Precision Measurements of the K^- and Σ^- Masses

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(Received 13 August 1987)

The masses of the K^- and Σ^- have been measured by use of exotic-atom x-ray transitions in Pb and W. The transitions used were free from contaminant nuclear γ rays and strong-interaction effects. Two K^- transitions and three Σ^- transitions for each target satisfied these criteria. The results are $M_{K^-}=493.636\pm0.011$ MeV and $M_{\Sigma^-}=1197.532\pm0.057$ MeV. The precision of these measurements is greater than that of any previous experiment. When combined with previous measurements they form a consistent set.

PACS numbers: 14.40.Aq, 14.20.Jn, 36.10.Gv

Kaonic and Σ-hyperonic exotic atoms in lead and tungsten have been studied with use of the Brookhaven National Laboratory Alternating Gradient Synchrotron. The magnetic moment of the Σ^- obtained from the analysis of the earliest portion of these data has been published, and a final manuscript is in preparation.[1] In this Letter we present the results of the analysis of all the data for the K^- and Σ^- masses.

Kaons of 680 MeV/c momentum from the separated C4 beam ($\pi^-:K^-=9:1$) were slowed down in a Cu degrader and brought to rest in a laminar target consisting of sheets of natural Pb or W immersed in liquid hydrogen. The Σ^- hyperons were produced by kaons stopped in liquid hydrogen through the reaction $K^-+p \rightarrow \Sigma^-\pi^+$, which has a 46% branching ratio. The resulting monoenergetic π^+ ($T_{\pi^+}=83$ MeV) served as a tag for Σ^- production ($T_\Sigma=12.4$ MeV). The π^+ were stopped in scintillation range spectrometers in which the subsequent e^+ from the decay chain $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ were detected. The Σ^- were brought to rest in the target material where they formed exotic atoms. Kaonic atoms were formed by beam K^- which stopped in the target sheets. Additional experimental details are given in Ref. 1.

X rays from the Σ^- and K^- atoms were detected by three intrinsic reverse-electrode Ge detectors housed in a single cryostat beneath the target. The x-ray events were sorted into tagged and untagged histograms depending on whether a π^+ was detected in coincidence with a stopped K^- and a signal from one of the Ge detectors. Figure 1 shows a Pb untagged spectrum with intense kaonic x-ray lines while in the tagged spectrum, Fig. 2, the Σ^- x-ray intensities are enhanced.

The Ge-detector resolution, line shape, and absolute energy calibration were determined continuously under beam conditions by our monitoring radioactive sources. The calibration sources[2] used were ^{57}Co, ^{133}Ba, ^{192}Ir, and ^{137}Cs. During beam-on periods chance coincidences between incident beam π^- and γ rays from the radioactive sources were recorded as beam-on calibration events. During the beam-off portion of the Alternating Gradient Synchrotron beam spill, histograms from the 122.061- and 661.660-keV lines from ^{57}Co and ^{137}Cs were accumulated. Every ten minutes these histograms were fitted on-line to provide a check on the gain stability of the systems and the results were written to tape for use in the off-line analysis as a two-point stabilization correction.

The relative intensities of the unresolved x rays result-

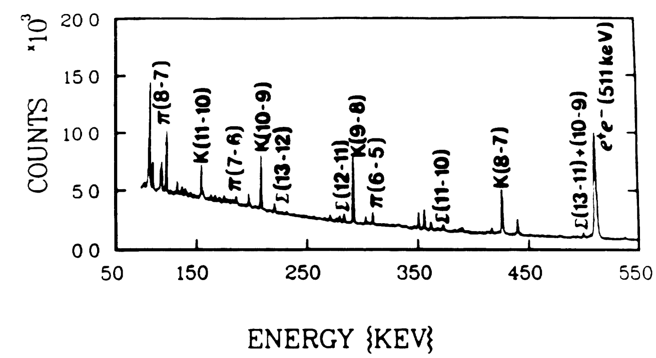

FIG. 1. Untagged Pb x-ray spectrum showing intense kaonic x-ray transitions.

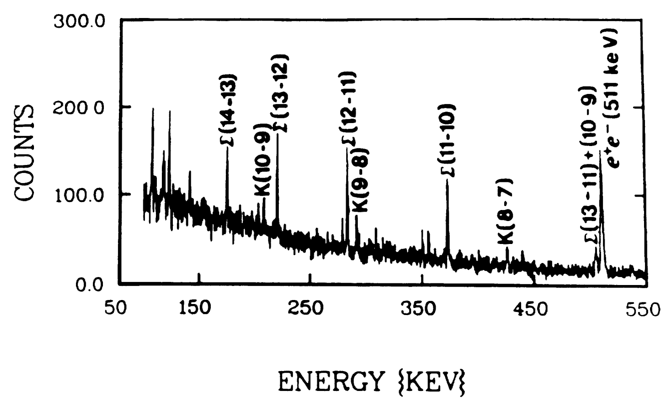

FIG. 2. Tagged Pb x-ray spectrum for the same energy region.

ing from noncircular transitions (those between levels with $l < n-1$) were determined from computer simulations of the atomic cascade. The initial populations of states and the strong-interaction absorption parameters (based on an optical-model potential[3]) used in the calculation were constrained by the yields of six $\Delta n = -1$ and three $\Delta n = -2$ transitions in Σ^-Pb, five $\Delta n = -1$ and two $\Delta n = -2$ transitions in Σ^-W, four $\Delta n = -1$ and five $\Delta n = -2$ transitions in K^-Pb, and four $\Delta n = -1$ and three $\Delta n = -2$ transitions in K^-W. The intensities of the K^- (Σ^-) noncircular components were typically determined to $\pm 10\%$ ($\pm 20\%$) which introduced less than ± 1 keV (± 5 keV) uncertainty in the measured masses.

There were two sets of gain-shifted data from each target, each consisting of three spectra from the different detectors. The measured x-ray transition energy was determined separately from each of these six spectra. For each target, fits of the K^- x-ray peaks in the untagged spectra and Σ^- x-ray peaks in the tagged spectra were made for each detector. The amplitude ratios of the noncircular transitions relative to the circular transition were held fixed to the values predicted by the cascade calculation. The energy separations between the circular and noncircular transitions (and fine-structure components for Σ^- transitions) were held fixed to calculated values as discussed below. The instrumental widths of all the components of a given $n_i \rightarrow n_f$ peak were assumed to be equal, but this width was a free parameter in the fit. The centroid of the circular transition and the background parameters were also free parameters. If additional γ- or x-ray lines were present in the fit region they were represented by Gaussians with the full width at half maximum and the amplitude as free parameters. The χ^2 per degree of freedom for a fit over a region of 200 channels (18 keV) was typically ≤ 1.0. To search for nuclear γ rays which might fall in the region of the K^- and Σ^- x rays, π^- were stopped in each target. The K^- and Σ^- x-ray transitions used in the analysis were chosen such that they had no measurable strong-interaction energy shift and no contaminant x rays or γ rays within one full width of the x ray of interest. The $K^-(10 \rightarrow 9)$ transition was not used in this analysis because of the close proximity of the $K^-(13 \rightarrow 11)$ transition.

Once the channel number for the centroid of a circular transition was obtained, it was necessary to convert this channel number to energy. Detailed studies of the integral nonlinearities in the detector-amplifier-ADC (analog-to-digital converter) systems revealed that nonlinearities as large as 2 parts in 10^4 were present in some energy regions. The deviation of the gain from a linear function was parametrized by up to a sixth-order Legendre polynomial for each of the three detectors.

If an x-ray transition fell in a region where the residuals from a linear fit varied linearly, and calibration lines existed with energies that bracketed the line of interest, a linear interpolation was used to determine the energy of the circular transition. The statistical error on the transition energy from the centroid uncertainties was calculated. The weighted average of the six measurements of the transition energy was calculated. If the χ^2 per degree of freedom for this set was greater than 1.0, the statistical error on the average was scaled up[4] by the factor $(\chi^2/\nu)^{1/2}$ where ν is the number of degrees of freedom. The uncertainty from energy uncertainties on the calibration lines[2] was then added in quadrature to the statistical error on the centroid. This was taken to be the total error on the transition energy. This procedure was used for the W$K^-(11 \rightarrow 10)$, Pb$K^-(9 \rightarrow 8)$, and Pb$\Sigma^-(12 \rightarrow 11)$ transitions. For all other transitions used in the K^- and Σ^- mass measurements it was necessary to use the Legendre-polynomial fit to the nonlinearities to determine the energy.

To determine the K^- (Σ^-) mass it was necessary to calculate the transition energy with the assumption of a value for the K^- (Σ^-) mass and then to compare that energy to the measured value. The Klein-Gordon (Dirac) equation was integrated with potentials for a finite nuclear-charge distribution, vacuum polarization,

TABLE I. Calculated and measured circular $E1$ transition energies in kiloelectronvolts. The K^- and Σ^- masses used in the calculation were 493.6364 and 1197.5323 MeV, respectively. For the Σ^- transitions, the magnetic moment was taken to be $-1.161\mu_N$. In the analysis, the fine-structure splitting was held fixed to the calculated value. The two calculated energies listed for Σ^- transitions are the $j_i = l_i + \frac{1}{2} \rightarrow j_f = l_f + \frac{1}{2}$ and $j_i = l_i - \frac{1}{2} \rightarrow j_f = l_f - \frac{1}{2}$ energies, respectively. The $\Delta j = 0$ transitions are of negligible intensity and were ignored. The measured energy, which corresponds to the lower-energy member of the fine-structure doublet, is to be compared with the calculated $j_i = l_i + \frac{1}{2} \rightarrow j_f = l_f + \frac{1}{2}$ transition energy.

Transition	$E_{\text{calculated}}$		E_{measured}
K^-Pb(11→10)	153.891		153.903±0.008
K^-Pb(9→8)	291.583		291.5800±0.0044
K^-W(11→10)	125.208		125.251±0.024
K^-W(9→8)	237.171		237.206±0.035
Σ^-Pb(14→13)	174.707	174.846	174.736±0.028
Σ^-Pb(13→12)	220.322	220.546	220.315±0.018
Σ^-Pb(12→11)	283.308	283.682	283.280±0.044
Σ^-W(14→13)	142.107	142.199	142.091±0.047
Σ^-W(13→12)	179.205	179.353	179.183±0.019
Σ^-W(12→11)	230.426	230.673	230.454±0.021

and electron screening following Borie.[5] Higher-order relativistic recoil corrections as well as static polarization of both the orbiting particle and the nucleus were included as perturbations using the wave functions obtained from the Klein-Gordon (Dirac) equation. The circular and noncircular transition energies were calculated initially with use of the tabulated masses.[4] For the Σ^- transitions, a magnetic moment of $\mu(\Sigma^-) = (-1.161 \pm 0.021)\mu_n$ was assumed in order to calculate the fine-structure splitting. This value of the magnetic moment was obtained by our taking the weighted average of the published Σ^- magnetic moment from four experiments.[1,6-8] The values of the Σ^- magnetic moment which were obtained in the two early exotic-atom measurements were not included for reasons discussed in Ref. 1.

The measured mass was then determined by our iteratively scaling the mass used in the energy calculation by the ratio of the corresponding measured to calculated energies. This process was iterated until the value obtained for the mass changed by less than 0.1σ per iteration. Table I contains the calculated and measured transition energies. Estimated errors in the calculated transition energies are ± 1 eV.

Studies were made of the systematic errors arising from uncertainties in the instrumental resolutions of the detectors, in the calculated energy levels, and in the noncircular-transition contributions. In background studies we carefully examined the energy regions of interest for contaminant nuclear γ rays by stopping π^- in our Pb and W targets. In the analysis of the Σ^- data, the sensitivity of the measured mass value to the populations of the fine-structure states and to the value of the magnetic moment, the latter being varied by $\pm 2\sigma$, was also studied.

For each transition the systematic and statistical errors on the transition energy were added in quadrature to obtain a total error. The mass values obtained from the several transitions in the two targets are given in Table II. The error obtained on the weighted average of the separate K^- mass values was scaled up by $\sqrt{2.31}$ (see Table II) to give the total error.[4]

The masses thus obtained were

$$M_{K^-} = 493.636 \pm 0.011 \text{ MeV}$$

and

$$M_{\Sigma^-} = 1197.532 \pm 0.057 \text{ MeV}.$$

A weighted average of this work and all measurements of M_{K^-} listed in Ref. 4 gives 493.6443 ± 0.0088 MeV with a χ^2 per degree of freedom of 0.77. The mass value of 493.657 ± 0.020 MeV obtained by Cheng *et al.*[9] differs from our measurement in several ways. In the current work only two transitions in Pb and two in W satisfied the criteria discussed above. If the measured K^-Pb energies reported in Table I are compared with the values of 153.892 ± 0.011 keV and 291.577 ± 0.013

TABLE II. Experimental mass measurements from each transition in megaelectronvolts. If the χ^2 per degree of freedom was greater than 1.0, the error listed with the weighted average is the statistical error scaled up by a factor of $(\chi^2/\nu)^{1/2}$.

Transition	M_{K^-}	Transition	M_{Σ^-}
K^-Pb(11→10)	493.675±0.026	Σ^-Pb(14→13)	1197.731±0.192
K^-Pb(9→8)	493.631±0.007	Σ^-Pb(13→12)	1197.492±0.098
K^-W(11→10)	493.806±0.095	Σ^-Pb(12→11)	1197.412±0.186
K^-W(9→8)	493.709±0.073	Σ^-W(14→13)	1197.397±0.396
		Σ^-W(13→12)	1197.388±0.127
		Σ^-W(12→11)	1197.677±0.109
Average	493.636±0.011 $\chi^2/\nu=2.31$	Average	1197.532±0.057 $\chi^2/\nu=0.968$

keV obtained by Cheng *et al.*,[9] one finds agreement to better than 1σ. Although both of these values are lower than those reported in Table I, the mass reported in Ref. 9 is higher than that reported here. This is a reflection of the improvements in calculations of the kaonic-atom levels. The calculational method of Borie,[5] which was improved and expanded by Phillips,[10] improves the accuracy of the calculations for these levels to ± 1 eV.

The present measurement of M_{Σ^-} does not seem to be in good agreement with the previous exotic-atom measurement of Dugan *et al.*[11] However, it should be noted that in the analysis of Ref. 11 the value $\mu(\Sigma^-) = -1.40\mu_N$ was used, and for the $\mathrm{Pb}\Sigma^-(12 \rightarrow 11)$ transition a 3% noncircular-transition intensity was assumed. With the improved signal-to-noise ratio of the present work, this noncircular intensity was determined to be 12%. The untagged $\mathrm{Pb}\Sigma(12 \rightarrow 11)$ transition (equivalent to the data of Ref. 11) was analyzed for M_Σ with the magnetic moment and noncircular contribution assumed in Ref. 11. The value thus obtained for M_{Σ^-} was in reasonable agreement with the value quoted by Dugan *et al.*[11] We conclude that the data of Ref. 11 need to be reanalyzed with use of the current value of $\mu(\Sigma^-)$ and a larger noncircular contribution.

We wish to acknowledge the important efforts of R. Pehl, F. Goulding, D. Landis, and N. Madden of the Lawrence Berkeley Laboratory, and R. Trammell of EG&G Ortec in developing the Ge detector and associated electronics. We wish to thank Dr. D. Lowenstein and the Brookhaven Alternating Gradient Synchrotron staff for their support, and R. Meier for the design and construction of the laminar liquid-hydrogen target. We wish to thank Dr. L. Rosen of the Los Alamos Meson Physics Facility for the use of the LAMPF test beam and Dr. N. J. Colella, Dr. K. L. Giovanetti, and Dr. R. D. Hart for their help on the earlier phases of the experiment. We thank Dr. J. Reidy for helpful discussions on absolute energy calibrations, and Dr. E. Borie, Dr. M. Leon, and Dr. R. Seki for the use of their computer codes. This work was supported in part by the National Science Foundation and the U.S. Department of Energy.

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