Prediction of Aharonov-Bohm Oscillations on the Quantum Hall Plateaus of Small and Narrow Rings

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It is shown, by use of a Landauer-type formulation of the quantum Hall transport, that in small and narrow two-dimensional rings, the Hall quantization of ρ_{xy} is accompanied by a finite, but small, ρ_{xx}, which exhibits periodic Aharonov-Bohm oscillations. These oscillations arise as a subtle consequence of the possible elastic backscattering of the current-carrying electrons in narrow channels.

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Cutting a hole in a two-dimensional (2D) sample has no effect on the phenomenon of quantum Hall effect (QHE),[1] namely, the existence of plateaus of quantized ρ_{xy} and vanishing ρ_{xx}, even though one might naively expect some Aharonov-Bohm (AB) interference effects.[2] This is an extreme example of a theorem (Prange[3] and Laughlin[4]) which states that localized impurities do not disturb QHE so long as all the extended states are occupied. This partly follows from the fact that in the QHE conditions only a forward scattering of the electron is allowed, which causes no loss of current and hence results in a vanishing (residual) resistance, ρ_{xx}. In this Letter it is shown that for small rings, however, it should be possible to see AB effects concurrent with QHE, because in narrow channels the impurities *can* give rise to a backscattering of the current-carrying electrons. This backscattering is made possible by the proximity of the extended edge states carrying current in opposite directions.

Consider Fig. 1. The physical sample consists of an ideal sample plus impurities, where the ideal sample is characterized by a uniform confining potential, which is taken, for simplicity, to be monotonically increasing as one moves away from the thick line in Fig. 1. This potential confines the electrons to a channel about the thick line. The following discussion will be within the framework of the recently developed Landauer-type formulation for the treatment of elastic scattering in quantum Hall transport.[5,6] A zero temperature will be assumed. Following Refs. 5 and 6, it is imagined that the physical sample is connected through ideal leads to two particle reservoirs, one on either side. The boundary conditions are such that the reservoir on the left emits R electrons (i.e., electrons moving toward right) up to energy μ_1 into all the available Landau levels (LL's) and absorbs all the L electrons (i.e., the electrons moving toward left) incident on it, while the reservoir on the right emits L electrons up to energy μ_2 and adsorbs all the incoming R electrons. Assume that $\mu_1 > \mu_2$. Since the direction of electronic motion is determined by the potential gradient, there is a spatial separation of the R and the L electrons. For a magnetic field in the positive Z direction, the R electrons are confined to the upper half of the leads, while the L electrons are confined to the lower half. The electron energy increases as one moves away from the center (thick line), and the R and L electrons with energies μ_1 and μ_2, respectively, define the edges of the sample. Since the reservoirs emit electrons into all the states below μ_1 or μ_2, I am effectively only considering the quantum Hall transport when all the available LL's are completely filled, which is a condition for QHE (quantized ρ_{xy} and vanishing ρ_{xx}).

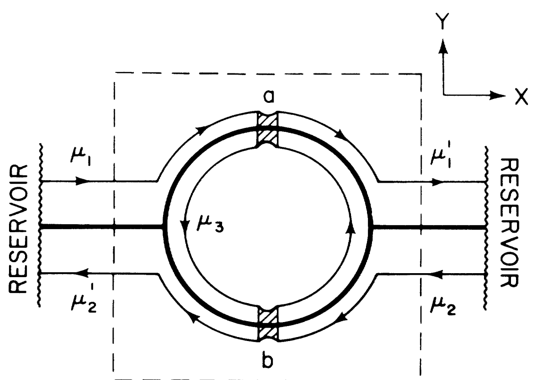

FIG. 1. Schematics of quantum Hall transport with the physical sample enclosed in the dashed box.

The localized scattering potentals which do not give rise to backscattering are irrelevant to the following discussion for the reason mentioned earlier, and I replace them by the ideal sample. The relevant impurities are the ones which extend across the width of the sample since they can backscatter the electrons near one edge (μ_1) into the empty states above μ_2 at the other edge. Such an impurity, for example, can be a bottleneck on the ring, as shown in Fig. 1, where the two edges come within a few magnetic lengths of each other, and thus produce an enhanced reflection amplitude. The Pauli principle forbids any scattering of electrons with energy below μ_2 since all the states below μ_2 are occupied, and only the electrons in the energy range $\mu_2 < E < \mu_1$ can be scattered. I shall assume that $\mu_2 \rightarrow \mu_1^+$, which is the limit of very small current flow, so that the transmission coefficients are constant over this small energy range. It can be shown[7] that the electrons below μ_2 carry no *net* current, and that the current carried by the edge electrons in the energy range $\mu_2 < E < \mu_1$ satisfies the correct quantization condition provided there is no scattering. It is assumed that there is no backscattering in the leads (indicated by their greater channel width compared to the channel of the ring), and that there is only one relevant scatterer on either branch shown at a and b, with the transmission and reflection amplitudes t_a, r_a, and t_b, r_b, respectively.

For simplicity, let us first consider the high magnetic field limit so that only the lowest LL is occupied and the problem reduces mathematically to a single-channel problem,[5,6] since for a given energy there is only one incoming and one outgoing state on either side of the sample. If the transmission coefficient of the physical sample enclosed in the dashed box in Fig. 1 is denoted by T_{tot}, then the current is given by

$$I = (e/h)(\mu_1 - \mu_2) T_{\text{tot}}, \tag{1}$$

which leads to the correct quantization condition for ρ_{xy} for perfect transmission ($T_{\text{tot}} = 1$). The chemical potentials of the R electrons on the right-hand side of the sample and the L electrons on the left-hand side of the sample are modified because of scattering.[8] These are given by $\mu_1' = \mu_2 + (\mu_1 - \mu_2) T_{\text{tot}}$ and $\mu_2' = \mu_1 - T_{\text{tot}}(\mu_1 - \mu_2)$, where the chemical potential is defined to be the energy such that the number of electrons (occupied states) above it is equal to the number of holes (unoccupied states) below it. Then

$$\rho_{xx} = \frac{\mu_1 - \mu_1'}{eI} = \frac{\mu_2' - \mu_2}{eI} = \frac{h}{e^2} \frac{R_{\text{tot}}}{T_{\text{tot}}}, \tag{2}$$

$$\rho_{xy} = \frac{\mu_1 - \mu_2'}{eI} = \frac{\mu_1' - \mu_2}{eI} = \frac{h}{e^2}, \tag{3}$$

where $R_{\text{tot}} = 1 - T_{\text{tot}}$. Thus, although ρ_{xy} still remains quantized, ρ_{xx} becomes T_{tot} dependent. On the assumption of no inelastic scattering (or an infinite-range phase coherence), the total reflection coefficient is given by

$$R_{\text{tot}} = \left| \frac{r_a r_b}{1 - t_a t_b e^{i(2\pi\phi/\phi_0)}} \right|^2, \tag{4}$$

where $\phi_0 = hc/e$ and ϕ is the flux through the area enclosed by the inner perimeter. With this reflection coefficient, ρ_{xx} is oscillatory with period ϕ_0. The origin of these oscillations is clear. The electron gets reflected by first getting reflected at a to the inner edge of the ring, then making one or many complete circles (each time picking up an AB phase of $2\pi\phi/\phi_0$) and finally getting reflected at b to the lower edge. An interference between these different paths causes AB oscillations in R_{tot} and hence in ρ_{xx}.

According to Eq. (4) *both* r_a and r_b must be nonzero for an observation of AB oscillations on the quantum Hall plateaus. This may be somewhat surprising at first sight for the following reason: Consider the case when there is no backscattering in the lower branch of the ring, i.e., $r_b = 0$. Then the electron can get transmitted across a either directly or by first tunneling into the inner edge, making n complete loops, and finally tunneling out to the upper edge on the other side of a. One would intuitively expect these different paths to interfere and produce AB oscillations. However, since we know that the electron *must* finally get transmitted across a, which implies that the total transmission *must* be 1, there *cannot* appear any interference effects or deviations from QHE in this case.

Now let us consider the consequences of a finite phase coherence length, L_ϕ. The probability of an electron's retaining its phase memory after one complete circle is $\exp(-L/L_\phi)$ where L is the perimeter of the inner loop. Thus in Eq. (4) the AB phase factor must be multiplied by $\exp(-L/L_\phi)$. Taking $t_a = t_b \equiv t$, one gets to the lowest order in R and $\exp(-L/L_\phi)$

$$\rho_{xx} = R^2[1 + 2e^{-L/L_\phi}\cos(2\pi\phi/\phi_0)], \tag{5}$$

where $T = |t|^2$, $R = 1 - T$, and near perfect transmission is assumed so that $R \to 0$. Thus small and narrow rings are required for an experimental observation of the AB oscillations, since these are suppressed by a large value of L/L_ϕ as well as by a large channel width, which implies $R \to 0$. Some experiments on rings have been reported[9] which do not show any periodic AB oscillations on the quantum Hall plateaus, but these rings have large L/L_ϕ as well as fairly large channel widths. During an estimation of L_ϕ it must be kept in mind that the reflected electrons are not in thermal equilibrium with the sample, and are "hot."

These considerations can be easily generalized to the situation when many LL's are occupied. Since, for strong confinement, the edges of the different LL's are very close to each other (in space), it is a reasonable assumption that because of inter-LL scattering all of the LL's will locally equilibrate to the same chemical potential —which reduces the problem effectively to a single-channel problem. In principle, because of a small difference in the fluxes enclosed by the inner edges of the different LL's (which are very slightly displaced relative to each other, with the lowest LL edge enclosing the smallest area), there will be as many periodicities as the number of occupied LL's, but in practice the difference in the periodicities may be too small to be observed. Unlike the case when only the lowest LL is occupied, now ρ_{xy} is not strictly quantized in the presence of elastic backscattering[6] and will also show AB oscillations; however, in strong magnetic fields one expects the oscillations in ρ_{xy} to be much weaker than those in ρ_{xx}. Also, until now I have only discussed noninteracting electrons, but the physical description presented in this work can also be generalized to include electron-electron interaction, and, in particular, to treat fractional QHE, by our taking the scattering states in the (ideal) leads to be the appropriately charged quasiparticles.[1]

There are several interesting differences from the earlier AB experiments on normal metal rings and the low-field experiments on semiconductor heterostructure rings. In these experiments in the lowest order, the wave function of the incident electron splits into two at one end of the ring and rejoins at the other end producing AB interference. The AB effect discussed here is a relatively much weaker effect since it entails at least two reflections, and in the QHE regime, when $\rho_{xx} \rightarrow 0$, the reflection coefficient is very small $(R \rightarrow 0)$. It follows that a nonzero value of ρ_{xx} is necessary to see the AB effect. A perfect transmission $(R=0)$ implies an absence of AB oscillations, since then the current-carrying edge electrons are completely unaware of the presence of an aperature in the middle of the sample. Also, until now we have completely neglected the aperiodic manifestation of the AB phenomenon[10] which involves an interference between different paths an electron can choose *within* one branch of the ring. The reason is that in the QHE regime all the states of the LL's are completely occupied which severely restricts the electron scattering, and hence the number of allowed paths. Thus the aperiodic AB fluctuation pattern should be suppressed, as also found experimentally.[9] This means that the ratio of the area of the annulus to the area of the sample, which must be large to see the ϕ_0 oscillations in the metallic rings,[11] is not an important parameter here. The magnetic field through the sample is not a nuisance, but rather is responsible for the physics here. Moreover, the relevant flux here is more precisely defined than in the earlier experiments in which there is some uncertainty[2] in ϕ with its limits given by the fluxes enclosed by the inner and the outer loops of the ring.

Although I have only considered the zero-temperature situation explicitly, it is clear that the AB oscillations will appear even at finite temperatures so long as $L_\phi \gtrsim L$. A more serious assumption is that only the case of a single scatterer on either branch of the ring has been considered; in case of many scatterers the (random) phase factors coming from various paths will tend to cancel the effect. Thus the best samples to see the AB oscillations are ones in which the two edges come close only in a small region on either branch. It should be possible to make such samples with the presently available technology. It should also be pointed out that the single-scatterer model may be valid even for a narrow ring with a uniform distribution of impurity scatterers, since it has been argued[12] that on the QHE plateaus, in general, only one impurity dominates the backscattering process.

In conclusion, I have given a Landauer-type description of QHE in rings and have shown that in certain conditions the near-zero ρ_{xx} will exhibit AB oscillations, while ρ_{xy} will remain more or less quantized. Crucial to this prediction is the physical importance of the edge currents for quantum Hall transport in narrow channels. There has been some evidence for edge currents in narrow channels at low current values, notably by Kane, Tsui, and Weimann.[13] An observation of the predictions made here will not only be further evidence in support of the edge-current picture, but will also be an experimental proof of the validity of the Landauer-type formulation of quantum Hall transport in which the elastic backscattering of the electrons is partly responsible for the finite value or ρ_{xx}, or, in other words, for the breakdown of dissipationless transport.[6]

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