# Enhancement Effects of the *P*-Conserving *T*-Invariance Violation in Neutron Transmission

V. E. Bunakov[(a)]
*Max-Planck Institut für Kernphysik, 6900 Heidelberg, West Germany*
(Received 23 March 1988)

The theoretical estimates of the *P*-conserving *T*-invariance violation in neutron transmission are reexamined. It is shown that the previously omitted factors of dynamical enhancement caused by the *T*-nonconserving interaction of the two closely lying resonances might sum up to an overall amplification of the effect by 3–5 orders of magnitude.

PACS numbers: 25.40.Dn, 11.30.Er, 24.30.He, 24.60.Dr

The *P*-conserving, *T*-nonconserving term in the elastic neutron-nucleus forward-scattering amplitude of the type $\boldsymbol{\sigma}\cdot(\mathbf{k}\times\mathbf{I})(\mathbf{k}\cdot\mathbf{I})$ (where $\boldsymbol{\sigma}$, $\mathbf{I}$, and $\mathbf{k}$ are the neutron and nuclear spins and neutron momentum, respectively) was first mentioned by Baryshevsky[1]; however, attention was not paid to the fact that since this term is *P* conserving it might be larger than the *P*- and *T*-nonconserving one. Direct measurements of this term in polarized-beam transmission through an oriented-nuclei sample were suggested by Kabir.[2] It was shown that this term causes the difference in total cross sections $\sigma_{\rightarrow}$ and $\sigma_{\leftarrow}$ for the transmission of neutrons with opposite polarizations, parallel and antiparallel to the $\mathbf{k}\times\mathbf{I}$ axis,

$$\Delta_T=\sigma_{\rightarrow}-\sigma_{\leftarrow}. \tag{1}$$

Thus one can observe in this experiment the effect of *P*-conserving *T*-invariance violation:

$$\beta=\frac{\Delta_T}{\sigma_{\rightarrow}+\sigma_{\leftarrow}}\approx\frac{\Delta_T}{2\sigma_{\text{tot}}}. \tag{2}$$

The magnitude of this effect was estimated independently by Barabanov[3] and Bunakov and Gudkov[4] to reach in the vicinity of an isolated compound *p* resonance its maximal value

$$\beta\approx\phi, \tag{3}$$

where $\phi$ is approximately given by the ratio of the variances of the matrix elements of *T*-nonconserving and *T*-invariant potentials.

In the present paper it is demonstrated that a major enhancement factor was overlooked in this analysis. Indeed the general expression for $\Delta_T$ was found to be (see, e.g., Ref. 4) (the angle between $\mathbf{k}$ and the target alignment axis is assumed to be $\pi/4$ while the beam-polarization and target-orientation parameters are unities)

$$\Delta_T=(4\pi/k)\,\text{Im}f_T \tag{4}$$

where

$$f_T=\frac{A}{k}\Bigg[3\sqrt{2}\begin{Bmatrix} I & 2 & 1\\ I-1/2 & J & I+1/2\end{Bmatrix}(\langle I+\tfrac{1}{2},1|R^J|I-\tfrac{1}{2},1\rangle-\langle I-\tfrac{1}{2},1|R^J|I+\tfrac{1}{2},1\rangle)$$
$$+(-1)^{2I}\left[\frac{3}{2I}\right]^{1/2}(\langle 1+\tfrac{1}{2},2|R^J|I-\tfrac{1}{2},0\rangle-\langle I-\tfrac{1}{2},0|R^J|I+\tfrac{1}{2},2\rangle)\delta_{J,I-1/2}$$
$$+(-1)^{2I}\left[\frac{3}{2(I+1)}\right]^{1/2}(\langle I+\tfrac{1}{2},2|R^J|I-\tfrac{1}{2},0\rangle-\langle I-\tfrac{1}{2},0|R^J|I+\tfrac{1}{2},2\rangle)\delta_{J,I+1/2}\Bigg], \tag{5}$$
$$A=(2J+1)\left[\frac{5I(2I-1)}{(2I+1)(2I+3)}\right]^{1/2}(-1)^{I+3/2-J}.$$

Here $R=1-S$, $S$ being the scattering matrix. The notation $\langle s',l'|R^J|s,l\rangle$ is used for the matrix elements where $l$ and $l'$ are the orbital momenta of the initial and final channels, $s$ and $s'$ are the corresponding channel spins, and $J$ is the compound-nucleus spin.

One might now recollect the general expression obtained by Mahaux and Weidenmüller[5] for the quantity $\delta S_{\lambda\mu}=S_{\lambda\mu}-S_{\mu\lambda}$ arising in the presence of the *T*-nonconserving part $H'$ in the Hamiltonian. In the case of two isolated ($\Gamma\ll D$) resonances (say the *p*-wave ones) this expression becomes

$$\delta S_{\lambda\mu}=-2(2\pi)^{1/2}\left[\frac{\Gamma_{1\mu}^{1/2}\text{Im}\langle\psi_E^{(\lambda)}|H'|\psi_i\rangle-\Gamma_{1\lambda}^{1/2}\text{Im}\langle\psi_E^{(\mu)}|H'|\psi_2\rangle}{E-\epsilon_1}+\frac{\Gamma_{2\mu}^{1/2}\text{Im}\langle\psi_E^{(\lambda)}|H'|\psi_1\rangle-\Gamma_{2\lambda}^{1/2}\text{Im}\langle\psi_E^{(\mu)}|H'|\psi_2\rangle}{E-\epsilon_2}\right]$$
$$+2i\frac{\Gamma_{1\mu}^{1/2}\Gamma_{2\lambda}^{1/2}\mathcal{H}_{12}+\Gamma_{2\mu}^{1/2}\Gamma_{1\lambda}^{1/2}\mathcal{H}_{21}}{(E-\epsilon_1)(E-\epsilon_2)}. \tag{6}$$

Here $\Gamma_{i\alpha}^{1/2}$ are the amplitudes of the partial widths for the decay of the $i$th resonance into the channel $\alpha$ caused by the strong-interaction potential (they are essentially real for $\Gamma \ll D$). The amplitudes $\langle \psi_E^g | H' | \psi_i \rangle$ define the additions to these partial widths caused by the $T$-noninvariant interaction $H'$. The quantities $\mathcal{H}_{12} = -\mathcal{H}_{21} = \langle \psi_1 | H' | \psi_2 \rangle$ define the mixture of compound resonances 1 and 2 with eigenenergies $\epsilon_i = E_i - i\Gamma_i/2$ caused by the perturbation $H'$. One can easily see that only the first term of (6) was taken into account in the analysis of Refs. 3 and 4. Consider now the last term of this expression. From the general considerations of $T$ reversibility, one can assume the matrix elements $\mathcal{H}_{12}$ to be purely imaginary:

$$\mathcal{H}_{12} = i v_T. \tag{7}$$

Therefore

$$\mathrm{Im}\delta S_{\lambda\mu} = -b[(E-E_1)\Gamma_2 + (E-E_2)\Gamma_1]/[(E-E_1)^2 + \tfrac{1}{4}\Gamma_1^2][(E-E_2)^2 + \tfrac{1}{4}\Gamma_2^2], \tag{8}$$

where

$$b = v_T(\Gamma_{1\mu}^{1/2}\Gamma_{2\lambda}^{1/2} - \Gamma_{2\mu}^{1/2}\Gamma_{1\lambda}^{1/2}) = v_T a. \tag{9}$$

Substituting now Eqs. (8) and (9) into Eqs. (5) and (4), I get for the contribution to $\Delta_T$ from the two near-lying $p$-wave resonances

$$\Delta \approx \frac{4\pi}{k^2} \frac{[(\Gamma_{1(-)}^n \Gamma_{2(+)}^n)^{1/2} - (\Gamma_{2(-)}^n \Gamma_{1(+)}^n)^{1/2}] v_T}{[(E-E_1)^2 + \Gamma_1^2/4][(E-E_2)^2 + \Gamma_2^2/4]} \equiv \frac{(4\pi/k^2) a_{12} v_T}{[(E-E_1)^2 + \Gamma_1^2/4][(E-E_2)^2 + \Gamma_2^2/4]}. \tag{10}$$

I have omitted for simplicity the geometrical factors and 6-$j$ symbols of Eq. (5). $\Gamma_{i(\pm)}^n$ stands here for the neutron partial widths of the $i$th resonance decay into the channels $I \pm \frac{1}{2}$.

Usually in the case of low-energy neutrons the cross section $\sigma_{\mathrm{tot}}$ is dominated by the tail of the nearest $s$-wave resonance:

$$\sigma_{\mathrm{tot}} \approx \frac{\pi}{k^2} \frac{\Gamma_s^n \Gamma_s}{(E-E_s)^2 + \Gamma_s^2/4}. \tag{11}$$

Therefore

$$\beta(E) \approx b_{12} \frac{[(E-E_1)\Gamma_2 + (E-E_2)\Gamma_1][(E-E_s)^2 + \Gamma_s^2/4]}{[(E-E_1)^2 + \Gamma_1^2/4][(E-E_2)^2 + \Gamma_2^2/4]\Gamma_s^n \Gamma_s}. \tag{12}$$

In the vicinity of the $i$th $p$-wave resonance $\beta(E \approx E$ displays a resonance behavior and reaches its maxim value:

$$\beta(E_i) \approx \frac{8a_{12}}{\Gamma_s^n} \frac{\Gamma_k}{\Gamma_i} \frac{(E_i - E_s)^2}{\Gamma_s \Gamma_i} \frac{v_T}{D}. \tag{13}$$

Here $D = |E_1 - E_2|$. Assuming all the $\Gamma_{i(\pm)}^n$ values in $a_{12}$ to be of the order of $\Gamma_p^n$ we can estimate $a_{12} \approx \Gamma_p^n$. The typical ratio $\Gamma_p^n/\Gamma_s^n$ is defined by the ratio of $p$ and $s$ penetrabilities: $\Gamma_p^n/\Gamma_s^n \approx (kR)^2$. Normally $\Gamma_i \approx \Gamma_k \approx \Gamma_s = \Gamma$. Thus the resonance values are

$$\beta_{\mathrm{res}} = \beta(E_1) \approx \beta(E_2) \approx (kR)^2 \frac{(E_p - E_s)^2}{\Gamma^2} \frac{v_T}{D}. \tag{14}$$

Between the two $p$ resonances ($E = \bar{E} \approx |E_1 + E_2|/2$) a lower value is obtained:

$$\bar{\beta} \approx \frac{8a_{12}}{\Gamma_s 6n} \frac{\Gamma_p}{\Gamma_s} \frac{(\bar{E} - E_s)^2}{D^2} \frac{v_T}{D} \approx (kR)^2 \frac{(\bar{E} - E_s)^2}{D^2} \frac{v_T}{D}, \tag{15}$$

or in the typical case of $|\bar{E} - E_s| \approx |\bar{E} - E_p|$,

$$\bar{\beta} \approx (kR)^2 v_T/D. \tag{16}$$

Thus we observe in $\beta$ the typical hindrance factor $(kR)^2$ (see Bunakov and Gudkov[4,6]) caused by the presence of $k^2$ in the $T$-nonconserving correlation considered and therefore by the necessity to have $p$ waves in both the initial and final channels. But in the neighborhood of the $p$-wave resonance it is compensated by the resonance enhancement factor $|E_p - E_s|^2/\Gamma^2$. While the $s$-wave resonance tail dominates in the total cross section [i.e., while $\sigma_s(E_p) \geq \sigma_p(E_p)$], this compensation is incomplete since in this case

$$\frac{\Gamma_p^n}{\Gamma_s^n} \frac{|E_p - E_s|}{\Gamma^2} \approx (kR)^2 \frac{|E_p - E_s|^2}{\Gamma^2} \leq 1.$$

However, it is easy to see [by just assuming $\sigma_{\mathrm{tot}} \approx \sigma_p$ instead of (11)] that as soon as the contribution of the $p$ resonance to $\sigma_{\mathrm{tot}}$ equals the $\sigma_s(E_p)$ background or exceeds it (the case of "isolated" $p$ resonance), the resonance enhancement factor exactly cancels the hindrance factor $(kR)^2$ so that

$$\beta_{\rm res} \approx v_T/D. \tag{17}$$

When $\sigma_p$ starts to dominate in $\sigma_{\rm tot}$ over a broader region, the resonance maximum in $\beta$ is also broadened over the whole region where $\sigma_p(E) \approx \sigma_{\rm tot}(E)$:

$$\beta(E) \approx \frac{v_T}{D} \frac{\sigma_p(E)}{\sigma_{\rm tot}(E)}. \tag{18}$$

The analysis on the same lines of the $s \leftrightarrow d$-wave terms in Eq. (5) shows that they are smaller than the $p \rightarrow p$ terms considered above by approximately a factor $(kR)^2$.

Let us now analyze the quantity $v_T/D$ in more detail. First I demonstrate that it contains the so-called dynamical enhancement factor $\sqrt{N}$ (where $N$ is the number of simple-structure components building up the compound-resonance wave function). To do this the scaling factor $\phi$ can be introduced between the $T$-noninvariant matrix element $v_T$ and the usual strong-interaction matrix element $v$ between the same states $\psi_1$ and $\psi_2$. To be exact, since the structure of the compound resonances involved is quite complicated, we can make only statistical estimates of the variances $\bar{v}_T$ and $\bar{v}$:

$$\bar{v}_T \approx \phi \bar{v}. \tag{19}$$

In order to estimate $\bar{v}$ roughly one can recall the usual expression for the spreading width of a single-particle resonance $\Gamma_{\rm SPR} = 2\pi \bar{v}^2 \rho_d$ (here $\rho_d$ is the density of the doorway states). In the statistical limit of the black nucleus the single-particle mode is spread over the characteristic distance $D_0$ between the single-particle states and distributed evenly over all the compound states with average spacing $\bar{D}$:

$$D_0 \approx \bar{v}^2/\bar{D}.$$

Therefore $\bar{v} \approx (D_0/\bar{D})^{1/2}$ and we get

$$\frac{\bar{v}_T}{D} \approx \phi \left( \frac{D_0}{\bar{D}} \right)^{1/2} \frac{\bar{D}}{D} \approx \phi \sqrt{N} \frac{\bar{D}}{D}. \tag{20}$$

Thus it is seen that the quantity (20) really contains the factor of dynamical enhancement $\sqrt{N}$ well known in the $P$-invariance violation for bound states (see Blin-Stoyle[7] and Shapiro[8]). A fair estimate of this factor which is confirmed by the experience of reaction theory for both strong (see, e.g., Mahaux and Weidenmüller[9]) and weak (see, e.g., Ref. 6) interactions is

$$\sqrt{N} \approx (10^3\,{\rm eV}^{1/2})/\bar{D}^{1/2}. \tag{21}$$

The more refined computations[10] with surface-$\delta$ forces also confirm this order-of-magnitude estimate.

Therefore for the medium-weight nuclei the dynamical enhancement factor is $\simeq 10^2$–$10^3$. So if we do not make special efforts to find the unusually close-lying strong $p$ resonances, we can observe in the medium-heavy nuclei

$$\beta \approx 10^3 \phi. \tag{22}$$

Since the existing upper limit on $\phi$ from the detailed-balance tests and neutron electric dipole moment $D_n$ seems to be not better than $10^{-3}$–$10^{-4}$ (see, e.g., Herczeg,[11] but mind that the estimates of $D_n$ are strongly model dependent and therefore quite unreliable) any experimental observation of $\beta$ with accuracy better than $10^{-1}$ will push this limit down. The situation looks even better if one finds experimentally two closely lying $p$ resonances with $D \ll \bar{D}$. Then the factor $\bar{D}/D$ in (20) might improve the situation by at any rate several orders of magnitude. One should point out that in the case of $T$-invariance violation considered one can exploit the $\bar{D}/D$ factor more liberally than in the majority of cases of the $P$-invariance violation in nuclear reactions where normally the strong $s$-resonance contribution to $\sigma_{\rm tot}$ in the denominators cancels the effects and does not allow an increase in it by pushing the mixing $s$ and $p$ resonances too close to each other (see Bunakov and coworkers[4,6,12]). In the case of $\beta$, pushing the two $p$ resonances closer together is prohibited only by the normal strong-interaction resonance repulsion. Obviously, when $D \leq \Gamma$ one needs to generalize the theory to the case of overlapping resonances. Work on this lines is being done presently.

But even without going to the extremes of $D \leq \Gamma$ it can be seen that the enhancement factors in Eq. (20) might sum up to a total amplification of $\simeq 10^5$, thus making the experimental searches of the resonance $\beta$ values perhaps the most sensitive probe of the $P$-conserving $T$-invariance violation.

The author greatly appreciates stimulating discussions with Dr. D. Davis who, but for his modesty, might be a co-author of this paper. He is also grateful to Professor H. A. Weidenmüller for his hospitality at the Max-Planck-Insitut.

[a]On leave of absence from Leningrad Institute of Nuclear Physics, 188350 Gatchina, Leningrad District, USSR.

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