Sawtooth Stabilization by Energetic Trapped Particles

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Recent experiments involving high-power radio-frequency heating of a tokamak plasma show strong suppression of the sawtooth oscillation. A high-energy trapped-particle population is shown to have a strong stabilizing effect on the internal resistive kink mode. Numerical calculations are in reasonable agreement with experiment.

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Experiments on the Joint European Torus (JET) that use high-power ion-cyclotron radio-frequency on-axis heating of a minority ion species[1] exhibit long sawtooth-free periods of up to 1.6 sec. The experiments involved heating of either ^{3}He or H in a background plasma of D or ^{4}He. In this Letter we show that a high-energy trapped-ion population such as that produced by the heating in JET can significantly decrease the growth rate of the resistive internal kink mode, leading to an increase of the sawtooth period.

Recently, the effect of an energetic trapped-particle population on magnetohydrodynamic modes in a tokamak has been explored with the use of a variational formalism.[2-6] The usual branch of the ideal internal kink,[7] unstable for plasma β greater than a threshold value, is stabilized by a trapped-particle population as long as the average toroidal precession rate of the particles is greater than the mode growth rate. On the other hand, it was found that for β near the internal-kink threshold value, the trapped particles resonantly destabilize a second branch of the internal-kink mode, with a real frequency given by the average precession frequency of the particle distribution, provided that the trapped-particle β, β_h, exceeded a threshold value. This branch is responsible for the fishbone calculation. The dispersion relation describing both branches of this mode was generalized to include resistive effects in Ref. 6, and takes the form

$$\delta W_c+\delta W_k+[8S^{-1/3}\Lambda^{-9/4}\Gamma(\tfrac{1}{4}(\Lambda^{3/2}+5))/\Gamma(\tfrac{1}{4}(\Lambda^{3/2}-1))][\Omega(\Omega+i\omega_{*i}/\omega_R)]^{1/2}=0, \quad (1)$$

where $\Omega=-i\omega/\omega_R$,

$$\Lambda=[\Omega(\Omega+i\hat{\omega}_{*e}/\omega_R)(\Omega+i\omega_{*i}/\omega_R)]^{1/3},$$

$\omega_R=S^{-1/3}\omega_A$ is the resistive frequency, S is the magnetic Reynolds number, ω_A is the shear Alfvén frequency

$$\omega_A=v_A/\sqrt{3}Rrq', \quad (2)$$

with v_A the Alfvén velocity, R and r the major and minor radii, respectively, and $q'=dq/dr$ with q the safety factor. The ω_* terms are diamagnetic frequencies with $\omega_{*i}=-(c/neBr)(dp_i/dr)$, $\omega_{*e}=(c/neBr)(dp_e/dr)$, and $\hat{\omega}_{*e}=\omega_{*e}+0.71(c/eBr)(dT_e/dr)$. The term in Eq. (1) involving the Γ functions arises from the inertial layer, and so all expressions are evaluated at the $q=1$ surface. The inclusion of the diamagnetic terms was carried out by Bussac *et al.*[8] and Ara *et al.*,[9] generalizing the work of Coppi *et al.*[10] The expression δW_c is the minimized ideal variational energy for the internal kink, first calculated by Bussac *et al.*,[7] and δW_k is the kinetic contribution coming from the trapped-particle distribution F,

$$\delta W_k=\frac{2^{3/2}}{B^2}m\pi^2\left[\int d(\alpha B)\int\frac{dE\,E^{5/2}K_2^2(\omega\partial/\partial E+\hat{\omega}_*)F}{K_b(\omega_d-\omega)}\right], \quad (3)$$

with $[y]=(2\int yr\,dr)/r_s^2$, r_s the $q=1$ radius, $\alpha=v_\perp^2/v^2$, $\hat{\omega}_*$ a differential operator associated with the diamagnetic drift frequency, and K_2 and K_b elliptic functions arising from bounce averaging. Details of the derivation of this expression are found in Refs. 3 and 5. The trapped-particle population contributes to the dispersion relation only in the domain $q<1$ in this formalism, which involves an expansion to lowest order in the inverse aspect ratio r/R. This is because the minimizing eigenfunction in this approximation is the usual internal-kink function, consisting of a single harmonic with poloidal and toroidal mode numbers equal to 1. A more complete calculation, with several poloidal harmonics, would include significant contribution from the entire trapped-particle population, also outside the $q=1$ surface. Numerical codes for the solution of the resulting dispersion relation for arbitrarily shaped cross-sectional geometry are being developed,[11] but until they allow a detailed analysis of the full trapped-particle contribution, comparison with experiment must be only approximate.

In the present work we examine the effect of a trapped-particle population on the resistive internal-kink branch, which is purely growing in the absence of kinetic effects. Without a trapped-particle population, $\delta W_k=0$, this branch of the solution to Eq. (1) possesses two well-known limits. We neglect the diamagnetic frequencies ω_{*e} and ω_{*i} for this discussion. They have the effect of decreasing the growth rate and giving the mode a real frequency; but they complicate the algebra, and the qualitative behavior of the solution can be understood without them. The numerical results presented include the diamagnetic terms.

For large S (high temperatures) the large argument limit of the Γ functions can be used to obtain

$$0=\delta W_c - i\omega/\omega_A, \tag{4}$$

giving a purely growing mode for $\delta W_c<0$, the ideal internal-kink mode. Note that this limit requires $\Omega \gg 1$, or $|\delta W_c|S^{1/3} \gg 1$. In the second limit, with $|\delta W_c|S^{1/3} \ll 1$, the dispersion relation is satisfied for $\Gamma((\Omega^{3/2}-1)/4)=\infty$, or $\omega=i\omega_R$. This is the usual $m=1$ tearing mode, which is of interest to us in the present work, and in the following we will take $\delta W_c=0$.

The behavior of the tearing-mode branch of the dispersion relation can be found by use of a model slowing-down trapped-particle distribution, which permits analytic evaluation of the kinetic contribution. If we introduce the distribution

$$F(E,\mu)=n(r)\delta(\mu/E-\alpha_0)/E^{3/2}, \quad E<E_m\ , \tag{5}$$

with μ the magnetic moment, E the energy, and $dn/dr \sim 1/\epsilon$, the kinetic contribution becomes[3]

$$\delta W_k = i(\beta_h/\epsilon)\Omega A \ln(1+i/A\Omega)\ , \tag{6}$$

with β_h the hot trapped-particle β, and $A=\omega_R/\omega_{dm}$ the ratio of the resistive growth rate to the maximum toroidal precession rate. The precession rate ω_{dm} is approximately given by the value for deeply trapped particles,

$$\omega_{dm}=E_m q/mrR\omega_0, \tag{7}$$

with ω_0 the gyrofrequency, $\omega_0=ZeB/mc$, and for this distribution the average precession rate is given by $\langle\omega_d\rangle=\omega_{dm}/2$. Thus the dispersion relation for the tearing mode becomes

$$8\Gamma(\tfrac{1}{4}(\Omega^{3/2}+5))/\Gamma(\tfrac{1}{4}(\Omega^{3/2}-1))+i(\beta_h/\epsilon)S^{1/3}\Omega^{9/4}A\ln(1+i/A\Omega)=0. \tag{8}$$

The trapped particles play an important role only for

$$(\beta_h/\epsilon)S^{1/3}A>1, \tag{9}$$

which is, within a factor of π, the fishbone threshold condition. Note that this is independent of the Reynolds number S. The combination $\beta_h A$ is proportional to the hot trapped particle density and magnitude of the charge, and independent of its mass and energy. Two limits are of interest. For small $\beta_h A S^{1/3}$ the solution can be obtained by expansion about $\Omega=1$, which is the solution for $\beta_h=0$, giving

$$\Omega=1-2iA\ln(1+i/A)(\beta_h/\epsilon)S^{1/3}/3\sqrt{\pi}. \tag{10}$$

The trapped particles are initially destabilizing, and more so for $A\approx 1$, i.e., when the precession frequency is resonant with the resistive rate. For large $A\beta_h S^{1/3}$ there are two roots, with $\Omega\to\infty$ and $\Omega\to 0$. For the small root, letting $\alpha=1/|\Omega|$ we find an equation determining α:

$$\alpha^{9/4}/\ln(\alpha)=\Gamma(\tfrac{3}{4})A(\beta_h/\epsilon)S^{1/3}/2\Gamma(\tfrac{5}{4}), \tag{11}$$

with $\Omega=(1/\alpha)e^{-i2\pi/9}$. This gives for the mode $\omega=(\omega_R/\alpha)e^{i5\pi/18}$, and it is strongly stabilized for $A(\beta_h/\epsilon)S^{1/3}\gg 1$. This asymptotic value depends only on the trapped-particle density, and is independent of energy. Note, however, that the analysis leading to the dispersion relation, Eq. (1), assumed that the temperature of the hot species is of order $(R/r)^2$ greater than that of the background plasma, i.e., trapped ions of at least 50 keV for the JET discharges. Note that hot electrons are also stabilizing since both ω_* and ω_d are negative.

The second root can be found by use of asymptotic expressions for the Γ functions, giving

$$\Omega=(\beta_h/\epsilon)S^{1/3}. \tag{12}$$

As long as $\langle\omega_d\rangle>\omega_R$, the first root, Eq. (11), is the tearing-mode root, and Eq. (12) describes the fishbone mode. If instead $\langle\omega_d\rangle<\omega_R$, the trapped particles destabilize the tearing mode and its as asymptotic limit is given by Eq. (12).

Thus we find that to stabilize the tearing mode it is necessary to produce a trapped-particle density which is large enough to destabilize the fishbone. The occurrence of the fishbone is, however, also contingent upon proximity to the internal-kink threshold.[3] Depending on plasma β and other equilibrium parameters, we must, in general, expect either sawtooth stabilization or fishbone oscillations, with fishbone oscillations dominating for high-β operation.

We have examined the solutions to Eq. (1) using a numerical code developed for the investigation of the fishbone mode.[5] Numerical values of temperatures, densities, diamagnetic frequencies, etc., were chosen to approximate the experiments done on JET. The kinetic contribution δW_k is generated by a Monte Carlo procedure. For minority species ion-cyclotron heating experiments, the hot trapped-particle distribution is well approximated by

$$F(E,\mu,r)=n(r)e^{-E/T}\delta(\mu/E-\alpha). \qquad (13)$$

The value of n is typically of the order of a few percent of the average plasma density, and the temperature T ranges from 70 to 150 keV depending on the ion-cyclotron radio-frequency power. For the experiments performed on JET with on-axis heating, the approximation of a single value of μ/E corresponds to a single value of the bounce angle, $\theta_b \approx \pi/2$. The details of the solution are not particularly sensitive to small changes in the particle distribution function.

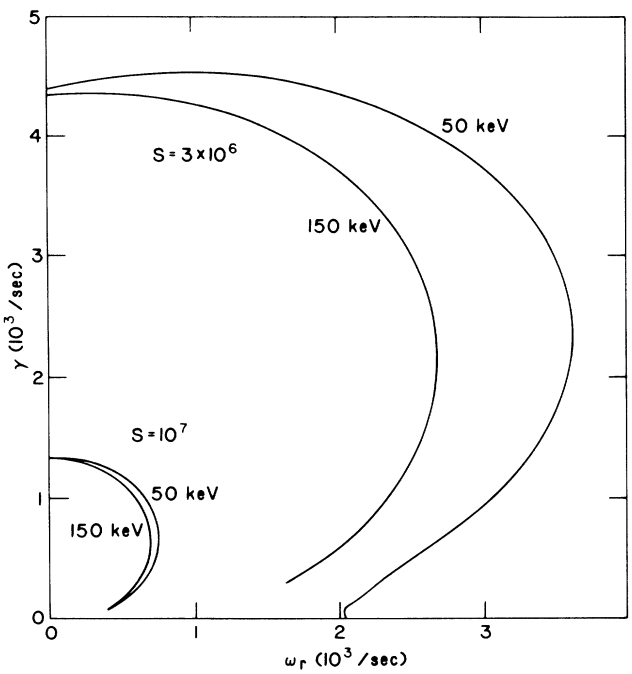

FIG. 1. The trajectories of solutions to Eq. (1) in the complex frequency plane as functions of trapped proton density, for two values of the magnetic Reynold number S, and different energies. Equilibrium parameters and Alfvén and diamagnetic frequencies were chosen to approximate JET.

Results of a Monte Carlo simulation are shown in Figs. 1 and 2 for a hydrogen minority species in JET. Shown in Fig. 1 are the trajectories in the complex frequency plane for an approximate JET equilibrium with $R=296$ cm, for two different values of the magnetic Reynolds number, $S=3\times10^6$ and 10^7. The trapped-particle density ranges from 0 to $10^{12}/\mathrm{cm}^3$. The particle distributions were of the form given by Eq. (13), and had temperatures of 50 and 150 keV. The toroidal field was $B=24$ kG and the average trapped-particle precession rates were $\langle\omega_d\rangle \approx 2\times10^4$/sec and 6×10^4/sec, respectively. The shear Alfvén frequency was $\omega_A=2\times10^6$/sec, and the diamagnetic frequencies were $\hat{\omega}_{*e}=-3\times10^4$/sec and $\omega_{*i}=2\times10^4$/sec. Without the trapped particles, the mode is almost purely growing but with a growth rate almost an order of magnitude smaller than the resistive value ω_R, a well-known effect of the diamagnetic terms. The trapped-particle population is destabilizing for small density if $A\approx 1$, and becomes stabilizing if $A(\beta_h/\epsilon)S^{1/3}>1$ provided that $\langle\omega_d\rangle>\omega_R$. We find this to be true for a wide range of values of ω_{*e}, ω_{*i}.

For $S=3\times10^6$ and 50 keV, $\langle\omega_d\rangle\approx\omega_R$ and the mode is barely stabilized. If either S or the energy is decreased, the mode is destabilized by the particles. Increasing the particle energy above 150 keV does not significantly change the results, which are independent of energy for $\langle\omega_d\rangle\gg\omega_R$. In Fig. 2 the mode growth rate is shown as a function of trapped-particle density for the trajectories of Fig. 1. The growth rate is decreased by a significant amount for values used in JET, where trapped-ion densities are estimated to be greater than

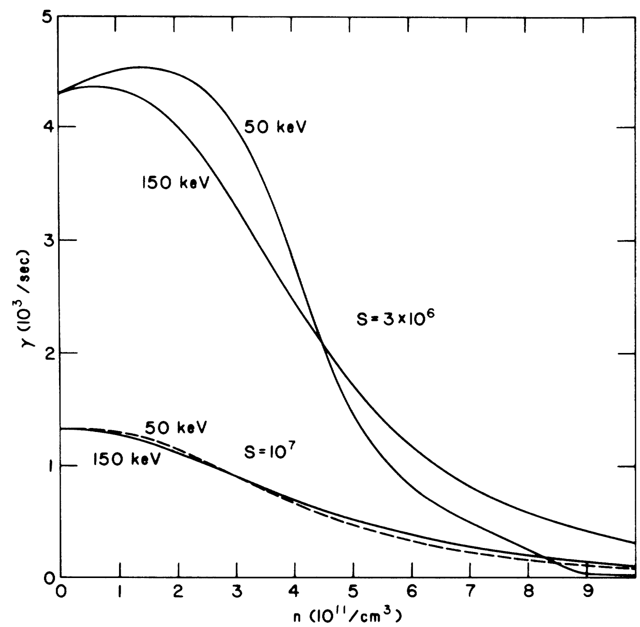

FIG. 2. Growth rate as a function of trapped-particle density for the trajectories of Fig. 1.

$10^{12}/cm^3$. From these curves it is clear that a significant lengthening of the sawtooth period is possible. We note that the trapped particles also stabilize the ideal internal kink,[5] and thus the qualitative features of this result are unchanged if a short-time-scale ideal model of the sawtooth is used. Notice that the temperature increase during heating is also stabilizing, since the growth scales as $S^{-1/3}$, but this effect is not as pronounced as that due to the particles.

Since the factor A is proportional to the magnitude of the charge Z, a ^{3}He trapped-ion population is more stabilizing than one consisting of hydrogen for equal density and energy, as observed on JET.

During neutral-beam heating of JET at lower toroidal field values (21 kG) and higher β, magnetic signals associated with the fishbone mode were observed.[12] The threshold value of β_h for the fishbone was calculated[13] to be 2×10^{-3}. Because of the efficient ejection of trapped ions by the fishbone, the trapped-particle population would be effectively limited to approximately this value, which corresponds, for the neutral-beam energy of 70 keV, to a trapped-particle density of $3\times10^{11}/cm^3$, too low a value to produce stabilization of the sawtooth. In general, since the stabilization condition implies that the fishbone threshold be exceeded, it appears, perhaps unfortunately, that at high-β operation the occurrence of the fishbone will not allow sawtooth stabilization with trapped particles. To our knowledge, fishbone oscillations have not yet been produced during ion-cyclotron radio-frequency heating, and so the nature of the transition from sawtooth stabilization to fishbone is still unexplored experimentally. The signature for this transition would be the occurrence of ion bursts with an associated drop in neutron production during the sawtooth period.

In conclusion, we find that the presence of a high-energy trapped-ion population introduces a stabilization of the sawtooth in a tokamak. Numerical calculations of the effect are in reasonable agreement with experiments on JET, giving almost an order of magnitude increase in the period. Although the stabilization of the sawtooth mode in low-β discharges is encouraging, at higher β the same trapped-particle population should destabilize the fishbone branch, as has been observed to happen with neutral-beam injection. The fishbone should be then expected to limit the trapped-particle population to a value too low to provide sawtooth stabilization.

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