Quenching of the Hall Effect

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We argue that for ballistic transport through a narrow conductor (of width W) a threshold magnetic field exists below which the Hall resistance vanishes. The field is of order $(h/e)k_F^{-1}W^{-3}$, and is reached when the transverse wavelength of quantum edge states becomes comparable to the width. This is offered as a mechanism for the quenching of the Hall effect discovered experimentally in a narrow two-dimensional electron-gas wire by Roukes *et al.*

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Ballistic motion may at first sight seem a trivial limit of electrical transport. Recent experiments, however, on high-mobility submicron devices have revealed a variety of unusual phenomena associated with ballistic transport in constricted geometries. Some of the most interesting effects are of a quantum-mechanical origin. We mention the Aharonov-Bohm oscillations in the magnetoresistance[1] and the quantized conductance of point contacts.[2] Some effects are not yet understood. One phenomenon which falls in both these categories is the *quenching* of the Hall effect, discovered by Roukes *et al.*[3] in a narrow conducting channel etched in the two-dimensional (2D) electron gas of a GaAs-AlGaAs heterostructure. In their narrowest channels at low temperatures and below a threshold magnetic field, an unexpected plateau of zero Hall resistance is found (unrelated to the quantum Hall-effect plateaus at much higher fields). Other groups[1,4,5] have noted low-field anomalies in the Hall resistance as well.

Although Roukes *et al.*[3] surmised the fundamental quantum-mechanical origin of their effect, what mechanism controls the threshold field remained a mystery. Our explanation is based on the differences in lateral extension of the magnetic quantum states at the Fermi level in a narrow channel (of width W). One has to distinguish between a high-field and a low-field regime, determined by the relative magnitude of W and the cyclotron orbit diameter $2l_{\text{cycl}}$ (with $l_{\text{cycl}} \equiv \hbar k_F/eB$, k_F being the Fermi wave vector and B the magnetic field). In the high-field regime $2l_{\text{cycl}} < W$, right- and left-moving electrons with the Fermi energy are spatially separated in edge states[6-8] at opposite boundaries. These current-carrying edge states can coexist with quantized cyclotron orbits in the bulk of the sample (Landau states)—when the Fermi level, as determined by the carrier concentration, coincides with a Landau level. Edge states correspond classically to electrons skipping along the boundary[9] (Fig. 1). The high-field regime has been discussed by Halperin[10] and MacDonald and co-workers,[11] who have shown how a Hall voltage arises because of differences in the population of right- and left-moving edge states. In the low-field regime $2l_{\text{cycl}} > W$ relevant to the experiments of Roukes *et al.*,[3] Landau states which are unperturbed by the boundaries no longer exist at the Fermi level. Concurrently, some edge states begin to interact with the opposite boundary. Prange[12] has calculated the magnetic quantum states in a thin-plate geometry. The differences in lateral extension of the states which follow from his calculation may be understood from the classical correspondence (Fig. 1). In addition to the skipping orbits (corresponding to edge states) we now also have trajectories which traverse the channel. The corresponding "transversing states" (also known as hybrid magnetoelectric subbands) interact with both boundaries. Because of the presence of these traversing states the arguments of Refs. 10 and 11 no

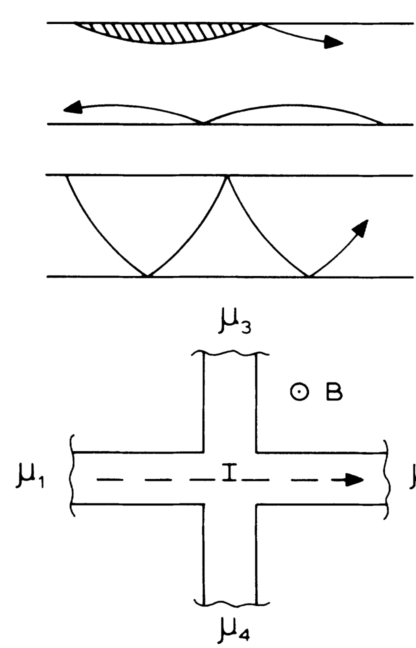

FIG. 1. Top: Skipping orbits, corresponding to edge states. The flux through the shaded area is quantized according to Eq. (2). Center: Traversing trajectory, corresponding to a traversing state (hybrid magnetoelectric subband). Bottom: Four-terminal conductor for Hall-resistance measurement.

longer apply, and anomalies in the Hall voltage can be expected to occur in the low-field regime.

Our explanation of the quenching of the Hall effect combines two considerations: (1) For a Hall voltage it is necessary that edge states exist at the Fermi level (see below). (2) Edge states are suppressed if their transverse wave length[6,12] $\lambda_t = (\hbar/2k_F eB)^{1/3}$ (in the direction perpendicular to the boundary) exceeds W. Note that in weak magnetic fields λ_t is much larger than the Fermi wave length. This implies that, in principle, quenching of the Hall effect is *not* restricted to samples with $k_F W \lesssim 1$—although in practice the threshold field may become unobservably small in much wider samples. (In the experiment,[3] quenching is observed for $k_F W \lesssim 20$.) Note also that our considerations apply as well to the wire geometry of Ref. 3 as to a thin-film geometry in parallel magnetic field (with Hall probes on opposite sides of the film).

The above argument gives a prediction for the threshold field B_{thres} which can be tested by comparison with the experiment. The minimum transverse extension $\Delta_{\min}$ of an edge state is of the order of λ_t. From the calculation of Prange[12] we estimate $\Delta_{\min} \approx 3\lambda_t$. (This value of $\Delta_{\min}$ includes the penetration of the wave function over a distance of about one transverse wavelength beyond the classical orbit.) The edge states are suppressed if $W \lesssim \Delta_{\min}$, which gives the threshold field

$$B_{\text{thres}} \approx 2(h/e)k_F^{-1}W^{-3}. \tag{1}$$

The value of the numerical coefficient in Eq. (1) is clearly dependent on the specific suppression criterion used, and is therefore somewhat uncertain, but the characteristic W^{-3} dependence of B_{thres} is not. It is worthwhile to see how this characteristic feature of the quenching mechanism follows semiclassically from the Bohr-Sommerfeld quantization rule,[13] applied to the flux enclosed by a skipping orbit. For an infinite-barrier confining potential the quantization condition is

$$BS = (n - \tfrac{1}{4})h/e, \quad n = 1,2,3\ldots, \tag{2}$$

where S is the area of the circle segment shaded in Fig. 1. A small circle segment of height Δ has area

$$S = \tfrac{4}{3}(2\Delta^3 l_{\text{cycl}})^{1/2}[1 + O(\Delta/l_{\text{cycl}})]. \tag{3}$$

Since $\Delta < W$ and $n \geq 1$, Eqs. (2) and (3) yield a threshold field of $(h/e)k_F^{-1}W^{-3}[1+O(k_F W)^{-2}]$, consistent with Eq. (1). (The smaller numerical coefficient is due to the fact that the penetration of the wave function beyond the classical turning point is neglected in this semiclassical approximation.)

Before comparing Eq. (1) with the experiment,[3] we will discuss in some more detail the special role which edge states play in establishing the Hall voltage in ballistic transport. We consider the geometry of Fig. 1, which resembles that of the experiment: a four-terminal conductor consisting of reservoirs at chemical potentials μ_i ($i = 1,2,3,4$), connected by 2D channels which are shorter than the mean free path. A magnetic field is applied, perpendicular to the 2D electron gas. A current I flows from reservoir 1 to reservoir 2, while reservoirs 3 and 4 are voltage probes[14] which draw no net current. The Hall resistance R_H is defined as $R_H \equiv (\mu_3 - \mu_4)/eI$. Büttiker[15] has derived the general Landauer[16] conductance formula for a four-terminal measurement, which relates the conductance to the transmission probabilities T_{ij} for an electron from reservoir j to reservoir i. In the present geometry his result is of the form

$$R_H = (h/e^2)D^{-1}(T_{31}T_{42} - T_{32}T_{41}). \tag{4}$$

The coefficient D is a subdeterminant of the matrix relating currents to chemical potentials (see Ref. 15), whose explicit expression is not needed here. One sees from Eq. (4) that a nonzero Hall resistance requires the *transmission asymmetry* $T_{31}/T_{41} \neq T_{32}/T_{42}$ or, in words, that the ratio of transmission probabilities to upper and lower voltage probes is different for electrons coming from the left or from the right.

In the high-field regime $2l_{\text{cycl}} < W$ the right- and left-moving states at the Fermi level are spatially separated at opposite edges of the wire. Such edge states have the largest possible transmission asymmetry ($T_{31}/T_{41} = \infty$, $T_{32}/T_{42} = 0$). The resulting Hall resistance is quantized,[10,11]

$$R_H(1/2N)h/e^2, \tag{5}$$

where N is the number of occupied (spin degenerate) Landau levels. In the low-field regime $2l_{\text{cycl}} > W$ current-carrying states appear which interact with both edges (traversing states). If we now make the reasonable assumption that traversing states have a much smaller transmission asymmetry than edge states, it follows from Eq. (4) that the Hall resistance is quenched when all edge states have disappeared. Quantum mechanically, this occurs at a *finite* magnetic field B_{thres}. A justification for the assumption of small transmission asymmetry for traversing states follows from the classical correspondence (Fig. 1): Since an electron moving on a traversing trajectory collides with the same frequency on the upper and lower edges (regardless of whether it comes from the left or the right), it has a ratio of transmission probabilities to upper and lower voltage probes which is left-right symmetric. This trajectory argument (comparable to the ray-optics description of propagation in a wave guide) breaks down if $k_F W \lesssim 1$, but should be reliable as a first approximation in wider channels. We stress that a simple relation like Eq. (5) between the Hall resistance and the number of occupied subbands N no longer exists in the low-field regime, when not all current-carrying states at the Fermi level are edge states.[17] This is why the attempt by the authors of Ref. 3 to explain the quenching by invoking such a relation could not succeed.

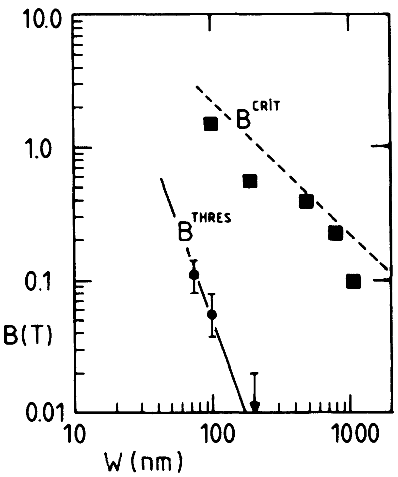

FIG. 2. Threshold field B_{thres} for the quenching of the Hall resistance and critical field B_{crit} for the onset of anomalies in the Hall effect, as functions of the width of a 2D electron-gas wire. Data points are estimated from the experiment of Roukes *et al.* (Ref. 3). The solid line is calculated from Eq. (1), with the value $k_F = 1.72 \times 10^8\ \text{m}^{-1}$ obtained from the classical Hall resistance (see Ref. 3). The dashed line corresponds to the criterion $W = 2l_{\text{cycl}}$ for the onset of classical size effects.

We now turn to a discussion of the experimental results of Roukes *et al.*,[3] which motivated the preceding analysis. In the experimental paper results have been presented for the Hall voltage and for the longitudinal voltage drop between two probes on the same side of the channel. Here we focus on the Hall measurements.[18] The data show that, upon a decrease in the width, deviations from the linear B dependence of the Hall resistance appear, in a field region bounded by $\pm B_{\text{crit}}$. A reasonably well-defined threshold field B_{thres} for the appearance of a Hall voltage can be obtained from the data for wires as narrow as 100 and 75 nm. We note that B_{thres} is typically an order of magnitude smaller than B_{crit}. For a wire 200 nm wide a clear quench plateau is no longer seen; we estimate from the data $B_{\text{thres}} \lesssim 0.02$ T. In Fig. 2 the experimentally obtained values for B_{thres} and B_{crit} are plotted as functions of the wire width. The solid line is the prediction for B_{thres}, calculated from Eq. (1). *It is apparent from this figure that the W^{-3} dependence of B_{thres}, which follows from our analysis, is supported by the data.* The remarkable numerical agreement evident in Fig. 2 is better than one might hope, in view of the uncertainties in the numerical coefficient in Eq. (1). Note also that a detailed comparison is sensitive to relatively small uncertainties in the actual value of the channel width (because of the W^{-3} power law). We also briefly comment on the data for B_{crit} plotted in Fig. 2. Although Roukes *et al.*[3] maintain that the anomalous Hall region, bounded by B_{crit}, approximately follows a $W^{-3/2}$ trend, it is clear from Fig. 2 that a W^{-1} trend is at least equally well supported by the experimental data. The latter behavior is expected from the condition $W = 2l_{\text{cycl}}$ for the onset of classical size effects (dashed line in Fig. 2).

In conclusion, we have proposed a mechanism for the quenching of the Hall effect in narrow conductors, consistent with data from Roukes *et al.*[3] A more extensive analysis of the anomalies observed beyond the threshold field should be feasible. Because of the strong coupling of the voltage probes to the current-carrying wire, a detailed knowledge of the local geometry and the transmission probabilities will then be necessary. We have therefore restricted ourselves in this Letter to the quenching of the Hall effect, which according to our analysis is a more fundamental and universal phenomenon.

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[1]G. Timp, A. M. Chang, P. Mankiewich, R. Behringer, J. E. Cunningham, T. Y. Chang, and R. E. Howard, Phys. Rev. Lett. **59**, 732 (1987); A. M. Chang *et al.*, Phys. Rev. B **37**, 2745 (1988); G. Timp *et al.*, to be published.

[2]B. J. van Wees, H. van Houten, C. W. J. Beenakker, J. G. Williamson, L. P. Kouwenhoven, D. van der Marel, and C. T. Foxon, Phys. Rev. Lett. **60**, 848 (1988); B. J. van Wees *et al.*, to be published; D. A. Wharam, T. J. Thornton, R. Newbury, M. Pepper, H. Ahmed, J. E. F. Frost, D. G. Hasko, D. C. Peacock, D. A. Ritchie, and G. A. C. Jones, J. Phys. C **21**, L209 (1988).

[3]M. L. Roukes, A. Scherer, S. J. Allen, Jr., H. G. Craighead, R. M. Ruthen, E. D. Beebe, and J. P. Harbison, Phys. Rev. Lett. **59**, 3011 (1987).

[4]J. A. Simmons, D. C. Tsui, and G. Weimann, Surf. Sci. **196**, 81 (1988).

[5]T. J. Thornton, private communication.

[6]R. E. Prange and T.-W. Nee, Phys. Rev. **168**, B779 (1968).

[7]M. S. Khaikin, Adv. Phys. **18**, 1 (1969).

[8]The observation of edge states by electron tunneling in a heterostructure has recently been reported by B. R. Snell, K. S. Chan, F. W. Sheard, L. Eaves, G. A. Toombs, D. K. Maude, J. C. Portal, S. J. Bass, P. Claxton, G. Hill, and M. A. Pate, Phys. Rev. Lett. **59**, 2806 (1987).

[9]The skipping orbits associated with multiple specular reflections from the 2D electron gas boundary have directly been observed in a transverse electron focusing experiment; see H. van Houten, B. J. van Wees, J. E. Mooij, C. W. J. Beenakker, J. G. Williamson, and C. T. Foxon, Europhys. Lett. (to be published).

[10]B. I. Halperin, Phys. Rev. B **25**, 2185 (1982).

[11]A. H. MacDonald and P. Streda, Phys. Rev. B **29**, 1616 (1984); P. Streda, J. Kurcera, and A. H. MacDonald, Phys. Rev. Lett. **59**, 1973 (1987).

[12]R. E. Prange, Phys. Rev. **171**, B737 (1968).

[13]A. M. Kosevich and I. M. Lifshitz, Zh. Eksp. Teor. Fiz. **29**, 743 (1955) [Sov. Phys. JETP **2**, 646 (1956)].

[14]H.-L Engquist and P. W. Anderson, Phys. Rev. B **24**, 1151

(1981).

[15]M. Büttiker, Phys. Rev. Lett. **57**, 1761 (1986), and IBM J. Res. Dev. (to be published).

[16]R. Landauer, IBM J. Res. Dev. **1**, 223 (1957), and Z. Phys. B **68**, 217 (1987).

[17]As shown experimentally in Ref. 2, such a relation between R and N does exist for the two-terminal resistance $R_{2t} \equiv (\mu_1 - \mu_2)/eI = h/2Ne^2$.

[18]The negative magnetoresistance seen in the longitudinal voltage drop is, as we have argued elsewhere, evidence for reduced backscattering in a magnetic field. See H. van Houten, C. W. J. Beenakker, P. H. M. van Loosdrecht, T. J. Thornton, H. Ahmed, M. Pepper, C. T. Foxon, and J. J. Harris, Phys. Rev. B **37**, 8534 (1988).