Two-Photon Correlated-Spontaneous-Emission Laser: Quantum Noise Quenching and Squeezing

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Previous studies have shown that the possible utility of the two-photon laser as a squeezed-state generator is negated by spontaneous-emission fluctuations. We here show that a two-photon correlated–spontaneous-emission laser can produce light which is both squeezed and free from spontaneous-emission fluctuations.

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The limiting sensitivity in modern optical interferometry is often determined by quantum noise. In the case of passive interferometry this "photon shot noise" has its origin in the vacuum fluctuations of the radiation field. In the case of active interferometry the "phase diffusion noise" is due to spontaneous-emission fluctuations.

The application of "squeezed states" (error in one quadrature below vacuum noise level) in passive interferometry has attracted substantial theoretical[1] and experimental[2] interest. It has also been shown both theoretically[3] and experimentally[4] that in active devices quenching of spontaneous-emission fluctuations can occur in the correlated-emission laser (CEL) and a substantial noise reduced below the Schawlow-Townes limit is possible. Clearly, in the sense of improving upon quantum limits of sensitivity, squeezing[5] and CEL operation are related but, until now, no such connection has been demonstrated.

We here present the quantum theory of the two-photon CEL and show, for the first time, that (1) it is possible to develop a two-photon laser which shows CEL quantum noise quenching; (2) this two-photon CEL can also display squeezing; and (3) it is an active device.

The two-photon laser has long been suggested as a squeezed-state generator,[6] but its utility has subsequently been negated by the observation that "any potential squeezing is destroyed by fluctuations resulting from spontaneous emission,"[5] because of the phase insensitivity of these events. Nevertheless, the recent experimental demonstration of stimulated-emission gain[7] and then cw operation in a two-photon maser[8] provide a stimulus for the present studies.

In fact, as mentioned earlier, we have shown spontaneous-emission noise quenching via the CEL. The conditions for noise quenching revealed the crucial role played by the atomic coherence in the achievement of CEL operation (active, phase-insensitive system).[3] A similar conclusion has been reached about squeezing. Spontaneous-emission fluctuations are squeezed in a single three-level atom (passive, phase-sensitive system) if it is prepared in a coherent superposition of its ground state and topmost state.[9]

Motivated by the preceding arguments emphasizing the importance of a phase-sensitive interaction *and* correlation between spontaneous-emission events, we are led to investigate the two-photon CEL. In this context we note that in an interesting series of recent papers the questions of amplitude squeezing[10] and the utilization of squeezed vacuum to quench spontaneous-emission phase noise[11] have been addressed. Our primary interest is in phase noise and we find complete spontaneous-emission noise quenching and squeezing in an active device which generates squeezed light of macroscopic intensity from ordinary vacuum.

We present the linear theory, sufficient for the discussion of phase noise,[12,13] of a two-photon laser[14-16] with three-level atoms in the ladder configuration as in Fig. 1. The atoms are injected into the cavity with initial populations ρ_{aa}, ρ_{bb}, and ρ_{cc} and initial coherences $\rho_{ab}=\rho_{ba}^*=|\rho_{ab}|e^{i\theta_{ab}}$, $\rho_{bc}=\rho_{cb}^*=|\rho_{bc}|e^{i\theta_{bc}}$, and $\rho_{ac}=\rho_{ca}^*=|\rho_{ca}|\times e^{i\theta_{ac}}$, where a, b, and c refer to the top, middle, and bottom levels, respectively. The explicit dependence of

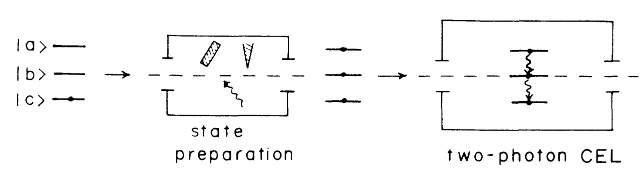

FIG. 1. Scheme of the two-photon correlated-emission laser. State preparation (first cavity) is separated from the laser operation (second cavity). Atoms in the first cavity are prepared in a coherent superposition by, e.g., passing through a foil or near a knife edge or optical pumping, and are injected into the second cavity where laser operation takes place.

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the results on θ_{ab}, θ_{bc}, and θ_{ac} renders the two-photon CEL laser a phase-sensitive device. By using standard technique of the quantum theory of the laser,[12] we obtain the master equation for the field density matrix ρ of the two-photon CEL as

$$\dot{\rho}=\{-is(\rho_{ba}\mathcal{L}_1+\rho_{cb}\mathcal{L}_2)[a,\rho]-(\alpha/2)[(\rho_{aa}\mathcal{L}_1+\rho_{bb}\mathcal{L}_2)(aa^\dagger\rho-a^\dagger\rho a)+(\rho_{bb}\mathcal{L}_1+\rho_{cc}\mathcal{L}_2)(\rho a^\dagger a-a\rho a^\dagger)$$
$$+\rho_{ca}\mathcal{L}_0\mathcal{L}_1(\rho aa-a\rho a)+\rho_{ca}\mathcal{L}_0\mathcal{L}_2(\rho aa-a\rho a)]-(\gamma/2)(\rho a^\dagger a-a\rho a^\dagger)+\text{H.c.}\}-i(\Omega-\nu)[a^\dagger a,\rho], \quad (1)$$

with $\mathcal{L}_j=\Gamma/(\Gamma-i\Delta_j)$, $j=0,1,2$, $\Delta_0=\omega_{ac}-2\nu$, $\Delta_1=\omega_{ab}-\nu$, $\Delta_2=\omega_{bc}-\nu$, where ω_{ij} is the transition frequency between levels i and j, Ω is the bare-cavity eigenfrequency, and ν is the operating frequency of the laser. In this simple model, $s=r_a g/\Gamma$ and the linear gain coefficient $\alpha=2r_a g^2/\Gamma^2$ depend on the atomic decay rate Γ, the atom-field coupling constant g (same for all levels), and the rate of atomic injection r_a. The physical significance of the various terms in Eq. (1) can be described as follows.[17] The first term, containing ρ_{ba} and ρ_{cb}, has the form of an injected signal. The term involving $\rho_{aa}\mathcal{L}_1+\rho_{bb}\mathcal{L}_2$ corresponds to the usual gain and that proportional to $\rho_{bb}\mathcal{L}_1+\rho_{cc}\mathcal{L}_2$, to absorption. The term containing the coherence ρ_{ca} will be seen to be responsible for CEL operation and squeezing. The term with γ represents cavity losses. The last term accounts for frequency pulling.

We can now convert this master equation into a Fokker-Planck equation for the Glauber-Sudarshan P representation of the density operator. Polar coordinates ($a|\mathcal{E}\rangle=\mathcal{E}|\mathcal{E}\rangle$, $\mathcal{E}=re^{i\phi}$) yield the phase-diffusion coefficient as

$$D(\phi)=(\alpha/8r^2)(\rho_{aa}\mathcal{L}_1+\rho_{bb}\mathcal{L}_2+\rho_{ca}\mathcal{L}_0\mathcal{L}_2e^{2i\phi})+\text{c.c.}. \quad (2)$$

If no atomic coherence is introduced into the system ($\rho_{ca}=0$) then (2) reduces to the standard diffusion coefficient of laser theory, and we see that the "ordinary" two-photon laser operation does not lead to quantum noise quenching. The Fokker-Planck equation yields the equation of motion for the phase ϕ as

$$\langle\dot{\phi}\rangle=\langle-(2r)^{-1}\operatorname{Im}(B+\alpha\rho_{ca}\mathcal{L}_0\mathcal{L}_2e^{i2\phi}r^{-1})+\Omega-\nu\rangle\equiv\langle d(\phi)\rangle, \quad (3)$$

where

$$B=\alpha r[(\rho_{aa}-\rho_{bb})\mathcal{L}_1+(\rho_{bb}-\rho_{cc})\mathcal{L}_2+\rho_{ca}\mathcal{L}_0(\mathcal{L}_1-\mathcal{L}_2)e^{i2\phi}]+2is\,e^{i\phi}(\rho_{ba}\mathcal{L}_1+\rho_{cb}\mathcal{L}_2),$$

and the phase drift has been defined as $d(\phi)$.

Now the total phase uncertainty in steady state is[18]

$$\langle(\delta\phi)^2\rangle=\langle(\delta\phi)^2\rangle_{\text{spont.}}+\langle(\delta\phi)^2\rangle_{\text{vacuum}}, \quad (4)$$

where the first (spontaneous-emission noise) term is found from our Fokker-Planck equation and the second (vacuum noise) term is given by $(4\bar{n})^{-1}$, where $\bar{n}$ ($\simeq r^2$) is the steady-state photon number. In order to connect the phase-diffusion coefficient to noise in the phase variable of a laser, we obtain the equation of motion for $\langle(\delta\phi)^2\rangle_{\text{spont.}}$ from the Fokker-Planck equation as

$$\langle(\delta\dot{\phi})^2\rangle_{\text{spont.}}=2\langle d(\phi)\delta\phi\rangle+2\langle D(\phi)\rangle. \quad (5)$$

In steady state, as seen from Eq. (4), the phase locks to the mean value $\phi=\phi_0$ for which $d(\phi_0)=0$. This value is stable if $\partial d(\phi_0)/\partial\phi<0$. Expanding $d(\phi)$ around ϕ_0 up to first order in Eq. (5) and using Eq. (4), one obtains the steady-state value of the phase variance to be

$$\langle(\delta\phi)^2\rangle=(4\bar{n})^{-1}+\langle D(\phi)\rangle|\partial d(\phi_0)/\partial\phi|^{-1}. \quad (6)$$

The first term on the right-hand side is the vacuum fluctuation noise; the second term is the additional spontaneous-emission noise. Under stable phase locking, the sign of $\langle D(\phi)\rangle$ will decide whether the additional noise adds to or subtracts from the vacuum noise. From Eq. (2), without atomic coherence, $\langle D(\phi)\rangle$ is always greater than zero and the added noise is always positive. Thus the Schawlow-Townes linewidth corresponds to the smallest attainable linewidth of an active device with incoherent atomic excitations. As shown below, under the appropriate (CEL) conditions we find $\langle D(\phi)\rangle=0$ or even $\langle D(\phi)\rangle<0$.

Next we find conditions under which the phase noise is below the Schawlow-Townes limit and then below the vacuum level. From Eq. (2) we see that the injected atomic coherence modifies the spontaneous-emission noise. The actual value of $\langle D(\phi)\rangle$ is controlled by the magnitude $|\rho_{ac}|$ and phase θ_{ac} of the atomic coherence and detunings. We shall focus our attention on two-photon resonance, $\Delta_0=0$. Let $\Delta=\Delta_1=-\Delta_2$ and $\mathcal{L}=\Gamma/(\Gamma-i\Delta)$. The expression of $D(\phi)$ in terms of these variables, from Eq. (2), is

$$D(\phi)=(4\bar{n})^{-1}\alpha|\mathcal{L}|^2\{\rho_{aa}+\rho_{bb}+|\rho_{ac}/\mathcal{L}|\cos[\theta_{ac}-2\phi+\arctan(\Delta/\Gamma)]\}. \quad (7)$$

The injected atomic coherence does not affect linear mode pulling. From (3) we obtain phase locking satisfying $d(\phi_0)=0$ which is stable if $\partial d(\phi_0)/\partial\phi<0$. For (1) $\Delta=0$, $\rho_{ba}\neq0$, $\rho_{cb}\neq0$ ($\theta_{ab}=\theta_{bc}=\frac{1}{2}\theta_{ac}$), the phase locks to $\phi_0=\theta_{ab}-\frac{1}{2}\pi$ and

$$D(\phi_0)=(4\bar{n})^{-1}\alpha(\rho_{aa}+\rho_{bb}-|\rho_{ac}|). \quad (8)$$

For (2) $\Delta\neq 0$, $\rho_{bb}=\rho_{ba}=\rho_{cb}=0$, the phase locks to $\phi_0=\frac{1}{2}\theta_{ac}-(\frac{3}{4}\pm\frac{1}{2})\pi\,\mathrm{sgn}\Delta$ and

$$D(\phi_0)=(4\bar{n})^{-1}\alpha|\mathcal{L}|^2(\rho_{aa}-|\rho_{ac}\Delta/\Gamma|). \tag{9}$$

Hence the total phase noise in steady state [cf. Eq. (6)] becomes

$$\langle(\delta\phi)^2\rangle=(4\bar{n})^{-1}+D(\phi_0)[2\alpha|\mathcal{L}|^2|\rho_{ac}\Delta|/\Gamma+(s/\sqrt{\bar{n}})(|\rho_{ab}|+|\rho_{bc}|)]^{-1}, \tag{10}$$

where we have neglected a term proportional to $\bar{n}^{-1}$ in the denominator. In the absence of driving terms ($\rho_{ba}=\rho_{cb}=0$) the second term in the denominator vanishes. $D(\phi_0)$ is given by (8) for case (1) and (9) for case (2).

Next we investigate several cases. First in (1)-(3) below, we deal with the resonance condition $\Delta=0$, then in (4) below we shall investigate the case $\Delta\neq 0$.

(1) If $|\rho_{ac}|=0$, then $D(\phi_0)=A/4\bar{n}$, where $A=\alpha(\rho_{aa}+\rho_{bb})$ is the rate of spontaneous emission and for $\rho_{bb}=\rho_{cc}=0$ it coincides with the linear gain $G=\mathrm{Re}(\partial B/\partial r)$. In this case we have just the usual laser linewidth.[12]

(2) A particularly interesting case is $\rho_{aa}+\rho_{bb}=|\rho_{ac}|$. From Eq. (8) we now see that $D(\phi_0)=0$. Thus, the contribution of spontaneous emission is completely suppressed, and from Eq. (10) the phase noise is entirely due to vacuum fluctuations.

(3) If $\rho_{aa}+\rho_{bb}-|\rho_{ac}|<0$, then the added noise, as seen from Eq. (10) with $\Delta=0$, is negative and the total noise is suppressed below the vacuum level. Because of the presence of driving terms in (1) (terms proportional to s), we are still dealing with an active device ($\bar{n}>0$) even if the linear gain $G=\alpha(\rho_{aa}-\rho_{cc})<0$. (When $\rho_{ba}=\rho_{cb}=0$ such driving terms vanish, as does the mean photon number $\bar{n}$.) For any given mean photon number $\bar{n}$ the total phase noise (10), as a function of initial atomic variables, has a minimum. For $(\gamma/\alpha)^{1/2}\ll 1$, this minimum is of the order

$$\langle(\delta\phi)^2\rangle_{\min}\sim(4\bar{n})^{-1}(\gamma/\alpha)^{1/2}\ll(4\bar{n})^{-1}, \tag{11}$$

when the photon number is of the order

$$\bar{n}\sim(2s/\alpha)^2=(\Gamma/g)^2. \tag{12}$$

Here we used the definition of s and α as given after (1). Thus a significant intensity, in the form of squeezed light, can build up when atomic coherences ρ_{ba} and ρ_{cb} are present.

(4) The linear gain of the two-photon CEL under phase-locking conditions can be identified from the Fokker-Planck equation, corresponding to (1) in the P representation, as

$$G=\alpha|\mathcal{L}|^2(\rho_{aa}-\rho_{cc})+2\alpha|\mathcal{L}|^2|\rho_{ac}\Delta|/\Gamma, \tag{13}$$

in which $\alpha|\mathcal{L}|^2(\rho_{aa}\rho_{cc})$ is the usual laser gain and the rest is an extra two-photon CEL gain. When $\rho_{aa}-|\rho_{ac}\Delta/\Gamma|<0$ and $\alpha|\mathcal{L}|^2(\rho_{aa}-\rho_{cc})>\gamma$ or $G>\gamma$, one has both squeezing of phase fluctuations and laser gain, i.e., a two-photon CEL which generates squeezed light in the cavity. Note that only in the off-resonance case ($\Delta\neq 0$) can one have both gain and phase squeezing simultaneously. The gain and squeezing are, however, competing factors. Substitution of (9) into (10) with $\rho_{bb}=\rho_{ba}=\rho_{cb}=0$ gives

$$\langle(\delta\phi)^2\rangle=(8\bar{n})^{-1}(1+\rho_{aa}\Gamma/|\rho_{ac}\Delta|). \tag{14}$$

For a given Δ/Γ phase noise decreases when $\rho_{aa}-\rho_{cc}$ decreases. Squeezing of phase noise occurs when $\rho_{aa}/|\rho_{ac}|<|\Delta|/\Gamma$. For perfect coherence [$|\rho_{ac}|=(\rho_{aa}\rho_{cc})^{1/2}$] this means that if $\rho_{aa}<\rho_{cc}$ it is still possible to have both squeezing and net gain ($G>\gamma$) due to the extra two-photon CEL gain. The minimum phase noise is achieved in the limit $|\Delta|/\Gamma\gg 1$ (but $2\alpha|\mathcal{L}|^2|\rho_{ac}\Delta|/\Gamma>\gamma$). From (14) it asymptotically tends to $\langle(\delta\phi)^2\rangle=\frac{1}{2}(4\bar{n})^{-1}$, i.e., half the vacuum noise level corresponding to 50% squeezing. This is our main result.

In conclusion, we have shown that the cascade two-photon laser with injected atomic coherence diplays CEL quantum noise quenching and generates squeezed light beginning from an initial vacuum state. The two-photon CEL may exhibit net gain and phase squeezing simultaneously. The total phase noise in the positive net gain region can be smaller than the vacuum noise level by at most a factor of $\frac{1}{2}$, as given by Eq. (14). In the region where $G<0$, but because of the driving terms ρ_{ba} and ρ_{cb} we are still dealing with an active device, the total phase noise, as given by Eq. (11), can be smaller than the vacuum noise level by a factor of order $(\gamma/\alpha)^{1/2}$, and when the cavity losses are negligible ($\gamma\simeq 0$) one would have perfect squeezing.

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