Finite-Temperature Phase Transitions in Lattice QCD for a General Number of Flavors

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Finite-temperature transitions in lattice QCD are studied for various numbers of flavors in the range $1 \leq N_f \leq 18$ on an $8^3 \times 4$ lattice by the Langevin simulation technique. It is found that the weakening of the transition at intermediate quark mass is a general feature for $N_f \geq 2$, but that the smoothing out of the transition observed for $N_f = 2$–4 does not occur for large numbers of flavors ($N_f \geq 10$). For $N_f = 1$ the transition weakens toward small quark mass m_q but remains first order down to $m_q a = 0.05$.

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While the basic feature of QCD is determined by the dynamics of the gauge field,[1] there are some cases where the reaction of quarks on the gauge field is expected to modify the qualitative behavior of the theory. The best studied example is the effect of quarks on the phase structure of QCD at finite temperature. It has been recognized that the first-order deconfining phase transition becomes weaker when dynamical quarks couple to the system and it may even disappear as quark mass m_q decreases.[2] On the other hand, for zero quark mass, the presence of quarks should trigger a transition in association with the recovery of the chiral symmetry at high temperatures. Numerical simulation reconciles these apparently contradictory statements in the way that the first-order phase transition, which disappears once as m_q decreases, will be recovered for a smaller quark mass.[3] Such a behavior has indeed been established for $N_f = 4$,[4] and evidence has been reported for $N_f = 3$[5] and 2.[6,7] The dependence on the flavor number in general, however, remains an interesting issue with this phase transition for several reasons. An effective-σ-model approach with the aid of the $4-\epsilon$ expansion predicts[8] that the chiral phase transition should be first order for $N_f \geq 3$, while for $N_f = 2$ it does not say definitely whether it should be first or second order, but the latter is preferred. The model predicts for $N_f = 1$ the absence of the chiral transition, for the determinant term in the Lagrangean explicitly breaks the chiral symmetry and hence washes out the phase transition which otherwise appears as second order. It is also of considerable interest to ask what is the behavior for large N_f,[9,10] where we expect an increasingly strong chiral perturbation from quarks but their effect at the same time weakens the deconfining phase transition. If we continue to increase N_f, the confinement will eventually be lost and a phase transition at zero temperature which separates the weak-coupling unconfined phase from the strong-coupling confining phase should appear.

In this Letter we report an initial analysis for the phase transition for a general number of flavors. We used the Langevin method,[11,12] and the number of flavors is effectively controlled by the strength of the bilinear noise term $-N_f \partial(\xi^\dagger \ln D \xi)$ with D the Kogut-Susskind Dirac operator. The method of simulation is essentially the same as reported in Ref. 6. We made typically $(2-5)\times 10^3$ iterations with the Langevin time step $\Delta\tau = 0.01$ at each parameter set on an $8^3 \times 4$ lattice, extending them to $(10-30)\times 10^3$ iterations where necessary. Some runs were also made on an 8^4 lattice.

$N_f = 1$.—The simulation is made for $m_q a = 0.4$, 0.2, 0.1, and 0.05. The average of Polyakov line $\langle \mathrm{Re}\,\Omega \rangle$, as presented in Fig. 1(a) ($50 \leq \tau \leq 100$), shows a jump at a certain value of β for each m_q, and this characteristic does not change from one m_q to another down to $m_q a = 0.05$. The magnitude of the jump $\Delta\langle \mathrm{Re}\,\Omega \rangle$ for $m_q a \gtrsim 0.2$ is comparable to that of the pure gauge system and decreases for smaller m_q (see Fig. 2). A similar abrupt jump is also observed in the chiral order parameter $\langle \bar{\chi}\chi \rangle$, with the magnitude of the jump more pronounced as m_q decreases. The behavior of the transition quite resembles the case with $m_q a = 0.1$ for $N_f = 2$ [see Fig. 1(b)] for which we have confirmed the first-order transition.[6]

To verify the first-order nature for the present case, we extended our runs in the transition region for $m_q a = 0.2$, 0.1, and 0.05 for which the transition may be weakening up to $\tau = 200$–300 and also made several new runs of similar length in steps of $\delta\beta = 0.01$. At $m_q a = 0.2$, the extended runs were quite stable with the average values of observables not changing from those of the short runs.

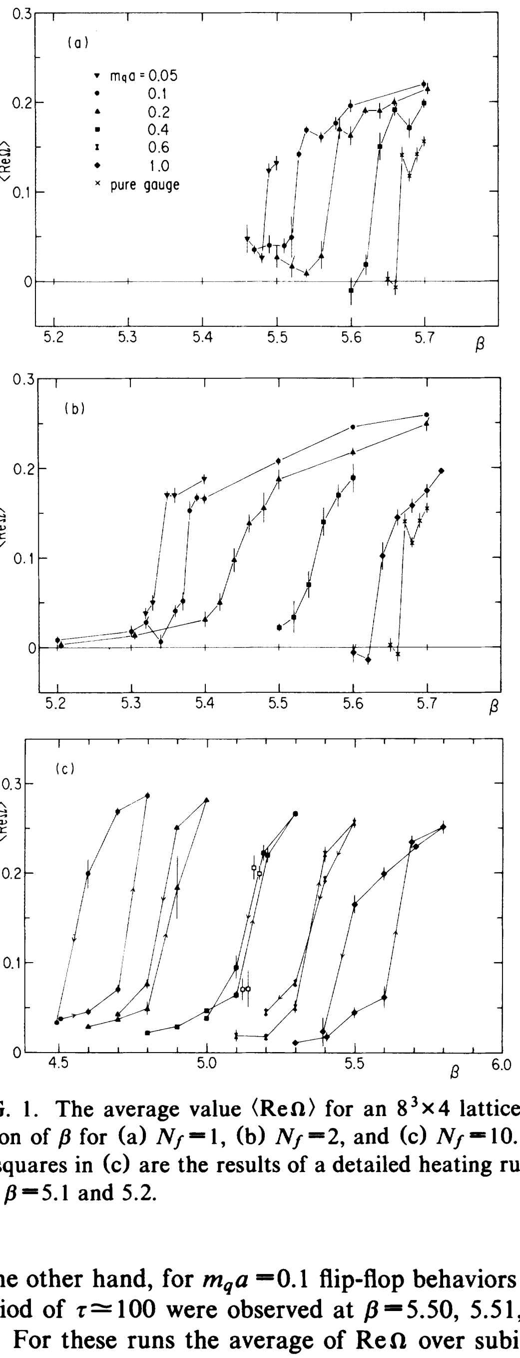

FIG. 1. The average value $\langle \mathrm{Re}\,\Omega \rangle$ for an $8^3 \times 4$ lattice as a function of β for (a) $N_f = 1$, (b) $N_f = 2$, and (c) $N_f = 10$. The open squares in (c) are the results of a detailed heating run between $\beta = 5.1$ and 5.2.

On the other hand, for $m_q a = 0.1$ flip-flop behaviors with a period of $\tau \simeq 100$ were observed at $\beta = 5.50$, 5.51, and 5.52. For these runs the average of $\mathrm{Re}\,\Omega$ over subintervals of $\tau = 50$ form two clearly separate clusters centering around $\langle \mathrm{Re}\,\Omega \rangle = 0.09$–$0.10$ and 0.04–0.05. Similar flip-flop behaviors with a somewhat more irregular pattern were seen for $m_q a = 0.05$ at $\beta = 5.47$ and 5.48.

The stable behavior at $m_q a = 0.2$ shows that the transition there is a relatively strong first-order transition. The appearance of flip-flops and the decrease of the amount of jump $\Delta \langle \mathrm{Re}\,\Omega \rangle$ across the transition at $m_q a = 0.1$ indicate that the transition weakens towards $m_q a = 0.1$. The transition, however, is still first order at $m_q a = 0.05$, as evidenced by the persistence of the flip-flop behavior. The transition might be weakening from

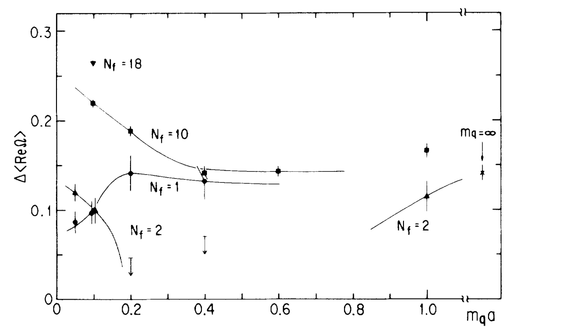

FIG. 2. The magnitude of the jump of the Polyakov line $\Delta \langle \mathrm{Re}\,\Omega \rangle$ across the transition as a function of $m_q a$ for various numbers of flavors N_f.

$m_q a = 0.1$ to 0.05, but it is not clear from our data whether it eventually disappears toward $m_q = 0$ reconciling with the original prediction of the σ-model analysis,[8] or remains first order.[13]

Our results show that the first-order phase transition persists at least down to $m_q a = 0.05$ with possible indications of weakening with decreasing m_q. This is rather different from the $N_f = 2$ case, where the transition becomes continuous for an intermediately light quark mass before turning first order again for $m_q a \lesssim 0.1$. For $N_f = 1$, the $Z(3)$ breaking effect of dynamical quarks is probably too weak to smooth out the first-order transition of the pure gauge system.

$N_f = 2$.—For $m_q a = 0.2$, 0.1, and 0.05, the results have been reported previously.[6] The transition shows first-order nature for $m_q a = 0.05$ and 0.1, and that for $m_q a = 0.2$ is of continuous transition. In the present analysis, the simulation has been extended to $m_q a = 0.4$ and 1.0. At $m_q a = 0.4$ the average value of Ω shows a continuous increase with β, while it exhibits an abrupt jump at $m_q a = 1.0$ [see Fig. 1(b)]. The continuous increase at $m_q a = 0.4$ is similar to that at $m_q a = 0.2$, but it appears to occur over a narrower interval. Thus the first-order deconfining transition, which persists at $m_q a = 1.0$, is smoothed out before $m_q a = 0.4$ and the dynamical quark continues to make the transition smoother at least down to $m_q a = 0.2$. From $m_q a = 0.1$ to 0.05, $\Delta \langle \mathrm{Re}\,\Omega \rangle$ increases as shown in Fig. 2, indicating that the transition becomes stronger towards $m_q = 0$ by the effect of the chiral phase transition.

$N_f = 10$.—We made thermal-cycle analyses with $\tau = 20$ at $m_q a = 0.1$, 0.2, 0.4, 0.6, and 1.0 taking averages over the last $\tau = 10$. As shown in Fig. 1(c), we have detected a clear hysteresis at $m_q a = 0.1$ and 1.0 which indicates first-order transitions at both the chiral and heavy-quark regions. The lack of hysteresis at other values of m_q shows that here again the transition is weakened at intermediate values of m_q. The increase of $\langle \Omega \rangle$ across the transition region, nonetheless, is very sharp at those values of $m_q a$. This feature and the fact that $\Delta \langle \mathrm{Re}\,\Omega \rangle$ continuously increases for smaller m_q (see

TABLE I. Critical value β_c for $8^3\times4$ lattice.

N_f \ $m_q a$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	1.0
1	5.475(5)	5.51(1)	5.57(1)		5.63(1)			
2	5.34(1)	5.375(5)	5.45(3)		5.54(2)			5.63(1)
4		5.10(10)	5.25(5)	5.35(5)		5.50(10)		5.65(15)
10		4.70(10)	4.85(5)		5.15(1)		5.35(5)	5.60(10)
12		4.50(10)						
18		4.25(15)						

Fig. 2) suggest persistence of a first-order transition. This is strongly supported by the additional runs made at $m_q a=0.4$ with a finer step in β ($\delta\beta=0.02$) which exhibited quite an abrupt jump between $\beta=5.14$ and 5.16 [see the open squares in Fig. 1(c)]; the sweep-to-sweep fluctuation was found to be much smaller (typically $\sim\frac{1}{10}$) than the jump. These results show that the transition remains first order for the entire range of the quark mass because of the strong chiral phase transition. The smoothing of the first-order transition at intermediate quark mass, which is typical of the cases $N_f=2$–4, is unlikely to occur at $N_f=10$.

Another interesting point with the phase transition at $N_f=10$ is whether it represents that for finite temperature or that for zero temperature, since the two-loop term of the renormalization-group β function changes sign at $N_f=8.05$ which may cause a first-order phase transition.[9] In the latter case the critical point β_c should not move when the temporal size of the lattice is changed, while it should for the finite-temperature transition. We made a thermal-cycle run on an 8^4 lattice at $m_q a=0.1$, and found that $\beta_c=4.9\pm0.1$ showing an increase of $\Delta\beta\simeq0.2$ compared with the critical value $\beta_c=4.7\pm0.1$ for an $8^3\times4$ lattice. This increase is consistent with what is expected from the β function for the finite-temperature phase transition for $N_f=10$ (i.e., $\Delta\beta=13\ln2/4\pi^2+\cdots\simeq0.22$ and the effect of scaling violation will decrease this value slightly).

$N_f=12$.—The simulation is made with $m_q a=0.1$ on an $8^3\times4$ and an 8^4 lattice. The qualitative features for an $8^3\times4$ lattice are quite similar to those for $N_f=10$. We also detected an increase of the critical coupling $\Delta\beta\simeq0.2$ when we moved from $8^3\times4$ ($\beta_c=4.5\pm0.1$) to 8^4 ($\beta_c=4.7\pm0.1$), and we conclude that the first-order transition we detected is of finite-temperature type.

$N_f=18$.—This number already exceeds the critical value $N_f=\frac{33}{2}$ for quark confinement in the continuum theory. The behavior of the system we found at $m_q a=0.1$, however, does not differ much from the cases for $N_f=10$ and 12. This is not surprising, because the lattice theory is always confining in the strong-coupling region where we expect a deconfining phase transition at finite temperature. We identify the phase transition of the present analysis with this type. The two-state signal was confirmed at $\beta=4.2$–4.3. There must be another type of phase transition at zero temperature separating the weak-coupling phase from the strong-coupling phase. To detect this, we have to use a much larger lattice and larger β.

It is conceptually well understood that dynamical quarks weaken the deconfining phase transition as they work as a source violating the center $Z(3)$ symmetry.[2] We may expect that such an effect is stronger for larger N_f; namely, we anticipate the smoothing at a larger m_q: The perturbative analysis in $1/m_q$ predicts that it happens at $m_q a\sim\exp[(1/N_t)\ln N_f]$ with N_t the temporal lattice size. This expected smoothing, however, did not occur in our simulation. For $N_f=10$ we rather found indications that the transition remains first order for all values of m_q. This should be ascribed to the strong chiral transition which takes over before the deconfining transition is smoothed out with decreasing m_q.

The role of the chiral phase transition is indeed apparent in the following places: (i) the increase of $\Delta\langle\mathrm{Re}\,\Omega\rangle$ for $N_f\gtrsim10$ towards smaller m_q as compared with the pure gauge system; (ii) larger value of $\Delta\langle\mathrm{Re}\,\Omega\rangle$ with increasing N_f; (iii) stronger transition for intermediate quark mass at $N_f\gtrsim10$; (iv) the recovery of first-order phase transition and the increase of $\Delta\langle\mathrm{Re}\,\Omega\rangle$ for smaller m_q for $4\gtrsim N_f\geq2$.

On the other hand, the role of the chiral transition is not conspicuous for $N_f=1$. While the transition remains

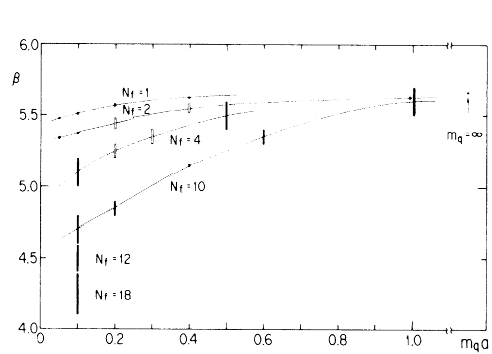

FIG. 3. The critical value β_c as a function of $m_q a$ for various numbers of flavors on an $8^3\times4$ lattice. Solid bars, discontinuous transitions; open bars, transitions that appear continuous. The data for $N_f=4$ are taken from Ref. 3.

first order down to $m_q a = 0.05$, an increase of its strength as observed for $N_f \gtrsim 2$ close to the chiral limit is not seen in this case. This might be a reflection of the absence of chiral transition, as predicted by the original σ-model analysis.[8]

Our results are summarized in Fig. 3 and Table I, where the critical coupling β_c is shown versus m_q for various N_f for an $8^3 \times 4$ lattice. Open bars in the figure show the region of transition for which the change of physical quantities appears continuous. As already noticed in earlier publications,[3,6] the smoothing of the transition due to dynamical quarks is visible for $N_f = 2-4$. Such a smoothing at intermediate range of the quark mass is not seen for $N_f = 1$ nor for large N_f ($\gtrsim 10$). Our observations thus lead us to conclude that the smoothing of the first-order transition is a phenomenon visible only in a limited range of number of flavors in general QCD dynamics.

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