Slowing-Down-Tail Enhancement of the Neoclassical Energy Flux of α's

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The retention of both drag and pitch-angle scattering of the energetic slowing-down-tail α's by the background ions can dramatically increase the radial energy flux of the α's in an ignited tokamak and can prevent the α energy from being deposited into the background plasma via electron drag. The condition that the enhanced neoclassical fluxes have negligible impact on the ignition can impose a more severe design constraint than drift loss.

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In a fusing tokamak plasma the neoclassical particle loss of α's is an important concern because of the desire to maintain an outward flux in order that the plasma can be refueled.[1] The neoclassical α energy flux is normally presumed to be less interesting because the thermalized Maxwellian or ash α's are less dense than the background ions so that their energy flux is smaller.[2] However, the preceding picture assumes that the neoclassical fluxes associated with the energetic slowing-down-tail α's are negligible. In the calculation that follows, the energy flux associated with the tail α's is evaluated and can be quite large. Indeed, if the tail and ash densities were comparable, the neoclassical energy flux of the tail α's would be far larger than that of the ash α's.

The energetic-tail energy flux might be thought to be small because the characteristic collision time of these α's is the slowing-down time τ_s which is much longer than the α-ion collision time τ_{ai}. However, the typical speed of a tail α is large and roughly on the order of the speed v_0 at which the tail α's are produced. Consequently, if one replaces the thermal speed of the ash α's $v_a \equiv (2T/M_a)^{1/2}$ by $v_0 \gg v_a$ and τ_{ai} by τ_s and observes that the weighting factor $\frac{1}{2}M_a v^2$ in the energy flux is on the order of $\frac{1}{2}M_a v_0^2$ rather than the temperature T, one obtains a neoclassical tail-α thermal conductivity of $\chi \sim (\epsilon^{1/2}/\tau_s)(v_0/\Omega_{pa})^2(v_0/v_a)^2$. The quantities ϵ and Ω_{pa} are the inverse aspect ratio and the gryofrequency of the α's in the poloidal magnetic field B_p. The definitions of τ_s and τ_{ai} are standard[3] and given in the text to follow.

Because $v_0^4/\tau_s \gg v_a^4/\tau_{ai}$ the tail-α thermal conductivity is much larger than that of the ash α's. For comparable ash and tail densities the tail α's are by far the dominant neoclassical energy-loss channel. Moreover, for similar densities the particle flux of the tail α's would be comparable to that of the ash α's even though the particle flux is not weighted by the extra $\frac{1}{2}M_a v_0^2$ factor. Estimating the neoclassical tail-α diffusion coefficient gives us $D \sim (\epsilon^{1/2}/\tau_s)(v_0/\Omega_{pa})^2$, which is roughly that of the ash because $v_0^2/\tau_s \sim v_a^2/\tau_{ai}$. Consequently, the tail-α particle flux will nearly always exceed the neoclassical electron particle flux.[1]

Early attempts[4-6] to evaluate the neoclassical transport associated with the tail α's are incomplete because pitch-angle scattering by the background ions is neglected. As a result of this neglect, the solution fails for the tail α's near the trapped-passing boundary. Recent work[7] retains pitch-angle scattering while treating drag as small. To remove the limitations of these treatments both drag and pitch-angle scattering must be retained. The retention of both effects is the crucial feature that distinguishes the present treatment from the prior work. The result obtained herein verifies the preceding rough estimates when v_0 is not too large and/or ϵ is sufficiently small.

To evaluate the tail-α energy flux it is convenient to employ a moment description so that only the lowest-order neoclassical gyroradius correction to the slowing-down-tail distribution function is required. Consequently, the appropriate moment of tail-α kinetic equation

$$\partial f/\partial t + \mathbf{v}\cdot\nabla f + (Z_a e/M_a)(\mathbf{E} + c^{-1}\mathbf{v}\times\mathbf{B})\cdot\nabla_v f = C\{f\} + (S/4\pi v^2)\delta(v - v_0) \tag{1}$$

is required, where f is the tail-α distribution function, S the α-particle source rate, v_0 the speed at which the tail α's are produced ($v_0 > v_c$), $\delta(v)$ a Dirac δ function, and $C\{f\}$ the $v_a^2 \ll v^2 \ll v_e^2 = 2T_e/m$ limit of the Fokker-Planck collision operator[3,5] given by

$$C\{f\} = \tau_s^{-1}\nabla_v\cdot[\mathbf{v}v^{-3}(v_c^3 + v^3)f + \tfrac{1}{2}v_b^3 v^{-3}(v^2\mathbf{I} - \mathbf{vv})\cdot\nabla_v f]. \tag{2}$$

In the preceding, v_e is the electron thermal speed,

$$\tau_s = 3M_a T_e^{3/2}/4(2\pi m)^{1/2} Z_a^2 e^4 N_e \ln\Lambda_e, \quad v_c^3 = \{3\pi^{1/2}T_e^{3/2}\sum[Z^2 N\ln\Lambda/M]\}/[(2m)^{1/2}N_e\ln\Lambda_e],$$

and

$$v_b^3 = \{3\pi^{1/2}T_e^{3/2}\sum[Z^2 N\ln\Lambda]\}/[(2m)^{1/2}M_a N_e\ln\Lambda_e],$$

where m, T_e, N_e, and $\ln\Lambda_e$ are the electron mass, temperature, density, and Coulomb logarithm, and T is the temperature of the thermal α ash and background ions. The sum in v_c^3 and v_b^3 is used to indicate that only the background ion species of mass M, density N, charge number Z, and Coulomb logarithm $\ln\Lambda$ are to be summed over. Self-collisions of the tail α's are neglected since the tail density is assumed small compared to N_e. The speed v_c is the speed at which electron and ion drag are equal, while the speed v_b carries a subscript b as a reminder that it is the characteristic speed associated with the pitch-angle scatter of the tail α's by the background ions.

The slowing-down-tail distribution function f_0 is the isotropic solution of

$$C\{f_0\}+(S/4\pi v^2)\delta(v-v_0)=0 \tag{3}$$

for $v_0 \gtrsim v \gg v_a$ and is found to be[3,5]

$$f_0=S\tau_s/4\pi(v_c^3+v^3). \tag{4}$$

The solution f_0 is non-Maxwellian because the source S is at $v_a \ll v_0 \ll v_e$ and f_0 extends from $v \gtrsim v_a$ up to v_0 where it abruptly vanishes.

If Eq. (1) is multiplied by $(M_\alpha c/Z_\alpha e)ER\boldsymbol{\theta}\cdot\mathbf{v}$ where $E=\frac{1}{2}M_\alpha v^2+Z_\alpha e\Phi$, $\mathbf{E}=-\nabla\Phi$, R is the major radius, and $\boldsymbol{\theta}$ the unit vector in the toroidal direction, then integration over all $\mathbf{v}$ and flux-surface averaging yields to lowest order

$$Q\equiv\left\langle\int d^3v\, fE\mathbf{v}\cdot\nabla\psi\right\rangle = (M_\alpha c/Z_\alpha e)\left\langle\int d^3v\, ER\boldsymbol{\theta}\cdot\mathbf{v}C\{f-f_0\}\right\rangle. \tag{5}$$

In the preceding the angle brackets denote a flux-surface average, ψ is the poloidal flux function, and Eq. (3) and the linearity of C are used to remove the source term.

The energy flux Q must be evaluated in the banana regime because the tail α's are so energetic and collide so infrequently. Consequently, the gyrophase average or neoclassical portion of $f-f_0$ may be written in the normal fashion in the banana regime[7,8] by taking

$$f-f_0=(M_\alpha cI/Z_\alpha e)v_\parallel \partial f_0/\partial\psi+h, \tag{6}$$

where $\mathbf{B}\cdot\nabla h=0$, $I=RB_t$ with B_t the toroidal magnetic field, and classical modifications are neglected. Then h may be determined to the requisite order in a $\epsilon^{1/2}$ expansion by making the usual assumption that the pitch-angle derivative of $f-f_0$ in $C\{f-f_0\}$ is large and localized to the vicinity of the trapped-passing boundary.[7] The banana solubility constraint then gives

$$\partial h/\partial\lambda=(M_\alpha cIv^2/2Z_\alpha eB_0\langle v_\parallel\rangle)\partial f_0/\partial\psi \tag{7}$$

for the passing α's and $h=0$ for the trapped ones, where $\mu=M_\alpha v_\perp^2/2B$ is the magnetic moment and $\lambda=2\mu B_0/M_\alpha v^2$ is the pitch angle variable with B_0 the field on axis.

The tail-α energy flux may then be evaluated by noting that $E\approx\frac{1}{2}M_\alpha v^2$ in Eq. (5) since $Z_\alpha e\Phi/M_\alpha v^2 \sim (Z_\alpha e\Phi/T)(v_a/v_0)^2 \sim (v_a/v_0)^2 \ll 1$. The employment of v and λ as the variables for $C\{f-f_0\}$ in Q and the integration by parts to express the drag and pitch-angle contributions in terms of $\partial(f-f_0)/\partial\lambda$ gives

$$Q=(\pi M_\alpha^2 cI/Z_\alpha eB_0\tau_s)\left\langle\int dv\, v^2[v_b^3+3(v_c^3+v^3)]\int d\lambda\,\lambda\,\partial(f-f_0)/\partial\lambda\right\rangle. \tag{8}$$

I make the usual assumptions that $\epsilon=r/R_0\ll 1$ and $B_p/B\ll 1$, introduce $B=B_0R_0/R$ with R_0 the major radius of the magnetic axis, insert (6) and (7), and recall that $1/\langle v_\parallel\rangle\to 0$ for the trapped α's so that[7,8]

$$\frac{3}{4}\left\langle\int_0^{B_0/B} d\lambda\,\lambda[(v/v_\parallel)-(v/\langle v_\parallel\rangle)]\right\rangle=1.46\epsilon^{1/2}, \tag{9}$$

and obtain

$$Q=-0.73\epsilon^{1/2}(M_\alpha cI/Z_\alpha eB_0)^2M_\alpha(4\pi/\tau_s)\int_0^{v_0}dv\, v^3[(v_c^3+v^3)+v_b^3/3]\partial f_0/\partial\psi. \tag{10}$$

For the 3.5-MeV tail α's

$$v_0^3\gg v_c^3, v_b^3. \tag{11}$$

Therefore the remaining integral can be approximated by

$$4\pi\int_0^{v_0}dv\, v^3[(v_c^3+v^3)+v_b^3/3]\partial f_0/\partial\psi\approx(v_0^4/4)\partial(S\tau_s)/\partial\psi \tag{12}$$

since $|(\partial/\partial\psi)\ln(S\tau_s)|\sim|(\partial/\partial\psi)\ln v_c^3|$. As a result, Q is well approximated by

$$Q\approx-0.18\epsilon^{1/2}(M_\alpha cI/Z_\alpha eB_0)^2M_\alpha v_0^4\tau_s^{-1}\partial(S\tau_s)/\partial\psi, \tag{13}$$

which can be comparable to the background-ion heat flux. Equation (13) is ϵ^{-1} larger than the drag-dominated result of Refs. 4–6 and $(v_0/v_b)^3$ larger than the pitch-angle-scatter–dominated form of Ref. 7.

To check the validity of the rough estimate for the tail-α thermal conductivity the large aspect-ratio result $\partial/\partial\psi=(R_0B_p)^{-1}\partial/\partial r$ is employed. Then χ becomes

$$\chi \equiv Q/[(R_0B_p)^2 T\partial(S\tau_s)/\partial\psi] = 0.36(\epsilon^{1/2}/\tau_s)(v_0/\Omega_{p\alpha})^2(v_0/v_\alpha)^2, \quad (14)$$

with $\Omega_{p\alpha}=Z_\alpha eB_p/M_\alpha c$. Since $v_b^3/\tau_s=(3\pi^{1/2}/4)v_\alpha^3/\tau_{\alpha i}$ where

$$\tau_{\alpha i}\equiv 3M_\alpha^{1/2}T^{3/2}/[4(2\pi)^{1/2}Z_\alpha^2 e^4 \sum Z^2 N \ln\Lambda],$$

$$\chi\sim(\epsilon^{1/2}/\tau_{\alpha i})(v_\alpha/\Omega_{p\alpha})^2(v_0^4/v_b^3 v_\alpha)$$

so that the tail χ is $(v_0^4/v_b^3v_\alpha)\simeq 10^2$ times larger than the ash thermal conductivity!

In obtaining the preceding results both pitch-angle scattering and drag are retained. The dominance of electron drag at large $v\sim v_0\gg v_c\sim v_b$ resulted in the approximation (12) being valid (and the final results being insensitive to impurities). Pitch-angle scattering by the background ions led to the form (7). However, further justification for the use of (7) is required.

In neoclassical treatments where there is only one characteristic speed (the species thermal speed) the usual localization argument[7,9] requires $\epsilon^{1/2}\ll 1$ in order for pitch-angle scattering to dominate over energy scattering. For the tail α's, however, there are two characteristic speeds that are relevant ($v_c\sim v_b$ and v_0) and they make the drag $(v_0/v_b)^3\gg 1$ larger than pitch-angle scattering. As a result, the $\epsilon^{1/2}(v_0/v_b)^3\ll 1$ restriction of Ref. 7 *appears* to be necessary to neglect the drag. However, Eq. (7) is valid more generally and only requires $\epsilon(v_0/v_b)^3\lesssim 1$. To see that this is so the drag dominated solution for h of Refs. 4–6 is required, namely

$$h_d=-(M_\alpha cI/Z_\alpha e\langle B/v_\parallel\rangle)\partial f_0/\partial\psi = [M_\alpha cIv^2/2Z_\alpha eB_0(\partial\langle v_\parallel\rangle/\partial\lambda)]\partial f_0/\partial\psi. \quad (15)$$

The first crucial observation is that the pitch-angle derivative of h_d and $\partial h/\partial\lambda$ from (7) overlap for nearly all λ because

$$\frac{\partial}{\partial\lambda}\frac{1}{\partial\langle v_\parallel\rangle/\partial\lambda}=\frac{1}{\langle v_\parallel\rangle}[1+O(k^4,\epsilon k^2)],$$

where $k^2\equiv 2\epsilon\lambda/[1-(1-\epsilon)\lambda]$. As $k\to 1$, however, (15) fails because of its strong pitch-angle dependence. The evaluation of the correction to (15) because of pitch-angle scattering shows (15) to be valid only if $\epsilon(v_0/v_b)^3(1-k^2)^2\gg 1$ for $v\sim v_0$. Consequently, the second important observation is that not only does (7) remain valid closer to the trapped-passing boundary than (15), but for $\epsilon(v_0/v_b)^3\lesssim 1$ the region in which (15) is valid is negligible. In which case (7) only fails for the barely passing and trapped tail α's for which the banana regime ordering fails.

Hinton and Rosenbluth[9] have evaluated the corrections due to this boundary layer in which the streaming term $v_\parallel \mathbf{n}\cdot\nabla h$ must be retained in the kinetic equation. Their analysis is appropriate for the tail α's in the vicinity of the trapped-passing separatrix because drag can be neglected compared to streaming as long as $\epsilon^{1/2}v_0\tau_s/qR_0\gg 1$, where $v\sim v_0$ and $B_p/B_0\approx\epsilon/q$ are employed with q the local safety factor. Collisions then sustain the boundary layer about the separatrix by scattering barely passing and trapped tail α's in pitch angle by roughly the boundary layer width in a single bounce. With use of their results for $v\sim v_0$, the error introduced by the employment of (7) for all the passing tail α's and $h=0$ for all the trapped tail α's is of order $(qR_0v_b^3/\epsilon^{3/2}\tau_s v_0^4)^{1/2}\ll 1$.

Because the tail α's energy flux is larger than that of the ash α's when $(v_0^4/v_b^3v_\alpha)S\tau_s$ is greater than the ash density, the energetic α's may not deposit most of their energy in the background plasma via electron drag. To verify the preceding, the rate of radial energy loss divided by the rate that the α's supply energy may be estimated with (14) and $\partial Q/\partial\psi\sim\chi TS\tau_s/r^2$ to obtain

$$|(\partial Q/\partial\psi)/\tfrac{1}{2}M_\alpha v_0^2 S|\sim\epsilon^{1/2}(v_0/\Omega_{p\alpha}r)^2. \quad (16)$$

The enhanced neoclassical energy loss via the tail α's is a serious threat to ignition only if (16) is on the order of or greater than unity and $\epsilon(v_0/v_b)^3\sim 1$. Consequently, an acceptable radial energy loss via the tail α's requires $(\epsilon^{1/2}v_0/\Omega_{p\alpha}r)^2\ll\epsilon^{-1/2}$, which is more restrictive than the condition needed to avoid drift losses,[1] $\epsilon^{1/2}(v_0/\Omega_{p\alpha}r)^2\ll 1$. For larger values of $\epsilon(v_0/v_b)^3$ the results of Nocentini, Tessarotto, and Engelmann[4,5] pertain.

The preceding results and estimates presume that the α-particle distribution function has not been depleted by anomalous particle losses due to α-particle-destabilized shear Alfvén modes[10] since the Alfvén speed is typically in the range of v_c to v_0.

Although the preceding calculation focuses on the energetic tail α's, similar results would be obtained for neutral-beam-injected ions[11] of injection speed v_0 as long as $2T/M\ll v_b^2\ll v_0^2\ll v_e^2$ and tail-tail collisions are negligible. For smaller $v_0\lesssim v_b$ a more general evaluation of the velocity integral in (10) is required. The more general form will be given in a followup publication which will also evaluate the tail-particle flux.

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