Frequency Chirping of the Free-Electron Laser

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Pulsed free-electron lasers in which there is a dependence of the electron energy on the time of injection into the wiggler are expected to produce wide bandwidth but coherent chirped optical pulses with unique characteristics. Such pulses could be compressed in a dispersive medium to lengths on the order of a few optical periods. Chirping should also produce enhanced efficiency in the high-power trapped-particle regime.

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A free-electron laser (FEL) powered by an rf linac typically produces picosecond pulses of laser radiation of about the same length as the electron pulses delivered by the accelerator. When the laser pulses have been amplified to high power, the ponderomotive potential is sufficiently strong to trap a large fraction of the electrons. In this regime it has been predicted[1] and demonstrated[2] that tapering of the FEL wiggler can slow down the trapped electrons and substantially increase the FEL efficiency.

I have recently suggested[3] that chirping of the frequency $\omega_s(\tau)$ of the FEL, where $\tau = t - z/c$ is retarded time, provides an alternative mechanism for slowing down trapped electrons. Moreover, the highly chirped laser pulses have several potential applications.[3] In particular, they can be compressed in a dispersive optical medium to produce extremely short optical pulses.

Trapped electrons in the FEL oscillate about the trajectories of hypothetical "resonant particles," whose trajectories simply match the speed of the ponderomotive potential. The resonant-particle trajectories are given by

$$\int_0^z dz' k_q(z') = \int_{\tau_0}^{\tau} d\tau' \, \omega_s(\tau'), \tag{1}$$

where $k_q(z)$ is the wave vector of the tapered wiggler and τ_0 is the time of injection into the wiggler. Equation (1) defines a family $\tau(z,\tau_0)$ of nonintersecting trajectories, and $\tau(z,\tau_0)$ is a monontonically increasing function of z. The resonant electron energy $mc^2\gamma_r$ is given by

$$\gamma_r^2 = \omega_s(\tau)\Delta(z)/2ck_q(z), \tag{2}$$

where

$$\Delta(z) = 1 + [eB(z)/mck_q(z)]^2 = 1 + a_q^2 \tag{3}$$

accounts for the fact that a portion of the electron energy is converted into transverse oscillatory motion by the wiggler field [the rms value of this field is $B(z)$]. In the most interesting case where $\omega_s(\tau)$ is a decreasing function of τ, Eq. (2) gives energy extraction even in a uniform wiggler.

The frequency $\omega_s(\tau)$ is determined by Eq. (2) at $z=0$ and by the time variation of the entering electron energies. The key to chirping of the laser pulses is thus to make the electron energies depend on the time of entry into the wiggler. Additional reasoning is needed to determine ω_s for times at which no electrons enter, but this is a concern only near the ends of the electron pulse. Trapping is necessary in order to have a coherent laser field with radiation near a single frequency (and possibly its harmonics) for each value of τ. This is true as well for the unchirped tapered-wiggler FEL.

rf linacs can easily be adjusted to produce electron micropulses with a time-dependent variation in energy.[4] This is because electrons in different parts of the pulse experience different accelerations as a result of their different phases with respect to the rf field. Normally the accelerator is adjusted to minimize this effect, but one can increase the time-energy dispersion by shifting the electrons to rf phases where the field gradient is large and by using fairly long electron pulses (nearly a quarter of the rf period).

The long (typically 150 psec) electron pulses produced by the accelerator can be compressed by the achromatic bending magnets which inject the electron beam into the wiggler. Such compression is important even for the present Los Alamos FEL.[5] For the chirped-pulse FEL it will be a major factor in the determination of $\omega_s(\tau)$.

Let us consider an example aimed toward a possible experiment using (mostly) existing apparatus at Los Alamos. Suppose we have a uniform wiggler of length L and a frequency chirp

$$\omega_s(\tau) = \omega_0(1-q\tau)^2, \tag{4}$$

corresponding to a linear decrease of the entering-particle energies with time. From Eq. (1), the resonant-particle trajectories are

$$(3qk_q/\omega_0)z = (1-q\tau_0)^3 - (1-g\tau)^3. \tag{5}$$

For a square current pulse in the range $0 \le \tau_0 \le T$, and on the assumption that all of the electrons are trapped, the efficiency η is

$$\eta = 1 \frac{\int_0^T d\tau_0 \, \omega_s^{1/2}(\tau)}{\int_0^T d\tau_0 \, \omega_s^{1/2}(\tau_0)} \simeq \frac{\delta}{(1-qT)(1-qT/2)}, \tag{6}$$

where $\delta = qk_qL/\omega_0$, which is the limiting value of η for small qT, is assumed to be much less than 1. To correspond to the Los Alamos FEL[6] let us choose $k_qL/2\pi = 37$ and $2\pi c/\omega_0 = 10$ μm. Supposing that the incident energies decrease by 20% over a pulse length $T = 12.6$ psec, we calculate that $\eta = 2.7\%$. Since the experimental efficiency of the unchirped Los Alamos FEL with a uniform wiggler is 0.6%, this represents a large enhancement. A longer wiggler would give even higher efficiency.

Electron trapping in the FEL can be understood by the consideration of the single-particle equations used by Kroll, Morton, and Rosenbluth[1] to study the motion of trapped electrons in a tapered helical wiggler. With some changes in notation, these equations are

$$d\theta/dz = k_q - \omega_s \Delta/2c\gamma^2, \tag{7}$$

$$d\gamma/dz = -(\omega_s/c\gamma)a_q a_s \sin\theta. \tag{8}$$

Here

$$\theta = \int_0^z dz' k_q(z') - \int_0^\tau d\tau' \omega_s(\tau') \tag{9}$$

is the electron phase angle with respect to the ponderomotive potential and a_q and a_s are the dimensionless forms of the vector potentials of the wiggler and the laser (i.e., the vector potentials multiplied by e/mc). Both ω_s and a_s are in general functions of τ, while k_q, a_q, and Δ may be functions of z. (Actually a_s depends on z also, except in the limit of zero current, but this complication is not important for the present discussion.)

While we assume that a_q is of the order of unity, a_s is very small in essentially all cases of interest. Let us introduce a small parameter ϵ, such that ϵ^2 is of the order of a_s at saturation. An estimate based on the parameters of the Los Alamos FEL[7] at a peak intracavity power of 500 MW gives $\epsilon = 0.027$.

Equations (7) and (8) are valid (though certain small terms have been neglected) when there is both tapering and chirping, but have a particularly simple structure when there is only tapering. By choosing τ, rather than z, as the independent variable, one can obtain an alternative form of these equations which is particularly simple when there is only chirping. Using Eq. (9) to carry out the transformation, one obtains

$$d\theta/d\tau = 2c\gamma^2 k_q/\Delta - \omega_s, \tag{10}$$

$$d\gamma/d\tau = -(2\gamma\omega_s/\Delta)a_q a_s \sin\theta. \tag{11}$$

Trapped particles undergo oscillations about the resonant-particle trajectory, which has energy γ_r and phase θ_r, where γ_r is given by Eq. (2) and θ_r is given by

$$d\gamma_r/dz = -(2\gamma_r k_q/\Delta)a_q a_s \sin\theta_r \tag{12}$$

or

$$d\gamma_r/d\tau = -(2\gamma_r \omega_s/\Delta)a_q a_s \sin\theta_r. \tag{13}$$

Since $|\sin\theta_r| < 1$, Eqs. (12) and (13) impose limits on $\gamma_r^{-1}|d\gamma_r/dz|$ or $\gamma_r^{-1}|d\gamma_r/d\tau|$. Specifically, the distance over which energy can be extracted is on the order of ϵ^{-2} times the wiggler period $\lambda_q = 2\pi/k_q$, while the retarded time over which energy can be extracted is on the order of ϵ^{-2} times the optical $T_s = 2\pi/\omega_s$. As a consequence of Eq. (2), there are also the minimum distance and time over which k_q/Δ or ω_s can vary without detrapping the electrons. Since, in fact, the electrons slip one optical period for each wiggler period traversed, the length of wiggler needed to achieve an efficiency η is essentially the same for tapering and chirping. We can obtain a crude estimate of η by setting $\theta_r = \pi/2$ in Eq. (12), neglecting variations in a_q, a_s, or Δ, and integrating. One obtains $\eta = 1 - \exp(-2a_q a_s N/\Delta)$, where N is the number of wiggler periods. For the Los Alamos example presented above, we set $\eta = 2.7\%$ and $a_q = 0.76$ and calculate $\epsilon = a_s^{1/2} = 0.0277$. The laser power needed for trapping is $P = \epsilon_0 c^3 (m/e)^2 \Sigma \omega_s^2 \epsilon^4$, where Σ is the laser mode area. For a given resonator, Σ is inversely proportional to ω_s, so that P is proportional to $\omega_s \epsilon^4$. Thus, less power is required to trap electrons at the rear of the pulse, even though Σ is larger there. If $\Sigma = 3.30 \times 10^{-6}$ m^2 at the front of the pulse, then the power needed for trapping there is 532 MW.

By our considering small oscillations (so-called synchrotron oscillations) about the resonant-particle trajectories, it is easily shown that the period of these oscillations in z or τ is of order $\epsilon^{-1}\lambda_q$ or $\epsilon^{-1}T_s$. The laser field may also vary on this scale without causing detrapping. Moreover, the relative energy shifts $\gamma/\gamma_r - 1$ of trapped electrons are of order ϵ.

A detailed analysis of the chirped-pulse FEL requires coupling the electron equations of motion to the Maxwell wave equation for the laser field. I have formulated the relevant equations in a one-dimensional approximation.[3] If we neglect higher harmonics, assume the electron Lorentz factor γ to be large, and assume no energy spread uncorrelated with time, these equations become

$$\sigma(z,\tau)\frac{\partial}{\partial z}[\sigma(z,\tau)E_s(z,\tau)] = \frac{e}{2mc}\frac{\omega_s(\tau)}{\omega_s(\tau_0)}\frac{A_q(z)}{\gamma_r(z,\tau)}I(\tau_0)\frac{1}{2\pi}\int_0^{2\pi} d\theta_0 \exp[-i\theta(z,\tau_0,\theta_0)], \tag{14}$$

$$\frac{\partial\theta(z,\tau_0,\theta_0)}{\partial z} = 2k_q(z)[\gamma(z,\tau_0,\theta_0)/\gamma_r(z,\tau) - 1], \tag{15}$$

$$\frac{\partial\gamma(z,\tau_0,\theta_0)}{\partial z} = -\frac{1}{2c\gamma_r(z,\tau)}\frac{e^2}{m^2c^2}\{A_q(z)E_s^*(z,\tau)\exp[-i\theta(z,\tau_0,\theta_0)] + \text{c.c.}\}. \tag{16}$$

$I(\tau_0)$ is the current and θ is the electron phase angle with respect to the ponderomotive potential, while $E_s(z,\tau)$ and $A_q(z)$ are the slowly varying amplitudes of the laser electric field and the wiggler vector potential. The latter includes an additional factor (a difference of Bessel functions) if the wiggler is linearly polarized.[8] The variables z, τ, and τ_0 are related by Eq. (1), while $\gamma_r(z,\tau)$ is given by Eq. (2). The laser intensity is $|\sigma E_s|^2$, where

$$\sigma(z,\tau)=\left[\frac{\pi\epsilon_0 c^2 Z_{\rm R}}{\omega_s(\tau)}\right]^{1/2}\left[1+\frac{i(z-z_0)}{Z_{\rm R}}\right] \tag{17}$$

accounts approximately for diffractive spreading and phase shift.[9] $Z_{\rm R}$ and z_0 are the Rayleigh range and the position of the beam waist.

I have written a computer program to solve the above equations in a multipass resonator configuration. Preliminary calculations have yielded 2.5% efficiency with the Los Alamos uniform wiggler. These calculations assume a Gaussian electron pulse with peak current of 100 A and $1/e$ half-width of 6.17 psec. In this example $\gamma_r(\tau)$ varies from 42.0 to 25.2 and the laser wavelength varies from 10.0 to 27.8 μm, with the most rapid variation occurring when the current (and hence the laser power) is largest. This permits better energy extraction over the whole pulse than could be obtained by wiggler tapering. The saturated gain is 9% at a peak power of 0.67 GW, while the small-signal gain is 20%.

Operating the chirped-pulse FEL as an oscillator will require wide-bandwidth, low-dispersion optics. Moreover, the electron pulses should have nearly the same time dependence of the energy over many passes. The resonator length must be set (possibly servo controlled) to maintain synchronization of the electron and optical pulses and keep the electrons resonant with the ponderomotive potential. A 0.5-μm cavity-length detuning can significantly reduce the efficiency after 20 passes, if one starts from a resonant high-power pulse. On the other hand, the front of the optical pulse may erode eventually even with perfect synchronization because of laser lethargy.[10] It could be helpful to introduce a small amount of optical dispersion at the high-frequency end of the pulse to push it forwards and prevent this erosion. One of the major questions yet to be explored is the influence of chirping on optical pulse stability and the development of frequency sidebands.[11]

By tailoring the frequency $\omega_s(\tau)$ to match the index of refraction $n(\omega)$ of a dispersive optical medium, it should be possible for us to compress the optical pulses after they leave the FEL to a very short length, on the order of 2π divided by the bandwidth in ω. For the example of Eq. (4) this yields a pulse length of 93 fs or of 28 μm. The compression raises the intensity by a factor of about 136.

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