Model of L- to H-Mode Transition in Tokamak

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A new model of L- to H-mode transition in tokamak plasmas is presented. Nonambipolar particle losses determine the consistent radial electric field near plasma periphery. A "cusp-type catastrophe" among the radial electric field, particle flux, and edge gradients is found. At the transition, plasma loss can take multiple values for one profile of density and temperature near the edge. A critical edge condition for the transition is obtained.

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After the discovery of the H mode in ASDEX,[1] the transition between the L and H modes has been observed in many other machines.[2-7] The H mode has made a breakthrough in the improvement of the plasma parameters, and has cast a new problem of the anomalous transport across the magnetic surface. Although there have been several theoretical works on the modeling of the H mode,[8-20] the transition characteristics[8-20] have not been fully explained. The transition occurs in both the separatrix and limiter configurations,[5] and originates from the change of the transport inside the outermost magnetic surface. There must be a critical condition for the electron temperature and/or density, and/or their gradients, near the edge.[20] At the onset of the transition, the heat flux which is going out of the plasma surface, q_{out}, suddenly decreases while the temperature and density remain almost unaltered. A finite time is necessary for the change of the background plasma parameters.

These experimental observations suggest that the transition should have a property that the heat and particle fluxes have two or multiple values for one condition of the temperature and density. The radial electric field is a candidate to play the role to cause such a transition as a hidden variable.

In this article, we propose a possible mechanism of the L- to H-mode (H- to L-mode) transition associated with the changes in the particle flux and the convective energy loss near the plasma edge. Nonambipolar ion loss near the plasma periphery has been discussed by Hinton[11] and Ohkawa.[14] Extending their theory, we include the nonambipolar electron loss in order to obtain the consistent radial electric field. A bifurcation in the particle flux associated with the change of the radial electric field is found. By this mechanism, the particle and convective-energy fluxes can have multiple values for the same density and temperature condition near the periphery. This transition occurs in both the limiter and separatrix configurations. The critical edge condition for the transition is obtained.

We study the zero-net-current condition to obtain the radial electric field, E_r. The radial fluxes of the electrons and ions near the plasma periphery (inside the outermost magnetic surface) are calculated. The plasma surface is determined either by the limiter or by the separatrix.

The ion flux in the region of $|a-r| \lesssim \rho_p$ is attributed to the direct loss (r, minor radius; a, plasma minor radius; ρ_p, poloidal gyroradius, $v_T m_i qR/aeB_t$; q, safety factor; R, major radius; v_T, thermal velocity of ions; B_t, toroidal magnetic field). The ion loss is given by[11]

$$\Gamma_i (n_i/\sqrt{\epsilon}\tau_{ii})\rho_p F, \tag{1}$$

where the coefficient F is proportional to the relative number in the loss cone in the velocity space and $0<F<1$ ($\epsilon = r/R$, τ_{ii}^{-1} is the ion-ion collision frequency, and n_i is the ion density). In order to estimate F in the presence of E_r, we make two simple assumptions: (1) that the ions which satisfy the resonance condition $v_\parallel/qR = E_r/rB$ are lost directly, namely the loss cone shifts in the velocity space, and (2) that the ion distribution function f_i is close to Maxwellian, $f_i \propto \exp\{-v_\parallel^2/v_T^2\}$. We then have the estimation $F \propto \exp\{-(\rho_p eE_r/T_i)^2\}$, and the ion loss has the form

$$\Gamma_i = \frac{1}{\sqrt{\epsilon}} \frac{n_i}{\tau_{ii}} \rho_p \hat{F} \exp\left\{ -\left(\frac{\rho_p eE_r}{T_i} \right)^2 \right\}, \tag{2}$$

where the coefficient $\hat{F}$ is the contribution of the bounce average and may weakly depend on E_r.

The electron loss is affected by the microturbulence. This turbulence-driven loss can also be nonambipolar (NA). This is because the local momentum balance between the electrons and ions via waves[21,22] does not always hold near the edge: The wave can propagate across the plasma surface to the scrapeoff layer so as to take the electron momentum away. It is also nonambipolar if it is driven by the magnetic braiding.[23] We therefore take the NA part of the electron loss and write as

$$\Gamma_e^{(\mathrm{NA})} = -D_e n \left[\frac{n'}{n} + \frac{eE_r}{T_e} + \frac{\alpha T_e'}{T_e} \right], \tag{3}$$

where α is a numerical constant of the order of unity. D_e can be represented in a form of the local turbulence spectra.[21-23]

Imposing the zero-net-current condition, i.e., $\Gamma_i^{(\mathrm{NA})} = \Gamma_e^{(\mathrm{NA})} (\equiv \Gamma)$, we have the equation to determine E_r as

$$\exp\left\{ -\left[\frac{\rho_p e E_r}{T_i} \right]^2 \right\} = d \left\{ \lambda - \left[\frac{\rho_p e E_r}{T_i} \right] \right\}, \tag{4}$$

where $d = D_e \tau_{ii} \sqrt{\epsilon} / \hat{F} \rho_p^2$ and $\lambda = -T_e \rho_p (n_e'/n_e + \alpha T_e'/T_e)/T_i$. The convective energy loss associated with Γ^{NA} is given by $Q_{\mathrm{conv}}^{\mathrm{NA}} = (T_e + T_i)\Gamma$. The total particle flux, Γ_{tot}, is the sum of Γ^{NA} and the intrinsic ambipolar component. The total convective-energy loss is given by $Q_{\mathrm{conv}}^{\mathrm{tot}} = (T_e + T_i)\Gamma_{\mathrm{tot}}$.

Figure 1 illustrates the normalized flux $\hat{\Gamma}_{e,i}$ ($\hat{\Gamma}_{e,i} = \Gamma_{e,i} \tau_{ii} \sqrt{\epsilon} / \hat{F} \rho_p n_i$), as a function of $\rho_p e E_r / T_i$. (The coefficient d may have a weak E_r dependence. We assume that d is independent of E_r.) As is seen from Fig. 1, Eq. (4) predicts the transitions of E_r and Γ. When λ is small, Eq. (4) has one real solution. With an increase of λ or d, the bifurcation of the solution appears (dashed lines in Fig. 1). For a fixed value of d, the transition occurs at the critical value λ_c of λ.

Figure 2 illustrates the λ dependences of $\hat{\Gamma}$ and $\hat{Q}$ [$\equiv Q_{\mathrm{conv}} \tau_{ii} \sqrt{\epsilon} / \hat{F} \rho_p n_i (T_e + T_i)$]. There appears a cusp-type catastrophe[24,25] (Riemann-Hugoniot catastrophe). The solutions of Eq. (4) form a cusp-type surface in the space of [λ, d, E_r (or $\hat{\Gamma}, \hat{Q}$)].[24] When λ is below the critical value λ_c, the electric field is negative (i.e., is directed inward) and the fluxes are large. If λ exceeds λ_c, the electric field turns to positive and the fluxes are reduced. In the branch of lower fluxes, E_r has an approximate value $eE_r \rho_p / T_i \simeq \lambda$ ($\lambda > \lambda_c$). The L mode corresponds to the branch of the large loss flux, and the H mode is the branch of the reduced loss flux. The transition from the L to H mode takes place as $A \to B' \to C \to C' \to D$ and that from the H to L mode occurs at $D \to C' \to B \to B' \to A$. Because λ_c for the L- to H-mode transition is larger than that for the H- to L-mode transition, there is a hysteresis in the relation of Γ and λ as is shown in Fig. 2(a). The value λ_c is of the order of unity (about 1.5 in this case). This value is in the range of the experimental observation.[18]

After the transition $C \to C'$, the particle flux Γ and the convective-energy flux $Q_{\mathrm{conv}}^{\mathrm{NA}}$ become about $\frac{1}{10}$ times smaller. The loss at the edge, q_{out}, becomes smaller than the heat supply from the core plasma, q_s [$q_s = (P_{\mathrm{heat}} - P_{\mathrm{rad}})/4\pi^2 aR$, where P_{heat} and P_{rad} are the heating power and the radiation loss power, respectively]. Because of the sudden increase of $q_s - q_{\mathrm{out}}$, the gradients T' and n' start to grow. This reduction of loss occurs in a thin layer near the boundary, $|a-r| \lesssim \rho_p$, and gives rise to a formation of the temperature and density pedestal.

The normalized loss flux in the H branch in Fig. 2(a) is a decreasing function of the gradient. After the tran-

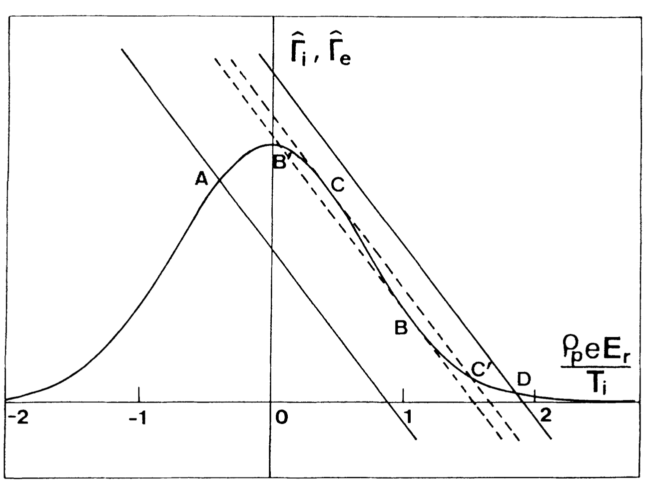

FIG. 1. Fluxes $\hat{\Gamma}_i$, $\hat{\Gamma}_e$ vs the radial electric field. Four cases of λ (with constant d) are shown. Dashed lines indicate the bifurcation condition.

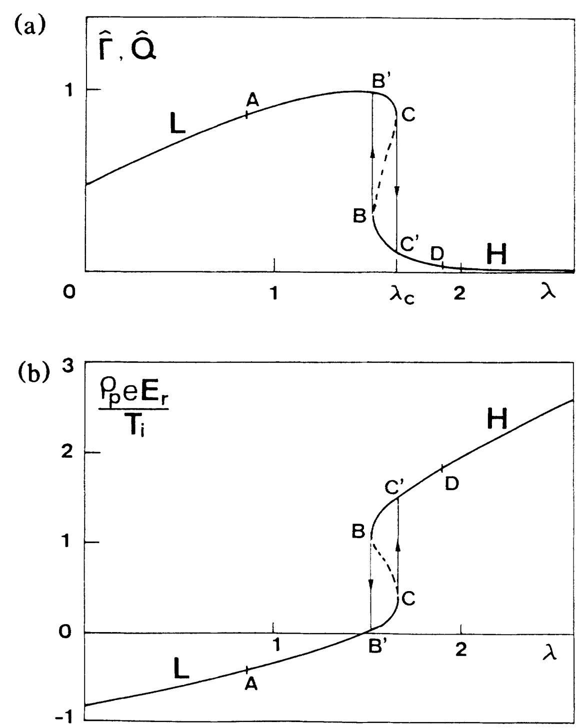

FIG. 2. Solutions for (a) the flux and (b) radial electric field as functions of λ (for the case of $d = 1.3$). Points A to D correspond to those in Fig. 1. Transition from the branch of large flux to that of small flux occurs at $\lambda = \lambda_c$.

sition $C \rightarrow C'$, a stationary state may not be realized, if there is no other loss and if the source is not affected by the transition. The gradient near the boundary continues to increase until the other losses increase and balance with the heat and particle inputs to the layer. There are the intrinsic ambipolar part of the particle flux, conductive-energy loss, charge-exchange loss, radiation loss, loss by the edge localized modes, etc. These losses determine the steepness of the edge gradients in the H phase.

In this model, the threshold value λ_c is predicted. The transition, in principle, can occur even for the zero additional heating power. The threshold power for the L- to H-mode transition is interpreted as the necessary power to reach the critical value λ_c. It depends on the confinement nature of the L mode. The peak value of $\hat{Q}_{\rm conv}$ does not coincide with the threshold power, because there are other losses. Our model predicts that gradients of the order of ρ_p^{-1} (i.e., the high edge temperature and density) are necessary for the L- to H-mode transition.

The result shows that the critical value λ_c is a decreasing function of d. The value of d increases as the ion temperature becomes high. To realize a high ion temperature, a reduction of the neutral-gas density, which requires a lower limit of the electron density, is also necessary.[11,14] The favorable location of the X point relative to the ∇B drift[11] may be explained in relation to the increment of λ and d. Furthermore, the electron edge temperature can be high in the same configuration, because the distance between the source and sink of the heat flux along the field line is longer compared to the case of the opposite ∇B-drift direction.[26] The value of d can also be increased by an increment of D_e. In such a situation, however, the absolute values of Γ and Q must be large to realize the critical value λ_c, i.e., the transition is hard to realize in comparison with the high-T_i case.

According to this model, the transition occurs at the plasma boundary. An improvement of the particle flux is expected and the convective loss to the scrapeoff layer should be reduced after the transition. Improvement of the particle confinement may further raise the edge temperature, and hence the conduction to the scrapeoff layer may be also reduced because of the collisionless neoclassical effect.[11] The reduction of H_α/D_α radiation is considered to be the result of the reduced convection and conduction losses.

Our model predicts the change of the radial electric field associated with the transition. An indication of the potential change has been observed.[27] The experimental data in Fig. 4 of Ref. 18 seem to support the picture that the loss takes two values for one temperature near the edge at the transition condition. The picture of the multiple flux solution is also supported in the observation that the transition is triggered by a small sawtooth. One of two successive sawteeth, for instance, can trigger the transition even if they carry heat flux of similar magnitude. The difference between the modifications of the background profile by the two sawteeth is not so much. However, the transport barrier is formed only by the second one, and the other pulse is not enough to raise the edge gradient to the critical value.

In the evaluation of λ_c, the crudest assumption lies in the form of F. It has also been pointed out that the radial gradient of E_r influences the ion orbit.[28] The zero-net-current condition may reduce to a differential equation. The precise form of F may differ from the one in Eq. (2). However, the assumptions do not change the general and important characteristics of F that $0 < F < 1$ and that $F \rightarrow 0$ as the loss cone deviates from the peak of the distribution function in the velocity space. The transition of E_r between positive and negative values is a consequence of the model assumption. The absolute values of E_r at the transition condition $\lambda = \lambda_c$ would also be subject to change if the estimation of F were more accurate. The precise evaluation of F in Eq. (2) and the improved estimation of λ_c are left for future work.

The mechanism for improvement of the core plasma confinement is beyond the scope of this article. Improvement in the core plasma may be a consequence of improvement of the edge confinement. Mechanisms to reduce the turbulence-driven anomalous loss in the case of the high edge temperature have been discussed.[15,17] The increment of the edge temperature due to reduction of the convective loss may stabilize the resistive modes so that further reduction of the conductive loss takes place after the transition.

We finally note that the transition can cause shear flow of the $\mathbf{E}\times\mathbf{B}$ motion. The Kelvin-Helmholtz instability can appear if $\partial E_r/\partial r$ becomes large enough.[29] The threshold gradient of E_r depends on various parameters[30] and a precise comparison requires future work.

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