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Acceptable Density Perturbations from Inflation Due to Quantum Gravitational Damping

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Nonperturbative quantum gravitational effects can act as a damping mechanism in primordial inflation. It is shown that this effect can reduce the value of $\delta\rho/\rho$ to an acceptable value of $\sim 10^{-4}$ in primordial inflation. The same effect reduces the gravitational-wave background by a factor 10^{-6}, making the primordial inflation viable.

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One of the major successes of the inflationary model is the generation of classical "seed" perturbations from quantum fluctuations.[1-4] Inflation predicts a nearly scale-invariant spectrum of the form

$$\left(\frac{\delta\rho}{\rho}\right)^2_{k,t_*(k)} \equiv \delta^2(\mathbf{k};t_*(k)) = \frac{\alpha}{|\mathbf{k}|^3}\ln(kH^{-1}), \quad (1)$$

where $\mathbf{k}$ is the comoving wave vector, H is the Hubble constant during inflation, $t_*(\mathbf{k})$ is the time at which $2\pi|\mathbf{k}|^{-1}a(t_*)=H^{-1}$, and $a(t)$ is the expansion factor. But, unfortunately, the generic value of the amplitude α turns out to be too large. To reduce α to acceptable values one has to fine tune the parameters of the inflationary potential in an unnatural manner.

In this Letter I will describe how the amplitude α can be reduced by a large factor ($\sim 10^{-6}$) by quantum gravitational damping. This damping mechanism is relevant only if the inflation is primordial—i.e., only if the energy density driving inflation $V_0 \simeq 10^{19}$ GeV.[4] Normally, such a primordial inflation would have produced too much of gravitational-wave perturbations and hence would have been ruled out.[5,6] However, the same damping mechanism reduces the gravitational-wave perturbations as well, thereby making the model viable.

To see how this damping works, recall the origin of (1) in the standard scenario (I will follow the analysis of Brandenberger.[7]) The $\mathbf{k}$ dependence arises essentially from the Fourier transform of the correlation (Green's) function:

$$[\delta\rho(\mathbf{k},t)]^2 \sim [\Delta\phi(\mathbf{k},t)]^2 = \int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}}\langle\phi(\mathbf{x},t)\phi(0,t)\rangle = \int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}}G(\mathbf{x},t). \quad (2)$$

For the scales which are of interest, $G(\mathbf{x},t)$ can be adequately approximated[7] by the free-field Green's function $[4\pi^2a^2(t)|\mathbf{x}|^2]^{-1}$ so that

$$[\delta\rho(\mathbf{k},t)]^2 \cong \text{const}\times\int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}}[a^2(t)|\mathbf{x}|^2]^{-1} \quad (3)$$

$$= \text{const}\times[a^2(t)|\mathbf{k}|]^{-1}. \quad (4)$$

We are interested in $\delta\rho(\mathbf{k},t)$ at the time $t=t_*(k)$, which is the epoch at which mode $\mathbf{k}$ "leaves the horizon." At $t=t_*$, $a=a(t_*)=(k/2\pi H)$ and hence we get

$$[\delta\rho(\mathbf{k},t_*(\mathbf{k}))]^2 = \text{const}\times k^{-3}. \quad (5)$$

The above analysis treats gravity as classical. I now ask, "How does quantum gravity modify the above result?" Since we do not have a quantum theory of gravity, such a question has no unambiguous answer. However, one can draw certain general conclusions.

The central quantity in the above analysis was the Green's function $G(x,y;g_{ik})$ evaluated in a given classical metric g_{ik}. If quantum fluctuations of gravity are taken into account, then one has to average $G(x,y;g_{ik})$ over all possible geometries. An exact calculation of such an average is, again, intractable. However, as a first approximation, one can average $G(x,y;g_{ik})$ over all geometries which are conformally related to g_{ik}. Such a calculation can be performed (see Padmanabhan[8]). The result replaces the Fourier transform of the Green's function,

$$G(\mathbf{k})=[2|\mathbf{k}|a^2(t)]^{-1}, \tag{6}$$

by the averaged value

$$\langle G(\mathbf{k})\rangle=[2|\mathbf{k}|a^2(t)]^{-1}\exp[-L_P/2\pi k^{-1}a(t)], \tag{7}$$

where $L_P^2=4\pi G/3$. [This result can be obtained by repetition of the analysis leading to Eq. (64) of Ref. 8, in the context of Friedmann-Robertson-Walker universes.] *The form of (7) implies that proper wavelengths smaller than the Planck length are exponentially damped.*

Even though (7) is derived by an approximate model for quantum gravity, it is very likely to reflect a generic feature of a full theory of quantum gravity. In support of this view, let me give three reasons: (i) Because of (7), modes with proper wavelengths far smaller than L_P have vanishing correlation. Such a lack of coherence[9] is to be expected because of random fluctuations of geometry at small scales. (ii) Equation (7) indicates that gravity acts as an "infrared regulator." Several model calculations support this view.[10,11] DeWitt, for example,[10] obtains exactly the same result as (7) by a partial summation of Feynman diagrams. This shows that (7) is not overly model dependent. (iii) The finiteness of string theories of gravity is essentially due to a natural small-distance cutoff at $\simeq L_P$. Thus it is reasonable to expect a result like (7) to arise from string models.[12] Because of these reasons, I expect quantum gravitational corrections to be generically of the type indicated by (7).

It is now easy to compute the modified $\delta\rho(\mathbf{k},t_*)$. Note that, at $t=t_*$, $2\pi k^{-1}a(t_*)=H^{-1}$. Thus our damping term in (7) reduces the original value of $\delta\rho/\rho$ by the factor

$$f=\exp(-\tfrac{1}{2}L_PH)=\exp(-\tfrac{1}{6}\pi\sqrt{32}\epsilon^2), \tag{8}$$

where I have taken the energy density V_0 driving the inflation as $V_0^{1/4}=\epsilon M_P$ where M_P ($\simeq 10^{19}$ GeV) is the Planck mass. Conventional analysis without fine tuning gives an amplitude α in the range of 1 to 50 when the fluctuations reenter the "horizon." Observationally one would like this number to be about 10^{-4} to 10^{-5}. Thus we need a factor f in the range 10^{-4} to 10^{-6}. This is easily achieved for ϵ in the range 1.8 to 2.2. Note that the only dimensionless parameter in the theory, ϵ, is of order unity and produces the correct $\delta\rho/\rho$. It should also be stressed that the scale-invariant nature of $(\delta\rho/\rho)^2 \propto \alpha k^{-3}$ is preserved by the correction term, because, at $t=t_*$, the exponential factor is independent of $\mathbf{k}$.

We also see from (8) that the damping factor is significant only for "primordial inflation," i.e., for $V_0^{1/4} \sim M_P$. (For grand-unified-theory inflation, $\epsilon \simeq 10^{-4}$ and $f-1\sim 10^{-8}$.) Usually such an inflation produces too many background gravitons, and hence will lead to unacceptably large temperature fluctuations in the cosmic microwave background radiation. However, for $\epsilon \sim 1$, the gravitational-wave modes are also damped by the same factor. To see this, recall that the characteristic fluctuations in the gravitational-wave modes are given by [see, e.g., Eq. (12) of Abbott and Harari[13]]

$$\Delta h^2(\mathbf{k})=\frac{k^3}{(2\pi)^3}\tfrac{1}{2}\int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}}\langle h_{ij}(\mathbf{x},t)h_{ij}(\mathbf{0},t)\rangle = \frac{k^3}{(2\pi)^3}\tfrac{1}{2}\mathcal{G}(\mathbf{k},t), \tag{9}$$

where $\mathcal{G}(\mathbf{k},t)$ is the spatial Fourier transform of the Green's function for a massless, spin-two graviton in the Friedmann-Robertson-Walker universe. Averaging over all conformally related Friedmann-Robertson-Walker models will now replace $\mathcal{G}(\mathbf{k})$ by $\mathcal{G}(\mathbf{k})f$ at $t=t_*$. Since $f\simeq 10^{-4}$ to 10^{-6} we easily bypass the constraint on primordial inflation.

Let me summarize the basic idea and the results. *Quantum gravity damps the propagation of modes with proper wavelengths smaller than L_P.* (I have given arguments to support this point of view; in a simple quantum gravity model[8] this feature can be explicitly demonstrated.) This damping mechanism reduces the amplitude of all (zero mass) fluctuations by a factor f given in (7). Since the factor is exponential, large reduction in the amplitude of fluctuations is possible even if the dimensionless constants are kept to be of order unity. It seems natural to consider a model in which the only scale is M_P and all dimensionless parameters are set to order unity.

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