Nonlocal Effects and the Raman Instability

H. C. Barr, T. J. M. Boyd,[a] and G. A. Coutts
University College of North Wales, University of Wales, Bangor, Wales
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The full Raman scattering equations have been solved in a linear density profile in an attempt to overcome the local nature of most previous analyses. The global solution obtained elucidates the transition between absolute and convective Raman instabilities. Feedback linking backward and forward Raman scattering can result in an instability that is *absolute* at all scattered frequencies. Moreover, thresholds for this absolute instability are lower by a factor of 4 than those for the convective instability where backward and forward scattering are treated as independent events.

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Stimulated Raman scattering has been a topic of interest in the physics of laser-plasma interactions ever since the scale length of the underdense coronal plasma in typical experiments became large enough for significant Raman growth to take place. This interest is sustained not only on account of the need to understand the physics of the interaction but by virtue of the threat posed by Raman scattering to effective inertial confinement. The plasma waves resulting from Raman decay generate hot electrons which materially affect target gain, on top of which the backscattered light reduces the efficiency of target illumination. Furthermore, high-gain targets are likely to generate more Raman backscatter than is the case at present.

Despite the attention given to stimulated Raman scattering over a number of years, discrepancies between theory and experiment persist. It is commonly observed in experiments that instability occurs at intensities at least an order of magnitude lower than those predicted theoretically.[1] This discrepancy has been attributed to a number of possible sources including the arbitrariness of the "threshold" in the case of the convective Raman instability—conventionally an amplification of $e^{2\pi}$. Again it has been suggested that local structure in the density profile in the form of plateaus would, if present, lower the threshold intensity for Raman scattering. Alternatively, if filaments are present in the underdense corona then the concurrent increase in scale length and enhancement of the light intensity within the filament would result in the appearance of Raman light below the conventional threshold. Of these possibilities, a simple change of the amplification factor to suit the circumstances is unappealing in that it is *ad hoc*. As for plateau formation, since it is attributable to a structure spreading from the quarter-critical density at typically ion sound speeds, one might expect to see wavelengths close to $2\lambda_0$ appearing at earlier times in the Raman spectrum. There is no evidence that this happens; indeed, many experiments have reported the onset of a broad spectrum of Raman wavelengths more or less simultaneously with gaps above λ_0 and below $2\lambda_0$. The gap below $2\lambda_0$ in particular has been the object of much attention and controversy and the anomaly persists.

It is harder to dismiss the possible effects of filamentation, resulting in lower Raman thresholds. Filamentary structures are present in many target plasmas, these structures being localized on a spatial scale finer than that associated with nonuniformities in the incident beam.[2] While there is some indirect evidence that filamentation can affect Raman emission, there is, at the same time, no data on observations of Raman scattering from plasmas in which filaments were detected and their characteristics diagnosed.

Given the persistence of both threshold and spectral anomalies it seems appropriate to reexamine some of the assumptions underlying the standard theory of stimulated Raman scattering (SRS). In this Letter we examine a nonlocal Raman model which allows for coupling between backward and forward scattering and shows that an *absolute* instability can occur for all scattered frequencies. In particular, in a monotonic density profile these assumptions have categorized Raman scattering as occurring in two distinct regimes. Below the quarter-critical density surface $n_c/4$, where n_c is the critical density for the incident laser light, a WKB analysis shows that for a linear density profile, only *convective amplification* is possible for both backward and forward scattering which are regarded as decoupled. This is the SRS-C regime. The conventional threshold is usually taken to be $(v_0/c)^2 k_0 L > 1$, where $2v_0$ is the quiver velocity of the electrons in the laser electric field, k_0 is the wave number of the incident light, and L the density scale length. This so-called threshold is, of course, arbitrary to a degree in that it is chosen to produce an amplification of $\exp(2\pi)$ over noise levels. This arbitrariness has led to suggestions that the amplification factor could be relaxed to diminish the discrepancy between theory and experiment.

Near the quarter-critical density, backward- and forward-scattering resonances become degenerate. At the scattered wave cutoff, WKB analysis breaks down, but by neglect of plasma-wave propagation ($v_e \rightarrow 0$), a second-order equation is retrieved from which absolutely unstable eigenstates are obtained with a threshold[3] $(v_0/c)^2(k_0L)^{4/3} > 1$. This is the SRS-A regime.

It has been common practice to regard backward and forward scattering as independent processes when in reality they are nothing of the kind, as Koch and Williams emphasized some years ago.[4] For laser light propagating in a plasma with a monotonically increasing density, the plasma wave amplified in the generation of backscattered light will propagate up the density gradient to the forward-scattering resonance. The separation in density of the backward- and forward-scattering resonances is very small, being of the order of $(v_e/c)^2$, where v_e is the electron thermal velocity. Thus unless the temperature is high enough and consequently the plasma-wave damping strong enough for the wave to decay between the two resonances, the forward propagating plasma wave will act as an enhanced noise source for the generation of forward-scattered light. If in turn the plasma is overdense to this light, it will reflect at some higher density and propagate back to the backscatter resonance point in the density profile where it will again be subject to amplification. These *nonlocal* effects create a feedback loop leading to temporal growth of the coupled backward- and forward-scattering instabilities in the manner described by Koch and Williams in 1984. Their approach is essentially in amalgam of *local* analyses of the two resonance regions with a WKB treatment of waves propagating between them, resulting in a global description.

By contrast the model used in this work is *global in its entirety*. We solve the full coupled wave equations for both plasma and scattered light waves, encompassing backward- and forward-scattering resonances with wave reflection. We write the one-dimensional equations corresponding to the backward and forward scattering of laser light normally incident to an arbitrary density profile $n(x)$, corresponding to a plasma frequency $\omega_p(x)$, as

$$[\partial^2/\partial t^2+\omega_p^2(x)-\partial^2/\partial x^2]v_s=v_0\partial E_p/\partial x, \quad (1)$$

$$[\partial^2/\partial t^2+\omega_p^2(x)-3v_e^2\partial^2/\partial x^2]E_p = \omega_p^2(x)(\partial/\partial x)(v_0v_s), \quad (2)$$

where v_s and v_0 define the electron quiver velocity in the field of the scattered and incident light waves, respectively. E_p is the plasma-wave electric field and all quantities are normalized to c and ω_0. In this model, damping terms are not included. We solve (1) and (2) by means of a code which treats the global SRS problem as opposed to resorting to a WKB treatment, except for the laser field. The equations are Laplace transformed in time and only resonant terms are retained to yield (a) temporally growing (i.e., absolutely unstable) eigenstates of the homogeneous SRS equations if these exist or, if this is not the case, (b) reflectivities (convective amplification) from the nonhomogeneous equations with a noise source retained.

The Raman equations are solved in an *inhomogeneous* plasma slab, $0 \le x \le l$ with boundary conditions chosen to continuously match fields in the *homogeneous* plasma extending for $x < 0$ and $x > l$. The interaction region is assumed infinite, though only resonant locally within the region $0 < x < l$. The driven equations are solved analytically within the homogeneous plasma $x < 0$, $x > l$ to provide four independent solutions. Of these in (a) above, only the two representing outward-propagating waves are chosen in each region. In the case of convective amplification, a variety of options for incoming waves is available to represent noise sources.

The code has been run to obtain temporally growing eigenstates in the linear density profile $n(x) = n(0) + xn_c/L$ as a function of scale length or maximum density, with reflections of forward-propagating waves included or excluded as the case may be. Note that L is the distance from vacuum to the critical density and is related to the more widely used, but density dependent, *local* density scale length by $L_{\text{local}} = n/(dn/dx) = \omega_p^2 L$. In the first instance, to test the dependence on the feedback mechanism, the code was run with variable maximum density but with the scale length held constant. This effectively controls the reflectivity of the forward-scattered light. Figure 1 plots the growth rates of several eigenstates versus maximum density for a plasma for which $L = 80\lambda_0$, $T_e = 1.7$ keV, and $v_0 = 0.1c$. To the right of the dashed line $\omega_s = \omega_{p\max}$, growth is a maximum when forward-scattered light is totally reflected; the addition of overdense plasma at the right-hand boundary does not affect the physics. If we reduce the reflectivity of forward-scattered light, the growth rate diminishes so that ultimately temporal growth is quenched and a convective instability ensues.

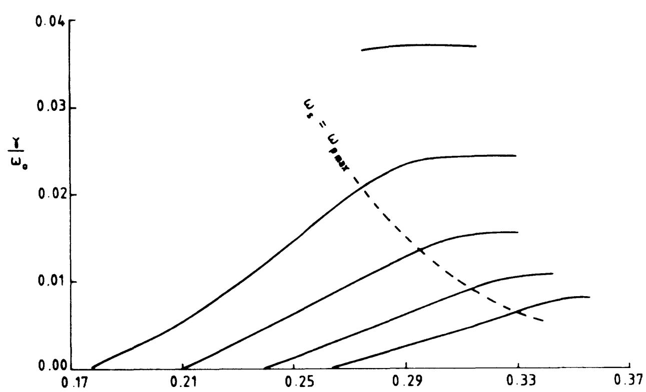

FIG. 1. Growth rates for several eigenstates plotted against maximum plasma density. Forward-scattered light is totally reflected to the right of the dashed line; away from it on the left, forward-scattered light is increasingly transmitted ($L = 80\lambda_0$, $v_e = 0.01c$, $v_0 = 0.1c$).

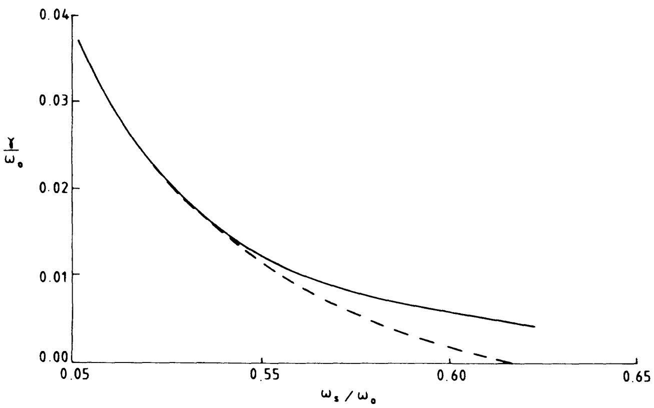

FIG. 2. Growth rate vs scattered frequency. The solid line assumes total reflection of forward-scattered light giving rise to absolute instability at all frequencies. The dashed line assumes a maximum density of $0.3n_c$. It diverges from the solid line at the frequency at which forward-scattered light may be transmitted ($L=160\lambda_0$, $v_e=0.01c$, $v_0=0.01c$).

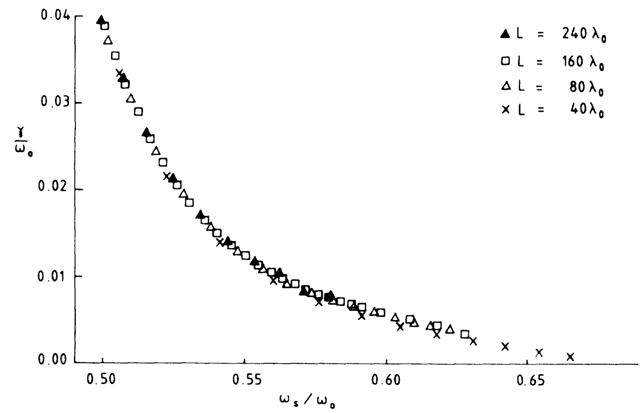

FIG. 3. Growth rates vs scattered frequency for scale lengths $L=40\lambda_0$–$240\lambda_0$. It shows the growth rate to be independent of scale length ($v_e=0.01c$, $v_0=0.1c$).

Figure 2 represents Raman growth rates as functions of the scattered frequency. The solid line corresponds to $\omega_{p\max} \gtrsim 0.63\omega_0$ so that there is total reflection of forward-scattered light for all frequencies shown. In the absence of damping, absolute growth occurs at any frequency. The dashed curve, on the other hand, corresponds to a maximum density of $0.3n_c$ with the result that the two growth-rate plots diverge at that frequency which allows scattered light to be transmitted. As the reflectivity is reduced, temporal growth is no longer possible so that beyond some critical frequency only a convective instability appears.

The code has been used to determine Raman characteristics for a range of plasma parameters. In particular we have allowed the density scale length to range from $L=40\lambda_0$ to $L=320\lambda_0$. Figure 3 shows the growth rate plotted against the scattered frequency for a range of scale lengths up to $240\lambda_0$. This shows that the growth rate appears to be, if not independent, at least insensitive to scale length well above threshold. The line spacings are approximately proportional to L^{-1} for ω_s away from $\omega_0/2$. We have also examined the growth rate as a function of v_0. For the first eigenvalue corresponding to emission near $\omega_0/2$, growth scales linearly with v_0. At higher scattered frequencies a nonlinear dependence, approximately proportional to v_0^2, is observed. At these frequencies, $\omega_s > \omega_0/2$, the analysis of Koch and Williams is valid and agrees with our results. All of these results correspond to cases in which total reflection of forward-scattered light is assumed.

In Fig. 4 we plot the minimum threshold (occurring when feedback is a maximum, i.e., total reflection) versus scattered-wave frequency. We see from this that the SRS-A threshold is reproduced correctly, confirming that the plasma-wave propagation is unimportant in this case. More significantly, we see that for $\omega_s > \omega_0/2$ the absolute-instability threshold is lower by a factor of about 4 than the SRS-C threshold, determined for an amplification of $e^{2\pi}$. Allowance of transmission of forward-scattered light quickly raises thresholds for absolute instability.

The relative thresholds are more weakly dependent on ω_s than previously believed to be the case. This effect is even more marked for an exponential density profile (as when a constant *local* density scale length is assumed) when thresholds are almost constant as a function of frequency. We have compared our results with thresholds evaluated on the basis of the Koch-Williams analysis and found good agreement.

Figure 5 shows a comparison with SRS-A theory. The SRS-A equations in which plasma-wave convection is neglected, depend only on the parameters $\lambda = (kv_0\omega_pL^{2/3})^2$ and $D=(2\omega_s-1)L^{2/3}$. [Note that our D is $D-B$ as defined in Eq. (53) of Ref. 3, normalized

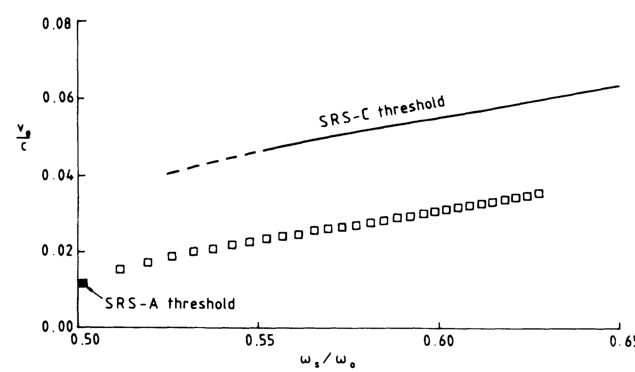

FIG. 4. Squares: Threshold v_0 vs scattered frequency. The first agrees with SRS-A theory which is valid near $\omega_0/2$. The solid line, valid away from $\omega_0/2$, is the SRS-C theory threshold ($L=160\lambda_0$, $v_e=0.01c$).

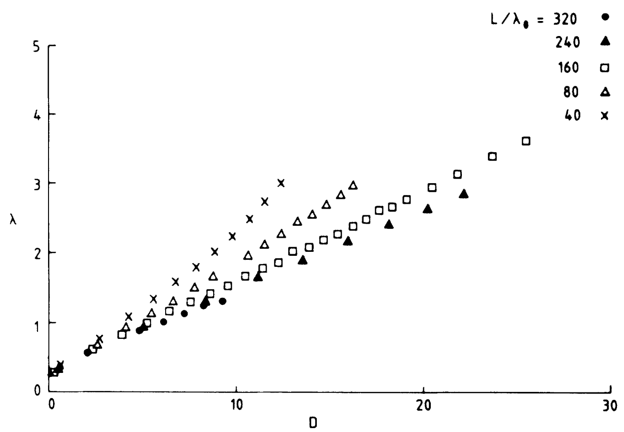

FIG. 5. Threshold plotted in terms of the normalized coordinates $\lambda=(kv_0\omega_p L^{2/3})^2$ and $D=(2\omega_s-1)L^{2/3}$. The theory of Liu, Rosenbluth, and White (Ref. 3) in which plasma-wave convection is ignored and hence dependent only on the parameters λ and D gives the correct scalings when ω_s is near $\omega_0/2$. At higher frequencies, plasma-wave convection contributes significantly to threshold values ($v_e=0.01c$, $v_0=0.1c$).

and with the density-independent definition of the scale length adopted in this Letter.] The parameter λ is seen to be independent of L near $\omega_0/2$, indicating the familiar $L^{4/3}$ scaling of the threshold. At higher frequencies plasma-wave convection cannot be neglected and shows increased thresholds in steeper profiles as one would expect.

The results described in the Letter have been obtained by use of a code which is applicable generally to warm inhomogeneous plasmas with any specified density profile and which allows for multiple resonances and wave reflection. Using this code we have elucidated the transition between SRS-A and SRS-C regimes. In situations in which backward and forward scattering are independent or weakly coupled, only *convective amplification* is possible, i.e., SRS-C theory is valid. For stronger feedback, temporal growth becomes possible and in the limit of total reflection of forward-scattered light, an *absolute instability* occurs at all frequencies. Under these conditions thresholds are approximately a factor of 4 below SRS-C thresholds.

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[a]Present address: Department of Physics and Astronomy, Wilder Laboratory, Dartmouth College, Hanover, NH 03755.

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