Lamb Shift in Singly Ionized Helium

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The Lamb shift ($n=2$) in $^4\mathrm{He}^+$ has been measured in a beam of metastable helium ions by means of the microwave resonance method. Our result is 14.0420(12) GHz which is in good agreement with the theoretical result based on the binding correction of Mohr, 14.0423(5) GHz.

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Tests of quantum electrodynamics (QED) based on measurement of the Lamb shift in hydrogen[1] are limited by theoretical uncertainty in the QED prediction arising from the uncertainty in the measured value of the proton's electromagnetic radius. One way to overcome this difficulty is to measure the Lamb shift in $^4\mathrm{He}^+$ since the charge radius of the α particle has been measured to an order of magnitude higher precision than the charge radius of the proton.[2] There are other advantages to Lamb-shift measurements in $^4\mathrm{He}^+$. The lack of hyperfine structure greatly simplifies the energy levels and $^4\mathrm{He}^+$ is more sensitive to the higher-order binding corrections to the self-energy.

We report here on a precision measurement of the $2s_{1/2}$-$2p_{1/2}$ Lamb shift in singly ionized helium ($^4\mathrm{He}^+$). Earlier measurements of this quantity utilized either the anisotropy method,[3] or the microwave resonance method in a helium discharge.[4] In the latter measurements, the microwave frequency was fixed and the resonance was swept with an external magnetic field. Our experiment also uses the technique of magnetic field sweeping of a microwave resonance but it is the first resonance measurement in He^+ to be done in a beam of metastable helium ions. The use of the ion beam rather than a discharge has allowed greater control over systematic effects. In particular, we have been able to study in detail effects caused by the dependence of the charged-particle dynamics on magnetic field.

Figure 1 shows the experimental apparatus. A 170-eV beam of $^4\mathrm{He}^+$ is extracted from an electron bombardment ion source[5] and travels along field lines of a large solenoid. The fraction of the beam in the $2s$ state at the detector is about 2×10^{-4}. The resonance experiment is designed around the $2s$ spin-up state (α). The first microwave resonator can be used to drive the $2s$ spin-down state (β) resonantly to the $2p$ state removing it from the beam. A second microwave resonator located in the central homogeneous part of the magnetic field is tuned to 24.6 GHz and drives the transition from the α state to the $2p$ spin-down state (f) which decays immediately to the ground state. In the metastable detector,[6] a large electrostatic potential is used to quench the α state and the resulting 304-Å radiation is detected with a large-solid-angle photodiode. The resonance is observed by our recording the fraction of α-state ions which survive the resonance region as a function of magnetic field. The Lamb shift is determined by our finding the center of the resonance and extrapolating to zero field using the well-known Zeeman effect. The width of the line, which is determined by the lifetime of the $2p$ state, is 0.86 kG or 15% of the central value. Thus a 100-ppm measurement of the Lamb shift requires splitting the line to one part in 1500.

The Lamb-shift measurement requires precise knowledge of the microwave frequency, which is locked to a 1-MHz crystal, and the magnetic field profile in the central region, which is measured with an NMR probe and locked via a temperature-compensated Hall-effect probe.[7] In addition, the magnet temperature, microwave power, and positive beam current are also locked during a run.

In order to be insensitive to both beam intensity variations and detector background (10%), we use a measuring scheme employing three states of the microwave power: microwaves off (I_0), microwaves on at half power (I_1), and microwaves on at full power (I_2). From integrated photocathode currents for each of the three states, a quantity we call the tristate ratio (R_3) is formed:

$$R_3=(I_0-I_1)/(I_0-I_2). \tag{1}$$

Because of long-term temperature drifts in the micro-

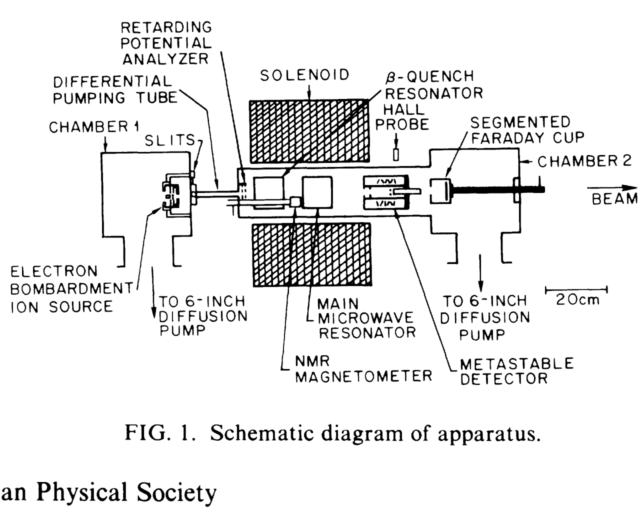

FIG. 1. Schematic diagram of apparatus.

wave-power measurement, we return to the central magnetic field every other point for a normalization point. The remaining data points are measured at randomly selected magnetic field values.

The data are adjusted with the normalization points and fitted by a theoretical expression based on the Hamiltonian of Brodsky and Parsons.[8] Five parameters are allowed to vary. They are the Lamb shift, the microwave-field intensity times transit time $(E^2\tau)$, the ratio of high to low microwave power, a linewidth factor, and a linear coefficient for $E^2\tau$ versus magnetic field. Figure 2 shows the fit to a typical resonance curve and the residuals obtained from it.

Two systematic effects contribute to a need to fit for a linear slope in $E^2\tau$. One is a field-dependent movement of the beam transverse to the beam axis and the other is dependence of the ion transit time on magnetic field. We have studied the beam movement using a segmented Faraday cup and the transit time using a retarding potential-energy analyzer. In addition, we studied the space charge of the beam, which effects the transit time, by measuring the distribution of positive ions and electrons in the beam. The diagnostic measurements give us an estimate of the size of the variation of $E^2\tau$ with field. Since the dominant effect is a linear slope, we fit the data for an $E^2\tau$ slope and use the diagnostic data together with Monte Carlo simulations to indicate the uncertainty in not including higher-order variations in $E^2\tau$.

Still, the Lamb shift obtained from the five-parameter fit to an individual resonance curve is typically in error by several megahertz because of an effect arising from interplay between the mode structure of the resonator and the helical trajectories of the ions. This effect was discovered during Monte Carlo simulations of metastable quenching in the resonator and was verified experimentally. In an ideal experiment, the phases of the cyclotron orbits of the ions in any portion of the beam will be randomly distributed in 2π. This random transverse motion leads to a distribution in the integrated microwave power seen by the beam. This effect broadens the resonance (by about 1%) and because the effect is field dependent it also shifts it by a small amount. To handle these effects, we allow the linewidth to be a free parameter in our fits and apply a small correction to our data. Of greater significance, however, is an effect caused by nonrandom transverse motion of the beam arising from some asymmetry in the apparatus such as a slight misalignment of the ion source relative to the magnetic field. Such "correlated" beam motion gives rise, in general, to a nonzero average cyclotron phase in a macroscopic beam sample. As the magnetic field changes, so does the pitch of the cyclotron orbits and this causes the beam to see an average microwave intensity that depends, in a rather complicated way, on magnetic field. The Monte Carlo simulations showed that any systematic errors caused by the correlated beam motion are periodic as a function of the transverse position of the resonator with a period proportional to half the wavelength of the radiation inside the resonator (0.61 cm). So in order to correct for this effect we put the resonator on a translation stage and in each run, resonance curves are obtained at several different transverse positions of the resonator, and the Lamb shift is obtained from a fit in which the centroids are characterized by a sinusoidal function of position, the average of which is the correct Lamb shift.

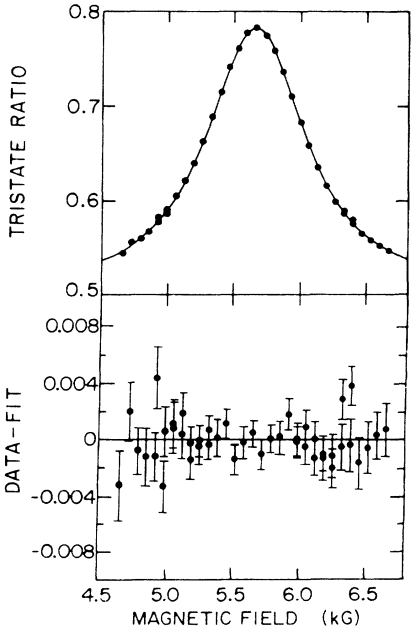

FIG. 2. Typical resonance curve (upper) and residuals from a five-parameter fit to the data.

Precision data were obtained in three runs which used different He^+ currents. Each run consisted of about twelve scans of the resonance each of which took about four hours to complete and was carried out at one of several different positions of the central resonator. From the values obtained for the fitting parameters and the resonator positions, corrected values for each of the five parameters are obtained with the fit by a sine curve. Values obtained for the Lamb shift are presented in Table I. Corrections have been applied for the following:

(1) Stark effect: The electric field on the ions arises chiefly from motion transverse to the magnetic field. The average static electric field is determined by our observing the quenching radiation due to these fields.

(2) Random transverse motion: This correction is based on Monte Carlo calculations and on our assigning an average transverse velocity to the beam from the measured lifetime of metastables in the apparatus.

(3) Electronic offsets: Small electronic offset currents were present that were integrated along with the metastable current causing a slight asymmetry in the line

TABLE I. Final results and corrections.

Run 1[a]	Run 2[a]	Run 3[a]	Notes
14041.1 ± 1.7	14043.5 ± 1.4	14043.2 ± 2.2	Error includes statistical error and resonator position uncertainty
−0.35 ± 0.17	−0.32 ± 0.16	−0.30 ± 0.15	Stark effect
0.18 ± 0.18	0.16 ± 0.16	−0.15 ± 0.15	Random transverse motion
−0.17 ± 0.17	−0.22 ± 0.3	−0.28 ± 0.28	Electronic offsets
14040.8 ± 1.7	14043.1 ± 1.4	14042.5 ± 2.2	Individual run results

The three combined: 14042.2 ± 1.0

0.0 ± 0.5	Transit-time residual
0.0 ± 0.5	Beam-motion residual
−0.12 ± 0.20	Magnetic field shape and uncertainty
−0.01 ± 0.01	ac Stark effect
0.00 ± 0.05	β state
−0.02 ± 0.01	Overlap correction
14042.0 ± 1.2	Final result with 1σ uncertainty

[a] He^+ currents for runs 1, 2, and 3 were 3.5, 4.4, and 2.8 μA, respectively.

shape. These currents were measured during each run.

(4) Transit-time and beam-motion residual: These are uncertainties from neglect of nonlinear variation in $E^2\tau$ with magnetic field.

(5) Magnetic field shape and uncertainty: The uncertainty includes both measuring uncertainty and uncertainty in going from NMR frequency to absolute field.

(6) ac Stark effect: This correction accounts for antiresonant α-f coupling which has been left out of the line-shape expression used to fit to the data.

(7) β-state correction: The presence of the nonresonant β-e transition can skew the line, but measurements indicate that less than 5% of the metastable beam is in the β state.

(8) Overlap correction: If there is a component of microwave polarization parallel to B, the nonresonant transition α-e will be driven. We have observed the α-e resonance and find its strength consistent with a 3° misalignment of the microwave resonator.

An extensive search was conducted to look for other microwave-dependent signals, but none were found. Of particular interest to us were signals that might arise from helium ions in higher excited states.[9] Several additional systematic effects were considered but found to be unimportant for this experiment including the Doppler effect, errors associated with the fitting function, electronic gain shifts and nonlinearity, and errors caused by a dependence of the electronics on magnetic field. Our final result 14042.0 ± 1.2 MHz has a 1-standard-deviation statistical error of 1.0 MHz and a systematic error of 0.7 MHz. It is consistent with the theoretical value[10] 14042.3 ± 0.5 MHz which uses Mohr's binding corrections.[11] With Erickson's binding corrections,[12] the difference between our result and theory is −3.1 ± 1.3 MHz. Our result differs from the most recent resonance measurement due to Narasimham and Strombotne[4] by −4.2 ± 1.7 MHz but agrees with the measurements of Lipworth and Novick (14040.2 ± 1.8 MHz)[4] and Patel, van Wijngaarden, and Drake (14042.22 ± 0.35 MHz).[3]

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[1] S. R. Lundeen and F. M. Pipkin, Phys. Rev. Lett. **46**, 232 (1981); G. Newton, D. A. Andrews, and P. J. Unsworth, Philos. Trans. Roy. Soc. London **290**, 373 (1979); Yu. L. Sokolov and V. P. Yakovlev, Zh. Eksp. Teor. Fiz. **83**, 15 (1982) [Sov. Phys. JETP **56**, 7 (1982)].

[2] E. Borie and G. A. Rinker, Phys. Rev. A **18**, 324 (1978).

[3] G. F. W. Drake, S. P. Goldman, and A. van Wijngaarden, Phys. Rev. A **20**, 1299 (1979); J. Patel, A. van Wijngaarden, and G. W. F. Drake, Phys. Rev. A **36**, 5130 (1987); G. W. F. Drake, J. Patel, and A. van Wijngaarden, Phys. Rev. Lett. **60**, 1002 (1988).

[4] E. Lipworth and R. Novick, Phys. Rev. **108**, 1434 (1957); M. Narasimham and R. Strombotne, Phys. Rev. A **4**, 14 (1971).

[5] R. W. Dunford and M. S. Dewey, to be published.

[6] J. C. Powelson, M. S. Dewey, and R. W. Dunford, Nucl. Instrum. Methods Phys. Res., Sect. A **260**, 403 (1987).

[7] M. S. Dewey and R. W. Dunford, Rev. Sci. Instrum. **56**, 1239 (1985).

[8] S. Brodsky and R. Parsons, Phys. Rev. **163**, 134 (1967).

[9] M. Dewey, Ph.D. thesis, Princeton University, 1984 (unpublished). Possible evidence for higher excited states was discussed, but this was unconfirmed by subsequent measurements.

[10] W. R. Johnson and G. Soff, At. Data Nucl. Data Tables **33**,

405 (1985); G. C. Bhatt and H. Grotch, Ann. Phys. (N.Y.) **178**, 1 (1987). We use 1.673(1) fm from Ref. 2 for the ^{4}He charge radius. This work has recently been challenged. See, for example, M. Eckhause, P. Guss, D. Joyce, J. R. Kane, R. T. Siegel, W. Vulcan, R. E. Welsh, and R. Whyley, Phys. Rev. A **33**, 1743 (1986). However, the theoretical value is unchanged if we use the result [1.674(12) fm] obtained from electron scattering: I. Sick, J. S. McCarthy, and R. R. Whitney, Phys. Lett. **64B**, 33 (1976).

[11] P. J. Mohr, in *Beam Foil Spectroscopy,* edited by I. A. Sellin and D. J. Pegg (Plenum, New York, 1976), pp. 89–95.

[12] G. W. Erickson, J. Phys. Chem. Ref. Data **6**, 831 (1977).