of the box.) One important feature that is apparent from Fig. 1 is that the majority of the string length is in the form of small loops in contrast to the initial condition where there are no small loops and 80% of the string length is in infinite strings. This is evidence that the infinite-string energy is indeed being transferred into small loops.

In an effort to "bracket" the scaling solution in the radiation era, we evolved several configurations with dif-

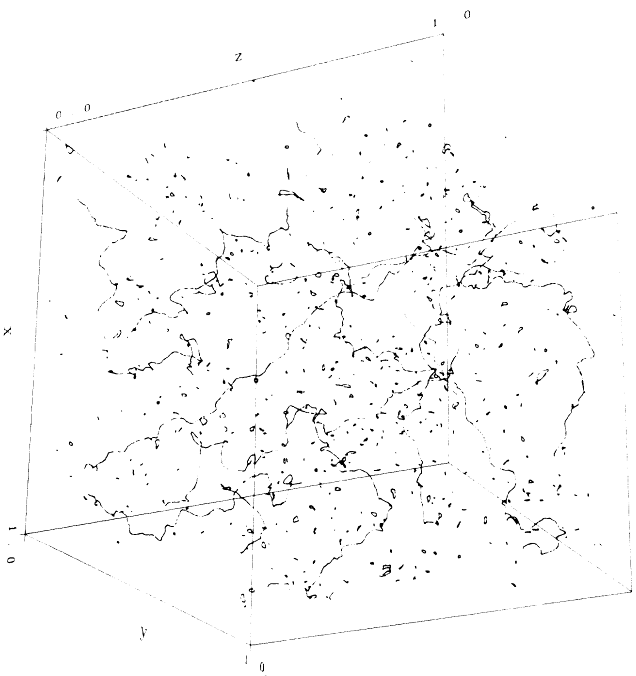

FIG. 1. A volume of size ct on the side that has been "cut" out of the final configuration of a run on a $28\xi_0$ box (see text).

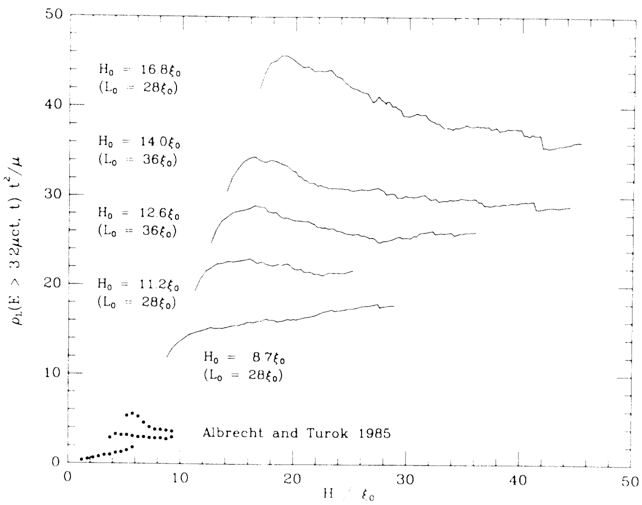

FIG. 2. Normalized energy density in long strings $\rho_L t^2/\mu$ as a function of the horizon size $H=2ct$ in units of the initial correlation length ξ_0, for various initial horizon sizes. The size L_0 of the computation is indicated in parentheses.

ferent initial horizon sizes, and thus different initial energy densities in long strings $\rho_L(t_0)$, to see if ρ_L scales as radiation as required for a scaling solution. (Long strings are defined to be of proper length $> 3.2ct$.) Figure 2 shows the behavior of $\rho_L t^2/\mu$ as a function of time for several different runs. It is apparent that the different runs seem to be converging towards similar (constant) values $\rho_L t^2/\mu \approx 28$, indicating a scaling solution.

We have done many tests and found that our results are independent of most of our numerical parameters. Results of runs with five sampling points per correlation length agree very well with those with ten or twenty points (the runs in Fig. 2 all used ten points per correlation length). We have tested to see at what stage our periodic boundary conditions might influence our results, and we have found no evidence of boundary effects if the run is stopped before $c\Delta\tau$ equals the co-moving box size. The one numerical parameter that does influence our results is the lower cutoff on the size of the loops that can be produced. Because we do not expect our evolution scheme to be very accurate for loops with very few points, we do not allow loops with fewer points than our lower cutoff to form. For the runs shown in Fig. 2, the cutoff is ten points.

The reason for this cutoff dependence is because loop reconnection to the infinite strings is much less efficient for very small loops than for larger ones. Thus, reducing the lower cutoff increases the efficiency of loop production (by decreasing the loop reconnections) and decreases the scaling-solution value of $\rho_L t^2/\mu$. Thus, our determination of $\rho_L t^2/\mu$ may suffer from a systematic error due to our small-loop cutoff. Figure 3 shows $\rho_L t^2/\mu$ for a series of runs in which we have varied the small-loop cutoff. We have referred to the cutoff as a fixed fraction of the ξ_0 because some of the runs in Fig. 3 have different numbers of sampling points per correlation length, and it is only the ratio of the cutoff to the number of points per correlation length that influences our results. Although it is clear from Fig. 3 that $\rho_L t^2/\mu$ shows some cutoff dependence, the runs with the cutoffs $\lambda_0=0.6\xi_0$ and $0.3\xi_0$ seem to be converging toward a cutoff-independent scaling-solution value of $\rho_L t^2/\mu \approx 20$.

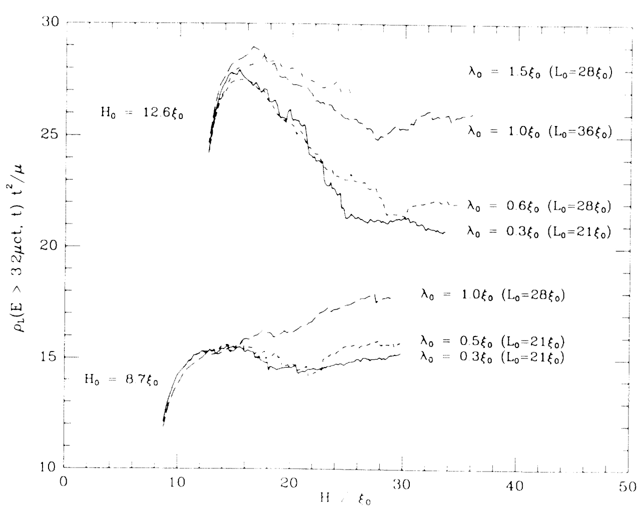

FIG. 3. Influence of the cutoff λ_0 on the energy density in long strings for two different initial densities which bracket the scaling solution.

With a conservative estimate of the cutoff dependence as well as other possible systematic errors, our value for the energy density in long strings is $\rho_L t^2/\mu = 20 \pm 10$. It should be stressed, however, that our result is very different from that of AT, who reported evidence for a scaling solution with $\rho_L t^2/\mu = 2.5 \pm 0.5$. Thus, our results cannot be considered to be consistent with theirs. The main reason for this difference is that the strings in the AT simulations lost a significant amount of energy through numerical errors.[14]

In fact, our results differ dramatically from the standard scenario that has usually been assumed to describe string evolution. The standard scenario for loop production holds that horizon-sized "parent" loops break off the infinite-string network and fragment into roughly ten "child" loops of roughly equal sizes, but we find that in addition to the horizon-sized parent loops, the infinite strings lose significant amounts of energy directly into small loops. The reason for this is presumably that the strings have a lot of short-wavelength structure in the form of the kinks that are formed whenever strings cross. Furthermore, we find that the fragmentation of the parent loops does not terminate with ten equal-sized child loops. Instead, we find that most, *but not all,* of the child loops are close enough to our lower cutoff on loop size (usually ξ_0) so that their chances of fragmenting further are significantly (or completely) suppressed. It is fortunate that the fragmentation probability is so high because it implies that the type of transient discussed in Ref. 7 which might mimic a scaling solution can probably be ruled out.

We have studied the evolution of a network of cosmic strings with a computer code that has a much larger dynamical range and which we believe is much more accurate than the code written by Albrecht and Turok. Our results strongly support the existence of a scaling solution in the radiation era, but the density of long strings at the scaling solution is $\rho_L = (20 \pm 10)\mu/t^2$. In more physical terms, ρ_L is 7×10^{-4} $(G\mu/10^{-6})$ times the radiation density. (The total string density has not been determined because it is dominated by the small loops.) This is almost an order of magnitude larger than the value reported by Albrecht and Turok. The source of this disagreement has been found to be numerical energy loss in the AT code. An improved version of the AT code, while still under development, now produces results that may be consistent with ours.[14] We find that a great deal of the loop production goes directly into small loops and that the large loops that are produced have a much higher probability to fragment into small loops than has previously been assumed. In fact, the bulk of the uncertainty in our value for ρ_L is due to possible systematic errors caused by the abundance of the small loops near the lower cutoff.

More details on our numerical algorithms, the testing of our code, and our results will soon be published.[12]

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