Comment on "Naked Singularities in Self-Similar Spherical Gravitational Collapse"

In a recent Letter,[1] Ori and Piran have shown that the self-similar spherical gravitational collapse of an adiabatic perfect fluid with a soft enough equation of state can give rise to a naked shell-focusing singularity.[2] This important work extends beyond dust[3] known fluid-collapse histories which have nakedly singular end states. For dust it is known[4] that the naked shell-focusing singularities are *not* strong-curvature singularities. The purpose of this Comment is to point out that the singularities produced by Ori and Piran *are* strong-curvature singularities.

Following the work of Clarke and Królak,[5] consider a null geodesic (N), affinely parametrized by λ, with four-tangent l^a, and terminating at the shell-focusing singularity where $\lambda=0$. If

$$\lim_{\lambda\to 0}\lambda^2 R_{\alpha\beta}l^a l_\beta \neq 0, \tag{1}$$

where $R_{\alpha\beta}$ is the Ricci tensor, then N terminates in a strong-curvature singularity in the sense of Tipler.[6]

For the example given by Ori and Piran[1] (and in their notation) it follows from Einstein's equations, the comoving condition, the equation of state $p=(\Gamma-1)\rho$, and the self-similar relations $\rho=D(y)/T^2$ and $e^{\psi}=(4\pi\times D)^{2(1-\Gamma)/\Gamma}$ that

$$\lambda^2 R_{\alpha\beta}l^a l^\beta = 2\Gamma(4\pi D)^{(2-\Gamma)/\Gamma}\left(\frac{\lambda}{T}\frac{dT}{d\lambda}\right)^2. \tag{2}$$

Now the appropriate N here is the Cauchy horizon $y=y_1=$const. The null geodesic equations for the Cauchy horizon integrate explicitly to give

$$T=R/y_1\propto\lambda^\delta,\quad \delta=\text{const}>0, \tag{3}$$

and so

$$\lambda^2 R_{\alpha\beta}l^a l^\beta = 2\Gamma\delta^2[4\pi D(y_1)]^{(2-\Gamma)/\Gamma}. \tag{4}$$

That is, the Cauchy horizon terminates in a *strong*-curvature singularity.

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[3] Reference 2; D. Christodoulou, Commun. Math. Phys. **93**, 171 (1984); R. P. A. C. Newman, Classical Quantum Gravity **3**, 527 (1986).
[4] See Newman (Ref. 3).
[5] C. J. S. Clarke and K. Królak, J. Geom. Phys. **2**, 127 (1986).
[6] See, for example, F. J. Tipler, C. J. S. Clarke, and G. F. R. Ellis, in *General Relativity and Gravitation,* edited by A. Held (Plenum, New York, 1980).

