Basis for a Characteristic Temperature in Nuclear Fragmentation

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The apparent characteristic temperature observed in the fragmentation of highly excited nuclei is studied by calculations using a simple model consisting of an expanding and emitting source. A characteristic temperature for emission, nearly independent of the initial temperature, is found in the calculations, and the underlying physics is interpreted. This temperature is consistent with the values found in experiments which seem independent of bombarding energy.

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There is increasing experimental evidence to indicate a characteristic temperature for the fragmentation of highly excited nuclear systems. This temperature, on the order of 5 MeV, has been seen in a wide variety of reactions. It has been measured by the determination of the relative yields of excited states in fragments of mass 5, 6, and 8, arising from reactions involving projectiles with incident energies of 35, 60, and 94 MeV/nucleon.[1-5] In each case the data suggest that the fragments arise from a system at a temperature of approximately 5 MeV. In addition, the isotope yields from ultrarelativistic proton collisions[6] are also consistent with a similar temperature.

In search of an explanation for a characteristic temperature I have investigated fragment emission within a schematic model. This model involves an expanding nuclear residue which statistically emits nuclear fragments while expanding. Although the model greatly simplifies the description of the expansion dynamics, it contains physical effects which are expected to be important in a more realistic treatment, and thus its use should provide insight into the real process.

The essential features of the model will be outlined here. The dynamical aspects of the process are treated by the calculation of the density of the residue, $\rho(t)$, at each instant in time. Here ρ is taken as the density of a uniform sphere of radius $R(t)$ containing $A(t)$ nucleons. Both the density and the number of residue nucleons change with time. The particle loss from the residue is calculated with emission rates governed by the phase-space considerations of the Weisskopf detailed-balance procedure.

There is a coupling between the emission process and the instantaneous density of the residue. First, in determining the emission rates, I model the excitation spectrum of the residue by a Fermi gas. The density determines the instantaneous Fermi energy for this, and hence the temperature associated with a given excitation energy. Second, the density determines the separation energy for the emitted fragments. The emission process, on the other hand, influences variations in the density of the residue. Since these are determined by the conservation of energy, they are coupled to the emission process through the changes in both excitation energy and compressional energy which are associated with the removal of particles.

I assume that the total energy is distributed in five forms: (a) collective kinetic energy of the residue, (b) collective compressional energy of the residue, (c) thermal excitation energy of the residue, (d) kinetic energy carried by the emitted particles, and (e) the separation energy associated with the emitted fragments. The emitting system evolves with conservation of the total energy, i.e., the sum of the rates of change for the five types of energy is zero. Each of these rates can be expressed in terms of the two primary functions of time, $A(t)$ and $\rho(t)$, and their derivatives. In order to be able to determine both $A(t)$ and $\rho(t)$, I imposed an additional constraint, namely, that the energy is conserved for the emission process alone in the frame of the expanding surface of the source.

The changes in the thermal excitation energy arise from two effects: particle emission and density variation. For the first, the emission rates are taken at constant density. For the second, the expansion is assumed to occur at constant entropy. In the emission, $C \rightarrow B + b$, it is convenient to divide the removal energy into two contributions: a part Q_b representing the mass difference taken at normal density and a part arising from the difference of the density from this normal value. The latter can be expressed in terms of the compressional energy per particle $\gamma(\rho)$, where this function is related to the equation of state. The total separation energy is then given by $Q_b - A_b\gamma(\rho)$.

The emission rates (dN_b/dt) are obtained, as in Friedman and Lynch,[7]

$$dN_b/dt\, dE_b \propto \pi R(t)^2 (E_b - V_{\mathrm{CB}})\Theta(E_b - V_{\mathrm{CB}}) \times e^{\{A_b[f^*(\rho,T)+\gamma(\rho)]-Q_b\}/T} e^{-E_b/T},$$

where f^* is the free energy per particle of the residue, and V_{CB} is the Coulomb barrier. To obtain the temperature T I assume that the instantaneous thermal excitation energy is that of a finite-temperature Fermi gas having a Fermi energy dependent on density: $\epsilon_{\mathrm{F}}(\rho)$

$=\epsilon_{F0}(\rho/\rho_0)^{2/3}$.

The changes in density are found to be governed by the thermal pressure, which produces a decrease (expansion), and forces represented by the collective compressional energy, which drive the residue toward normal density. Reaction forces between the emitted particles and the residue also produce changes in ρ.

Combining the ingredients outlined above, one can calculate the evolution of any system beginning from a choice of initial values for A, Z, ρ $\dot{\rho}$, ϵ_F, and T, and an equation of state represented by the function $\gamma(\rho)$. From this condition, the system is stepped through a series of small time intervals. Numerical solutions fall into two classes depending on the equation of state and the initial conditions. For one of these, the residue density decreases without limit; for the other, the density undergoes oscillations.

I have performed calculations with the above model to search for a basis for a characteristic temperature. The calculations started with ^{91}Zr at initial temperatures ranging between 15 and 25 MeV, a range in which the density expands without limit. A plot of temperature of the residue versus its density, obtained during the deexcitation process, is shown in Fig. 1(a) for T_{max} of 15, 20, and 25 MeV. These curves show a striking change as the density decreases. The temperature initially falls according to the isentropic expansion until there is a marked change at what appears to be a limiting temperature. Furthermore, one sees that this temperature is approximately the same for each initial temperature. Calculations with different equations of state showed similar behavior but with different limiting temperatures. Another interesting effect is to be seen in Fig. 1(b), which shows that the mass of the instantaneous residue declines sharply at the limiting temperature.

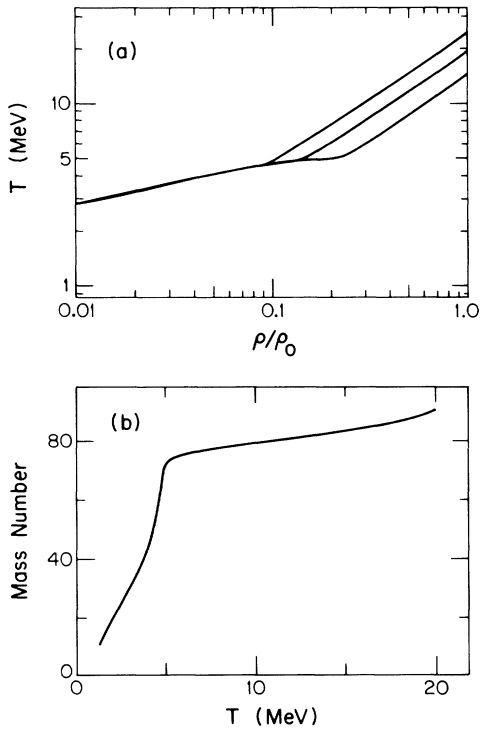

FIG. 1. (a) Temperature vs density, and (b) mass number vs temperature, for the instantaneous residues obtained during the deexcitation process.

Next I analyze the physics in the model which gives rise to the abrupt change in the temperature-density relationship. This analysis provides a means for estimating the specific value of the limiting temperature. Observe that the temperature falls *less* rapidly with reducing density below the critical value. Furthermore, the mass of the residue drops suddenly at precisely the same stage in the process. These two effects are closely related. Imagine an excited system which is expanding under the force of thermal pressure. As the system expands the temperature falls because of particle loss and density change. If the particle loss is small, and the expansion is otherwise isentropic, then $T\propto p^{2/3}$. During the expansion the compressional energy increases as the particles are drawn farther from the attractive influence of their neighbors. When a composite fragment particle is formed and leaves the expanded residue, the nucleons it contains must coalesce. This process releases energy to the fragment's surroundings, i.e., the residue. That energy heats the residue and consequently slows the rate of temperature fall. When particles are emitted from a residue at normal density there is generally a net *loss* of energy from the residue since they carry away both kinetic energy and the required separation energy. However, when the residue is sufficiently expanded coalescence heating becomes important. The heating effect is greatest for those fragments which, once formed, have the highest binding energy per particle. Thus, nucleons and loosely bound fragments do not contribute very much to this effect but their emission is influenced by it.

We can make a simple estimate of the conditions under which coalescence heating becomes important. By so doing we can estimate the limiting temperature. The Weisskopf statistical emission rate given above provides

$$dN_b/dt\propto e^{\{A_b[f^*+\gamma(\rho)]-Q_b\}/T}.$$

The free energy f^* is negative. When ρ equals ρ_0, $\gamma(\rho)=0$, and $\exp(A_bf^*/T-Q_b/T)$ has a small value, since the argument of the exponential is large and negative, especially for heavy fragments. As the density decreases the positive $\gamma(\rho)$ term becomes important. The limiting temperature occurs when the emission rate, especially for heavy particles, is high. This requires that $-f^*(T,\rho)\approx\gamma(\rho)$. With this relationship I estimate the limiting temperature.

For simplicity I take $\gamma(\rho)=(K/18)(1-\rho/\rho_0)^2$. The binding energy per particle at normal density is approximately 8 MeV. I require that this binding energy go to

zero when the density vanishes. A value of 144 MeV for the spring constant K will provide this condition. For K greater than 144 the binding energy per particle reaches zero at nonzero densities. Since positive values for this quantity are not physical at low densities I set $\gamma(\rho)$ to the constant value of 8 MeV for all values of density for which the parabolic form would exceed 8 MeV.

I next determine the values of T and ρ for the condition at which the emission rate becomes large. I assume that the early expansion to reach this condition is isentropic, so that $T=T_{\max}(\rho/\rho_0)^{2/3}$. Therefore the condition is determined by T alone, $-f^*(T)=\gamma((T/T_{\max})^{3/2}\rho_0)$. I evaluate $f^*(T)$ using a finite-temperature Fermi gas. For this case it can be shown that the function $[f^*(T)/T]$ depends only on (T/ϵ_F), where ϵ_F is the Fermi energy. Since ϵ_F is proportional to $\rho^{2/3}$, as is T, the function $f^*(T)/T$ is constant during the initial expansion. Thus $f^*(T)=(T/T_{\max})f^*(T_{\max})$.

Using this relationship with the equation above, we can estimate the limiting temperature for any $T_{\max}$ given K and ϵ_F. Limiting temperatures calculated by this means for $T_{\max}$ ranging from 15 to 25 MeV are shown in Fig. 2. The resulting $T_{\lim}$ is seen to be essentially independent of $T_{\max}$ but to depend weakly on K. We have used $\epsilon_F=25$ MeV, which provides an excitation energy per particle of $T^2/10$ for low energies. With this Fermi energy, and with $K=144$, we find that T is approximately 4.5 and essentially independent of the initial temperature. For ϵ_F taken from 20 to 30, there is about a 1-MeV spread in the corresponding values of T. It is also clear that the specific dynamics of the expansion has no effect on this estimate of the critical temperature. If the system is initially expanding from ρ_0 with a finite value of $\dot{\rho}$, it will reach the same critical temperature, as long as the process is isentropic.

The *density* at which this temperature is reached does vary with the initial temperature, $\rho=\rho_0(T/T_{\max})^{3/2}$.

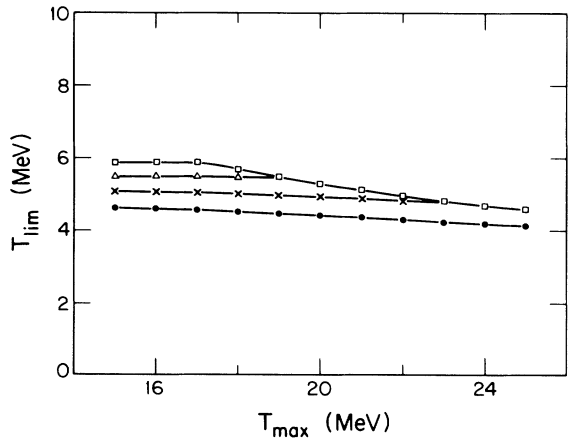

FIG. 2. Limiting temperature vs maximum temperature. The curves, lowest to highest, are for $K=150$, 175, 200, and 250 MeV, respectively, with $\epsilon_F=25$ MeV.

Thus, for the case of $K=144$, with $T=4.5$, we obtain densities of 0.08 and 0.16 ρ_0 for $T_{\max}$ of 25 and 15 MeV, respectively.

In brief, I have found that the rate for particle emission increases dramatically for an isentropically expanding system when a specific temperature is reached. That temperature is nearly independent of the initial temperature and only weakly dependent on both the equation of state (spring constant) and the Fermi energy for normal densities. The temperature is also independent of the initial rate of density expansion.

In detailed calculations I find that the mean temperature for the emission of most of the intermediate-mass fragments is very close to the characteristic temperatures obtained above. The temperature of the residue continues to fall after reaching this value, but the emission rate for heavy fragments decreases greatly *after* the critical condition is reached causing a sharp peak in the emission rate. This peaking may be understood to be a consequence of the coalescence heating. The emission rate of heavy fragments is governed by an exponential whose argument is proportional to the free energy f^*. To get a rough estimate of the behavior of f^* with time let us consider the low-temperature limit where

$$df^*/dt=2(f^*/T)(\dot{\rho})(dT/d\rho-\tfrac{1}{3}T/\rho).$$

For T varying like $\rho^{2/3}$, df^*/dt is positive. When $dT/d\rho\leq\frac{1}{3}T/\rho$, however, df^*/dt is negative. This reversal which occurs at the limiting temperature [see Fig. 1(a)] dramatically reduces the emission rate.

Finally, let us consider the measurement of the relative yield of states separated in energy by ΔE. The fragments in each of the states are emitted during the entire course of the evolution of the residue. If (dN/dT) is the rate for emission of the lower state, then $e^{-\Delta E/T}(dN/dT)$ is the rate for emission of the higher state when the spin degeneracy is the same. The ratio of the yields, R_Y, is then

$$R_Y=\frac{\int_{T_{\max}}^{T_{\min}}e^{-\Delta E/T}(dN/dT)dT}{\int_{T_{\max}}^{T_{\min}}(dN/dT)dT}.$$

The apparent temperature T_A is then defined by $e^{-\Delta E/T_A}=R_Y$. In general T_A need not equal $\langle T\rangle$. However, if dN/dT is strongly peaked, then the two will be close in value. Figure 3 shows the calculated temperature dependence of dN/dT for ^{6}Li fragments. When this distribution is used with ΔE values ranging from 5 to 15 MeV, the values of T_A and $\langle T\rangle$ are found to be nearly equal.

In summary, a simple model has been used to examine the possible basis for a characteristic temperature observed in the fragmentation of highly excited nuclei. The model involves the statistical emission of fragments from sources whose density varies with time. Calculations

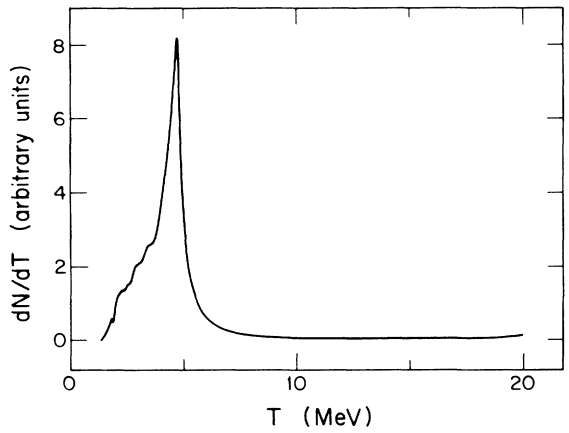

FIG. 3. Differential emission rate (dN/dT) vs temperature for ^{6}Li fragments, from ^{91}Zr, with $T_{max}=20$ MeV, $K=144$ MeV, and $\epsilon_F=25$ MeV.

provide evidence for a limiting temperature, nearly independent of the initial temperature. This feature appears to arise from coalescence heating. A procedure for estimating the limiting temperature has been established. The mean emission temperature and the apparent temperatures obtained from relative yields of excited states are nearly equal to the limiting temperature. This is due to a narrow emission window which is a consequence of coalescence heating. The values for the temperatures agree well with experimental results. Note that this value is set by two fundamental properties of nuclei, the equation of state (characterized by binding energy per particle of 8 MeV and spring constant K) and the density of excited nuclear states at normal densitites (characterized by the Fermi energy ϵ_F).

Although the model used is very simple, it is believed that it contains enough of the general features of physics operating in nature to permit extension of the conclusions to the results of experiments.

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