

# Comment on "‘Normal’ Tunneling and ‘Normal’ Transport: Diagnostics for the Resonating-Valence-Bond State"

In a recent Letter, Anderson and Zou<sup>1</sup> discussed the normal-state transport properties of a resonating-valence-bond system, in which the carriers are charged bosons (holons) and the spin excitations are neutral fermions (spinons), and compared their predictions to experimental data for high-temperature superconducting oxides. An important result of their analysis is the assertion that the resistivity in the Cu-O planes should be proportional to the temperature  $T$ . In the Comment we argue that the relation  $\rho \sim T$  is not the result of a simple low-order process involving charged bosons scattered by neutral fermions as implied in Ref. 1. In fact, the lowest-order process gives  $\rho \sim T^{3/2}$ , even in the presence of umklapp scattering.

The  $T^{3/2}$  dependence arises from the elementary process in which a holon scatters, creating a spinon-antispinon pair. For this process, the transport scattering rate is given by

$$\frac{1}{\tau} = \frac{\pi \hbar \beta |t|^2}{2n_B m_B} \sum_{\mathbf{k}, \mathbf{q}, \mathbf{q}'} q^2 n_{\mathbf{k}} (1 + n_{\mathbf{k}-\mathbf{q}}) f_{\mathbf{q}'} (1 - f_{\mathbf{q}'+\mathbf{q}}) \times \delta(\omega_{\mathbf{k}} + E_{\mathbf{q}'} - \omega_{\mathbf{k}-\mathbf{q}} - E_{\mathbf{q}'+\mathbf{q}}), \quad (1)$$

where  $\omega_{\mathbf{k}} = \hbar^2 k^2 / 2m_B$ ,  $E_{\mathbf{q}}$  is the spinon energy,  $n_B$  is the holon density, and  $n_{\mathbf{k}}$  and  $f_{\mathbf{q}'}$  are Bose and Fermi distribution functions. The low-temperature dependence of Eq. (1) arises from the integration over the momentum transfer  $\mathbf{q}$ . The spinons can make low-energy transitions between states for which the magnitudes  $|\mathbf{q}|$  and  $|\mathbf{q}+\mathbf{q}'|$  lie within a range of width  $T/v_F$  of the Fermi surface. This still allows  $0 \leq |\mathbf{q}| \leq 2q_F$ . However, from energy conservation, the final wave vector of the holon, with energy of order  $T$ , must lie in a region of radius  $|\mathbf{k}-\mathbf{q}| \sim T^{1/2}$ . The combined restrictions of momentum and energy conservation thus constrain  $\mathbf{q}+\mathbf{q}'$  to lie in a region around  $\mathbf{q}'$  for which the area is proportional to  $T^{3/2}$ . The complete evaluation of (1) yields

$$\frac{1}{\tau} = \frac{2\pi}{\hbar} g_F g_B |t|^2 T \left[ 3\zeta\left(\frac{3}{2}\right) \left( \frac{T}{2\pi m_B v_F^2} \right)^{1/2} \right], \quad (2)$$

for dilute holons at low temperature and spinons with a circular Fermi surface. Here,  $g_F$  and  $g_B$  are the spinon and holon densities of states and  $\zeta$  is the Riemann  $\zeta$  function. If the actual Fermi surface lies close to the Brillouin-zone boundary at some points, then this result will be modified by umklapp scattering, but only by a temperature-independent multiplicative factor. Equation (2) differs from the scattering rate quoted in Ref. 1 by the factor in square brackets. It also follows from Eq.

(2) and from the discussion in Anderson and Zou<sup>1</sup> that the  $c$ -axis conductance is proportional to  $T^{3/2}$  in lowest order.

If the carriers in high- $T_c$  superconductors are in fact bosons, then measurements of the scattering time suggest that their mean free path  $l = v\tau$  is shorter than their thermal wavelength  $\Lambda$ . For bosons in two dimensions,  $l/\Lambda = T\tau/2\hbar$ . Recent infrared measurements<sup>2,3</sup> imply that  $\tau \sim 1/T$  and  $\tau = (2-5) \times 10^{14}$  s, for  $T = 100$  K. Hence,  $l/\Lambda \approx 0.1-0.3$ , independent of temperature. If this small value is the result of strong holon-spinon scattering, then the Bose-Fermi mixture is effectively a strongly coupled quantum fluid and simple arguments based on well-defined Fermi and Bose quasiparticles are highly suspect. Alternatively, if the short boson mean free path results from impurity scattering, then one would expect impurity scattering to dominate the dc transport. Dilute point impurities give  $\rho \sim \sqrt{T}$ , at low  $T$ , and higher densities of impurities would be expected to yield stronger temperature dependences,  $\rho \sim T^n$ ,  $n > \frac{1}{2}$ . Whatever the cause, such a short mean free path cannot be derived from a simple argument based on low-order perturbation theory involving charged Bose and neutral Fermi quasiparticles.

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<sup>1</sup>P. W. Anderson and Z. Zou, Phys. Rev. Lett. **60**, 132 (1988).

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