Family Mass Hierarchy from Universal Seesaw Mechanism

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The "universal seesaw mechanism," which accounts for $m_{e,u,d} \ll m_W$, predicts $m_{\nu_L} m_{\nu_R} \approx m_e^2$. Combined with an axial-vector symmetry principle, this mechanism allows for the coexistence of heavy ($\lesssim m_W$) families without appeal to a hierarchy in Yukawa couplings and without introduction of additional mass scales. It then follows that (i) the physical right-handed heavy (light) ordinary fermions are mostly $SU(2)_R$ singlets (doublets), (ii) there is no analogous neutrino mass hierarchy, and (iii) $m_{PQ} m_{\nu_L} \approx m_W^2$ sets the Peccei-Quinn mass scale.

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The origin of quark and lepton masses is well established within the framework of the standard Glashow-Weinberg-Salam electroweak theory.[1] But ironically, this theory lacks the ability to account for their actual masses. With $m_W \approx 80$ GeV being the only spontaneously generated mass scale, it seems puzzling to have $m_e \sim 10^{-5} m_W$, especially when the top-quark mass $m_t \sim m_W$. Further, the neutrino sector receives a different treatment. And although it is possible to arrange for $m_{\nu_L} \lesssim 10^{-5} m_e$, in particular in the framework of the left-right-symmetric $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ extension,[2] the $m_{\nu_L} \ll m_e$ hierarchy is not correlated with $m_e \ll m_W$.

The "universal seesaw (US) mechanism"[3,4] is a device engineered to address the $m_{e,u,d} \ll m_W$ problem. To make it work, the fermionic representation was enlarged to include the $SU(2)_L \otimes SU(2)_R$-singlet partners. The Higgs system was simplified to its limits, containing none of the scalars which accompany conventional left-right-symmetric models. The major bonus of this scheme is the mass formula[3]

$$m_{\nu_L} m_{\nu_R} \approx m_e^2, \tag{1}$$

which follows without any further input. In other words, $m_{\nu_L} \ll m_e$ is correlated with $m_e \ll m_W$.

However, if in the standard model the question is why is the electron so light, in the US-mechanism scheme, the question is why is the top quark so heavy. A desirable situation would be to let some fermions acquire the Glashow-Weinberg-Salam mass scale, while protecting others by means of the US mechanism. In this Letter we demonstrate how such a situation can be realized without the introduction of additional mass scales beyond the ones mandatory for $m_e \ll m_W$, and without our appealing to a hierarchy in the Yukawa couplings.

Let us first briefly discuss the single-generation case in which two possible scenarios may emerge. The standard *complex* $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ representation

$$\begin{aligned} &q_L(3,2,1)_{1/3} + q_R(3,1,2)_{1/3}, \\ &l_L(1,2,1)_{-1} + l_R(1,1,2)_{-1}, \end{aligned} \tag{2}$$

is supplemented by the *real* piece

$$\begin{aligned} &U_{L,R}(3,1,1)_{4/3} + D_{L,R}(3,1,1)_{-2/3}, \\ &N_{L,R}(1,1,1)_0 + E_{L,R}(1,1,1)_{-2}, \end{aligned} \tag{2'}$$

such that every ordinary fermion has an $SU(2)_L \otimes SU(2)_R$-singlet companion with matching $SU(3)_C \otimes U(1)_Q$ assignments. The overall representation is unifiable.[5] The fermions couple to

$$\phi_L(1,2,1)_{-1} + \phi_R(1,1,2)_{-1}, \tag{3}$$

which provides a minimal Higgs system up to a possible scalar σ $(1,1,1)_0$. With $\langle \phi_{L,R} \rangle = v_{L,R}$ governing the spontaneous symmetry breakdown, we must have $v_L \ll v_R$. Notice that the conventional sources of quark and lepton masses, namely[2]

$$\Phi(1,2,2)_0 + \Phi(1,3,1)_{-2} + \Phi(1,1,3)_{-2},$$

are not introduced; these "old" scalars are nothing but bilinears[3] of the "new" scalars, and will appear as effective fields in the low-energy limit. Now, there is the *option* of incorporating an $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$-invariant mass term $\chi \bar{F}_L F_R$ $(F = U,D,E,N)$. This may represent a bare mass term, or alternatively $\chi \sim \langle \sigma \rangle$. The so-called "survival hypothesis" suggests $\chi \gg v_{L,R}$. The two possible scenarios that emerge are as follows:

(1) With the χ mass term present, a typical single-generation Dirac mass matrix takes the form

$$M_{(1)} = \begin{pmatrix} 0 & L \\ R & \chi \end{pmatrix}, \tag{4}$$

where $L,R \sim v_{L,R}$. This is the US mechanism, leading to (i) $m_f \sim LR/\chi \ll m_W$ being the lightest eigenmass, (ii) $f_R^{\rm phys}$ being mostly an $SU(2)_R$ doublet, and (iii) the superheavy fermions having a mass scale χ.

(2) If the χ term is absent as a result of some as yet unspecified symmetry principle, one has the diagonalized form

$$M_{(2)} = \begin{pmatrix} 0 & L \\ R & 0 \end{pmatrix}. \tag{5}$$

This in turn implies that (i) $m_f \sim L \sim m_W$, (ii) $f_R^{\rm phys}$ is an $SU(2)_R$ singlet, and (iii) the superheavy fermions have a mass scale R.

A pedagogical remark is in order. Once an extra symmetry is invoked to prevent some fermionic masses of order L from *dropping* into the LR/χ regime, the traditional role of a symmetry is apparently turned upside down. However, this should be regarded as a US-mechanism artifact. One must take into account the fact that, at the same time, that symmetry does protect some other fermions of mass R from the acquisition of the heavier χ scale.

We now attempt to merge scenarios (1) and (2) into a proper multigeneration scheme. The crucial point is that the submatrix $\hat{\chi}$ has to be singular. This hierarchy-motivated $\hat{\chi}$ singularity is enforced by means of a horizontal U(1) symmetry. Such a symmetry cannot be vectorial. If F_L^i and F_R^i ($i=1,\ldots,N$ is the generation label) had the same U(1) hypercharge, a bare diagonal contribution $\sum \chi_i \bar{F}_L^i F_R^i$ would have been generically allowed, making it unlikely for a singular $\hat{\chi}$ to emerge. In other words, *a singular $\hat{\chi}$ calls for an axial-vector horizontal*[6] *U(1)*. This conclusion is favored by (i) *grand unification*—if the f_L^i and $(\bar{f})_L^i \sim f_R^{i*}$ form an irreducible parent representation, they must have a common horizontal charge—and also by (ii) *the strong CP puzzle*[7]—a global anomalous $U(1)_A$ may play the Peccei-Quinn (PQ) role.[8] Unlike the single-generation scheme which is $SU(5)_L \otimes SU(5)_R$ embeddable,[5] the grand unification of the multigenerational extension is still an open question.

Let the N-generational US-mechanism Lagrangean be classically invariant under

$$\begin{aligned} f_{L,R}^i &\to e^{\pm i\theta x_i} f_{L,R}^i \quad (f=u,d,v,e), \\ F_{L,R}^i &\to e^{\pm i\theta y_i} F_{L,R}^i \quad (F=U,D,N,E). \end{aligned} \tag{6}$$

Let the yet unspecified $U(1)_A$ charges x_i, y_j be such that $x_i \neq x_j$ and $y_i \neq y_j$ for $i \neq j$. This way, the generations are distinguished. Moreover, as is well known, the minimal number of Higgs scalars is doubled.[8,9] Their corresponding hypercharges can be normalized to ± 1, that is,

$$\begin{aligned} \phi_L^{1,-1} &\to e^{\pm i\theta} \phi_L^{1,-1}, \\ \phi_R^{1,-1} &\to e^{\pm i\theta} \phi_R^{1,-1}. \end{aligned} \tag{7}$$

Next, define $K_{ij} \equiv x_i + y_j$, and momentarily concentrate on the quark sector. $K_{ij} = +1$ implies Yukawa vertices of the types $\bar{q}_L^i \phi_L^1 U_R^j$, $\bar{U}_L^j (\phi_R^{-1})^\dagger q_R^i$, $\bar{q}_L^i (\phi_L^{-1})^\dagger D_R^j$, and $\bar{D}_L^j \phi_R^1 q_R^i$. Similarly, $K_{ij} = -1$ implies Yukawa vertices $\bar{q}_L^i \phi_L^{-1} U_R^j$, $\bar{U}_L^j (\phi_R^1)^\dagger q_R^i$, $\bar{q}_L^i (\phi_L^1)^\dagger D_R^j$, and $\bar{D}_L^j \phi_R^{-1} d_R^i$. $K_{ij} \neq \pm 1$, on the other hand, means no corresponding Yukawa couplings. Notice that (i) we cannot allow identical entries in the same row or column of K, as this would violate our principle of distinguishing the generations; and (ii) at least N out of the N^2 available K entries must equal ± 1, as otherwise the associated mass submatrix ($m_{ij}=0$ if $K_{ij} \neq \pm 1$) would exhibit a physically unacceptable vanishing determinant. We also ignore trivial K patterns which allow a larger axial-vector symmetry and consider patterns which transform into each other by $x_i \leftrightarrow x_j$, $y_i \leftrightarrow y_j$, or $\phi^1 \leftrightarrow \phi^{-1}$ as equivalent.

At the pedagogical two-generation level ($N=2$), we are left with the single tenable pattern,

$$K = \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix}. \tag{8}$$

In particular, with a dot denoting entries $K_{ij} \neq \pm 1$, $\left(\begin{smallmatrix} \cdot & 1 \\ 1 & \cdot \end{smallmatrix}\right)$ and $\left(\begin{smallmatrix} 1 & \cdot \\ \cdot & -1 \end{smallmatrix}\right)$ are trivial, while $\left(\begin{smallmatrix} -1 & 1 \\ 1 & -1 \end{smallmatrix}\right)$ is self-inconsistent. Given the pattern (8), for which $x_1 - x_2 = y_1 - y_2 = 2$, the horizontal charges are determined up to $x_i \to x_i + a$, $y_j \to y_j - a$. This superfluous degree of freedom is to be explicitly broken at the χ sector. It then follows that the corresponding mass submatrix is of the Fritzsch type.[10] Such 2×2 submatrices constitute the off-diagonal blocks of the full 4×4 mass matrix in each quark sector.

The χ sector is next. Here the situation is more complicated, as each individual mass term may either be bare or alternatively $\sim \langle \sigma \rangle$. Let us first examine in detail the somewhat simpler σ-free case. The relevant structure to analyze is the symmetric $Y_{ij} \equiv y_i + y_j$. Its zeroes are the only entries which really matter. If $Y_{ij}=0$ for some i,j, we know that the bare-mass term $\bar{F}_L^i F_R^j$ and its symmetric $\bar{F}_L^j F_R^i$ are both present. These terms explicitly break the superfluous axial-vector degree of freedom mentioned earlier. Having in mind a singular $\hat{\chi}$, we are after a matrix Y with a single zero entry. The relevant patterns are $\left(\begin{smallmatrix} 0 & -2 \\ -2 & -4 \end{smallmatrix}\right)$ and $\left(\begin{smallmatrix} 4 & 2 \\ 2 & 0 \end{smallmatrix}\right)$, for which $y_1=0$ or $y_2=0$, respectively. Notice that with the zero entry specified, all other Y entries are fully determined (recall that $y_1 - y_2 = 2$). Altogether, two inequivalent models survive:

Model I: The hypercharges $x_1=3$, $x_2=1$, $y_1=0$, and $y_2=-2$ are associated with the mass matrix

$$M_{\rm I} = \begin{pmatrix} 0 & 0 & 0 & L \\ 0 & 0 & L' & l \\ 0 & R' & \chi & 0 \\ R & r & 0 & 0 \end{pmatrix}. \tag{9}$$

Model II: The alternative set $x_1=1$, $x_2=-1$, $y_1=2$, and $y_2=0$ gives rise to

$$M_{\rm II}=\begin{bmatrix}0&0&0&L\\0&0&L'&l\\0&R'&0&0\\R&r&0&\chi\end{bmatrix}. \qquad (10)$$

Here, $L,r\sim\langle\phi_{L,R}^{l}\rangle$ and $l,R\sim\langle\phi_{L,R}^{-1}\rangle$, and L',R' differ from L,R by Yukawa couplings. One can easily show that M_u and M_d are of the same form; they will only differ by $L\leftrightarrow l^*$, $r\leftrightarrow R^*$, and Yukawa coupling constants.

To extract the eigenmasses, it is useful to construct the quadratic $MM^\dagger$, and study its symmetric polynomials $S_n(m^2)$. With $L\sim L'\sim l$ and $R\sim R'\sim r$ subject to the basic US hierarchy $L\ll R\ll\chi$, we find $S_1(m^2)\sim\chi^2$, $S_2(m^2)\sim\chi^2R^2$, $S_3(m^2)\sim\chi^2R^2L^2$, and especially, $S_4(m^2)=\det^2M\sim R^4L^4$. Thus, for both models, the spectrum consists of two ordinary families of masses,

$$m_1\sim LR/\chi,\quad m_2\sim L, \qquad (11)$$

and their superheavy seesaw partners $m_1'\sim\chi$ and $m_2'\sim R$. The two mass matrices differ from each other by means of their associated mixing angles. To appreciate the point, consider the limit $\chi\to\infty$, where the heaviest fermion decouples and the lightest one becomes massless. In this limit, $\tan\zeta_I\to L/l$ while $\tan\zeta_{II}\to 0$, with ζ the mixing angle between the two ordinary fermions.

At the single-generation level, the US mechanism predicts[3] $m_{\nu_L}\sim L^2/\chi$ ($\ll m_e\sim LR/\chi$) with no further input. What is the multigenerational generalization of this powerful result? In particular for $N=2$, given the charged-fermion mass ratio $m_1/m_2\sim R/\chi$, with specified horizontal assignments, do we expect an analogous neutrinos mass hierarchy? The answer is contained within the 8×8 Majorana mass matrix (for model I)

$$M_\nu^{\rm I}=\begin{bmatrix}0&0&0&0&0&L_1&0&L_2\\0&0&0&0&L_1'&l_1&L_2'&l_2\\0&0&0&0&0&R_1&0&R_2\\0&0&0&0&R_1'&r_1&R_2'&r_2\\0&L_1'&0&R_1'&\chi_1&0&\chi&0\\L_1&l_1&R_1&r_1&0&0&0&0\\0&L_2'&0&R_2'&\chi&0&\chi_2&0\\L_2&l_2&R_2&r_2&0&0&0&0\end{bmatrix}. \qquad (12)$$

The basis used is $(\nu_L^1,\nu_L^2,\bar\nu_L^1,\bar\nu_L^2,N_L^1,N_L^2,\bar N_L^1,\bar N_L^2)$. If all Yukawa coupling constants are of the same order of magnitude, it is easy to verify that two eigenmasses are $\sim\chi$, while another four are $\sim R$. As far as the lightest two eigenmasses are concerned, observe that (i) $m_{1,2}\to 0$ as $L\to 0$ or $\chi\to\infty$, and (ii) $\det M_\nu\sim R^4L^4$. This means that

$$m_{\nu1}\sim m_{\nu2}\sim L^2/\chi, \qquad (13)$$

corresponding to (mostly) ordinary left-handed neutrinos. Thus, the associated surprising features are the following: (i) no mass hierarchy among the automatically superlight left-handed neutrinos, and (ii) no relatively light, that is $\sim R^2/\chi$, right-handed neutrinos.

A comment is in order. So far, models I and II are both physically reasonable. But from the neutrino point of view, model II is apparently pathological. $M_\nu^{\rm II}$ has its eigenmasses $\sim L^2/\chi$, $L,L,R^2/\chi,R,R,\chi,\chi$, suggesting a light neutrino accompanied by a double structure at the m_W scale. Thus, as far as the neutrino sector is concerned, it is model I which is preferred.

The fact that the family mass hierarchy originating from the US mechanism calls for a horizontal $U(1)_A$, which furthermore happens to be color anomalous ($\sum x_i+\sum y_j=N\neq0$), is nontrivial. The fundamental PQ ingredient is present. For the σ-free model discussed, the emerging PQ mechanism comes with its scale fixed at $m_{\rm PQ}\sim R$. We also know that $m_{\rm PQ}\lesssim10^{12}$ GeV, if the underlying cosmological reasoning[11] is to be trusted, telling us that $R\lesssim10^{12}$ GeV. However, as regards $m_e\ll m_W$, it is only the ratio R/χ that matters, thus pushing χ towards the grand-unification theory scale. Consequently, the neutrino mass drops into the $L^2/\chi\gtrsim10^{-4}$-eV regime. We are also familiar with the astrophysical constraint[7] $m_{\rm PQ}\gtrsim10^9$ GeV. As long as $m_{\rm PQ}\sim R$, this lower bound can be translated into $m_{\nu_L}\sim m_em_w/m_{\rm PQ}\lesssim0.1$ eV. Apart from upsetting those who favor light W_R or heavier neutrinos, there is apparently nothing terribly wrong with such a scenario. But still two questions bother us:

(i) Can we naturally have 10-eV neutrinos?

(ii) Is there a reason for $m_{\rm PQ}\lesssim10^{12}$ GeV based on the US mechanism?

The affirmative answers must be associated with our letting

$$m_{\rm PQ}\sim\chi\sim m_W^2/m_{\nu_L}. \qquad (14)$$

If the PQ scale is χ rather than R, then $m_{\nu_L}\simeq10$ eV$\leftrightarrow m_{\rm PQ}\simeq10^{12}$ GeV. Amazingly, such a correlation is an exact result within the Dine-Fischler-Srednicki extension[12] of the model in hand.

Let us introduce the scalar field $\sigma(1,1,1)_0$. It can play a triple role:

(i) The Dine-Fischler-Srednicki role[12]: $\langle\sigma\rangle$ spontaneously breaks $U(1)_A$ without affecting the underlying gauge invariance. It is worth recalling that the invisible horizontal axion (identical with the familon)[6,13] is accompanied by invisible[14] flavor-changing neutral currents.

(ii) The Chang-Mohapatra-Parida role[15]: $\langle\sigma\rangle$ spontaneously violates left-right symmetry, allowing for $R\gg L\neq0$. The latter was a problem in the original US mechanism.[3,4]

(iii) The US-mechanism role: $\chi \sim \langle\sigma\rangle$.

For (i) and (ii), a special cubic-interaction term of the form

$$\sigma\left[\phi_L^1(\phi_L^{-1})^\dagger - \phi_R^1(\phi_R^{-1})^\dagger\right] \qquad (15)$$

is mandatory. The parity-odd σ then transforms via $\sigma \to e^{-2i\theta}\sigma$ under the horizontal $U(1)_A$.

With the hypercharge of σ fixed, we can now replay the previous game, only with slightly modified rules. These are the ± 2 entries of Y which signal existing Yukawa couplings of the type $\sigma^* \bar{F}_L^i F_R^j$ and $\sigma \bar{F}_L^i F_R^j$, respectively. The zero entries, on the other hand, are forbidden now, as they may allow for undesirable bare-mass terms. Altogether, the Y patterns of interest are $\left(\begin{smallmatrix} -2 & -4 \\ -4 & -6 \end{smallmatrix}\right)$ for which $x_1=4$, $x_2=2$, $y_1=-1$, $y_2=-3$, and $\left(\begin{smallmatrix} 6 & 4 \\ 4 & 2 \end{smallmatrix}\right)$ with $x_1=0$, $x_2=-2$, $y_1=3$, $y_2=1$. For both, $x_1+x_2+y_1+y_2=2$. These patterns lead to generalized models I and II, respectively. Nevertheless, all the previous mass matrices are reproduced, and the same analysis is still valid. The only new ingredient is the invisible axion. In view of $m_{PQ} m_{\nu_L} \approx m_W^2$, interesting neutrino-axion physics[16] is expected.

Summarizing, our main concern was to show that not only can we have an electroweak model with natural $m_e \ll m_W$ and automatic $m_\nu \sim m_e W_L / W_R \ll m_e$, but also that it comes with a built-in family mass hierarchy. A horizontal axial-vector symmetry plays the apparently upside-down role of protecting some fermion masses of order m_W from dropping into the m_e regime. The model serendipitously displays a variety of exclusive features. In particular, (i) unlike standard left-right-symmetric models, the physical right-handed heavy (light) ordinary fermions are mostly $SU(2)_R$ singlets (doublets), (ii) no neutrino mass hierarchy is permissible, thus deviating from the conventional electronvolt-kiloelectronvolt-megaelectronvolt-type spectrum, and (iii) the Peccei-Quinn scale gets correlated with the neutrino mass scale via $m_{PQ} m_{\nu_L} \sim m_W^2$. These features are independent of the total number of generations. For simplicity reasons, however, the model has been discussed at the two-generational level. The three-generation model is by far more technical, and requires a detailed quantitative analysis to overcome the dilemma of dealing with (two light plus one heavy) families versus the (one light plus two heavy) alternative. It will be published elsewhere.[17]

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