## Gravitational Field of Superconducting Cosmic Strings

In a recent Letter[1] the effect that the current of a superconducting cosmic string has on its motion was studied. In particular, it was found that the energy-momentum tensor (EMT) of an uncharged straight superconducting string lying on the $z$ axis and carrying a current $j$ is given in cylindrical coordinates $(t,\rho,\phi,z)$ by

$$T^{\mu}_{\nu}=\mu\,\mathrm{diag}(1+j^2,0,0,1-j^2)\delta(\rho)/\rho. \quad (1)$$

$\mu$ is the string energy density and $j$ is restricted by $0\leq j\leq 1$.

In this Comment we argue that (1) cannot be the correct form for the EMT of a straight superconducting string. Our argument is based on the gravitational field produced by such an EMT source. The exterior gravitational field of this source can be found from Einstein's equations in vacuum in the following way. First, one might try a static solution. The most general vacuum static solution with cylindrical symmetry is

$$ds^2=-\rho^{2a}dt^2+d\rho^2+\alpha\rho^{2b}d\phi^2+\rho^{2c}dz^2, \quad (2)$$

where $a+b+c=a^2+b^2+c^2=1$ and $\alpha$ is an arbitrary parameter (Kasner solution). But such a solution does not correspond to a source of type (1). In fact, the Israel analysis of line sources[2] shows that (2) leads to an EMT of the form

$$T^{\mu}_{\nu}\propto\mathrm{diag}(1-a,0,1-b,1-c), \quad (3)$$

which shows that a static cylindrical metric has always hoop stresses ($T^{\phi}_{\phi}\neq 0$); unless $b=1$, in which case we have $a=c=0$, i.e., an ordinary cosmic string.[3] Note, incidentally, that (2) can represent the gravitational field of (1) in the limit of small current $j$ when one can neglect terms of order $j^4$; in such a case $a=j^2+O(j^4)$, $b=1+O(j^4)$, and $c=-j^2+O(j^4)$.

We could now try a time-dependent cylindrical solution. However, a cylindrical time-dependent metric with either one or two polarizations has also a Kasner-type singularity structure[4] and leads to a source of type (3).

One might look for more general solutions but since Belinsky, Khalatnikov, and Lifshitz claim[5] that the Kasner-type behavior is the general one—independent of the number of variables—one will always be led to line sources of type (3). Thus it appears that a line source like (1) is not compatible (as a unique source) with general relativity.

The suggestion that (1) cannot be the correct EMT for a superconducting string is confirmed in the analysis by Moss and Poletti,[6] who consider the gravitational field of a straight superconducting string by solving Einstein's equations without taking the line-source limit. They show that the magnetic stresses created by the string current outside the string cannot be neglected. They also obtain an effective EMT on the string in the zero-width limit, which has $T^{\rho}_{\rho}$ and $T^{\phi}_{\phi}\neq 0$. Therefore it differs from (3) and the above analysis applies. But such a source alone cannot be used to obtain the gravitational field of the string; the magnetic stresses outside the string must also be included.

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(Received 12 November 1987)
PACS numbers: 98.80.Cq, 04.20.Jb, 04.40.+c, 11.17.+y

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