Anderson and Zou Reply: We welcome the comment by Kallin and Berlinsky (KB) since, as they remarked, the problem of holon-spin scattering is indeed much more complex than the simple perturbation theory sketched in our paper[1] and worked out—correctly of course—by Kallin and Berlinsky for the case of free holons at very low density. Their result gives us the opportunity to discuss the question more fully.

That their calculation runs into trouble for finite holon density, even with neglect of holon-holon interactions, is seen by our looking at the reciprocal process of spinon scattering by holons. Every collision scatters both a spinon and holon and so we have, for the spinon mean free time, $n_B/\tau_B = n_F/\tau_F \simeq g_F T/\tau_F$. Thus the momentum uncertainty of a spinon is given by

$$(\hbar \Delta q)_F = \frac{\hbar}{l_F} \simeq n_B \hbar k_F \left[3\zeta(\tfrac{3}{2}) \left(\frac{T}{2\pi m_B v_F^2} \right)^{1/2} \right]. \quad (1)$$

For nearly zero $n_B = \delta$, this is small, to be sure; but for finite $n_B \sim 0.1$–0.5, as in real high-T_c materials, this is much higher than the momentum difference from the Fermi level, $\hbar k_F T/E_F \simeq \hbar k_F T/m_F v_F^2$. For $m_B < m_F$, and $\delta \sim 0.5$ (as we expect for 123 compounds), this alone would allow us to ignore conservation laws. This very large momentum uncertainty of spinons in doped materials is, of course, seen in the inelastic neutron-scattering experiments. It indicates that, as indeed remarked by KB, the real system is a strongly coupled holon-spinon soup whose true nature has yet to be worked out.

Another consideration also comes in at finite holon density, namely the large, long-range repulsive interactions between holons. In the limit of large U, holons must be simply k-space projection operators,[2] which project away the state of momentum k with either spin while causing a rearrangement of the Fermi sea of spinons which is not strongly dependent on the momentum of the holon. The square of such a projector is zero: One cannot occupy a holon state twice, at least in this limit. This implies long-range force in real space, in agreement with a conjecture of Laughlin.[3] In fact, most numerical calculations[4] show that holon density is nonzero over the entire Brillouin zone for the large-U Hubbard model. The situation was modeled in the paper of Wheatley, Hsu, and Anderson[5] by a k-space pseudopotential which gives the holons a flat spectrum in momentum space,

$$E_k = \begin{cases} \epsilon_0, & k < k_0, \\ tk^2, & k > k_0, \end{cases} \quad (2)$$

with $k_0 = 4\pi\delta^{1/2}/n_0$, δ being the holon concentration. This spectrum was used to calculate the superconducting transition temperature due to holon pair condensation. The predictions of this more realistic spectrum agree with the infrared data of Padamsee *et al.*[6] very well, better than expected. In this spectrum the average wave vector of the holons at low temperature is independent of temperature, $k \sim k_0$. Thus the phase space available for holon-spinon scattering is essentially proportional to T/J, where J is the spinon bandwidth as argued by Anderson and Zou.[1] The additional factor $T^{1/2}$ found by Kallin and Berlinsky appears only as $\delta \to 0$. Thus we believe that our conclusion that the spinon-holon scattering gives rise to the linear temperature dependence of the resistivity in Cu-O plane as well as the $1/T$ behavior of the out-of-plane resistivity is correct. Kallin and Berlinsky's calculation is correct for a dilute nondegenerate free Bose gas, but not relevant for the real system.

An even more difficult problem associated with the free-boson spectrum $E_k = k^2/2m$ is that such a two-dimensional free Bose gas will undergo single-particle condensation with T_c given essentially by the three-dimensional Bose condensation temperature,[7] which is much too high for the high-T_c superconductors. The long-range force in real space or short-range force in momentum space prevents the bosons from condensing into a zero-momentum state. Kallin and Berlinsky's comments showed the difficulty of calculating physical quantities to be compared with experiments. Our original estimates of the resistivity and tunneling are rather crude as remarked in the paper,[1] but we only wished to stress the physical ideas. In order to perform more accurate calculations, one needs to understand the fundamental nature of the holon and spinon spectra much better than we currently do. This problem remains wide open, and we welcome the comment of Kallin and Berlinsky as helping to make this clear.

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