Comment on "Self-Dual Fields as Charge-Density Solitons"

In a recent Letter, Floreanini and Jackiw[1] analyzed the quantization of two-dimensional self-dual fields. According to Floreanini and Jackiw,[1] one is to enforce a certain set of *unusual* equal-time commutation relations to secure that the Lagrange equations of motion are reproduced from the Hamiltonian formalism.

The Lagrangeans analyzed in Ref. 1 are first order in the velocities and, therefore, *necessarily* describe constrained systems. Moreover, these models only possess second-class constraints.[2] To quantize second-class systems the use of the Dirac-bracket quantization procedure is mandatory[2] and, therefore, the equal-time commutation relations are to be abstracted from the corresponding Dirac brackets. In this Comment we demonstrate that the unusual equal-time commutation relations defined by Floreanini and Jackiw[1] are nothing but the corresponding Dirac brackets for the models considered by these authors. To summarize, the agreement between the Hamiltonian and the Lagrangean description of the dynamics does not call for additional definitions.

We start by considering the nonrelativistic model in the Appendix of Ref. 1. The Lagrangean is ($C_{ij}=-C_{ji}$, $\det C\neq 0$)

$$L=\tfrac{1}{2}q_iC_{ij}\dot{q}_j-V(q), \qquad (1)$$

where i,j run from 1 to N. Notice that N must be even. After computing the canonically conjugate momenta $\{p_j\}$ one arrives at the conclusion that the system possesses N primary constraints $\{T_j\}$,

$$T_j\equiv p_j-\tfrac{1}{2}q_iC_{ij}\approx 0. \qquad (2)$$

The canonical Hamiltonian deriving from (1) reads $H=V(q)$. One can check that the Poisson bracket is

$$Q_{jk}\equiv[T_j,T_k]=C_{jk}\neq 0. \qquad (3)$$

As a consequence the Dirac algorithm[2] does not yield secondary constraints. Thereafter it follows from (3) that all constraints are second class.

For any two functions, f and g, of the phase-space variables, the Dirac bracket is defined as follows[2]:

$$[f,g]_D\equiv[f,g]-[f,T_j](Q^{-1})_{jk}[T_k,g]. \qquad (4)$$

The transition to the quantum theory is to be made by the abstraction of the equal-time commutation relations from the corresponding Dirac brackets,[2] namely,

$$i[f,g]_D\rightarrow[\hat{f},\hat{g}], \qquad (5)$$

where the caret indicates quantum operators. One then obtains

$$[\hat{q}_i,\hat{q}_j]=i(C^{-1})_{ij},$$
$$[\hat{q}_i,\hat{p}_j]=\tfrac{1}{2}i\delta_{ij},[\hat{p}_i,\hat{p}_j]=-\tfrac{1}{4}iC_{ij}, \qquad (6)$$

and, as consequence,

$$\dot{\hat{q}}_i=i[\hat{H},\hat{q}_i]=(C^{-1})_{ij}\,\partial\hat{v}/\partial\hat{q}_j. \qquad (6')$$

This last expression is in agreement with the Lagrange equations of motion deriving from (1).

We analyze next the two-dimensional field theory

$$L=\tfrac{1}{4}\int dx\,dy\,\chi(x)\epsilon(x-y)\dot{\chi}(y)-\tfrac{1}{2}\int dx\,\chi^2(x). \qquad (7)$$

The momentum canonically conjugate to $\chi(x)$ is given by $\pi(x)=\frac{1}{4}\int dy\,\chi(y)\epsilon(y-x)$, which implies that the system possesses an infinite continuous set of primary constraints $\{T(x)\}$,

$$T(x)\equiv\pi(x)-\tfrac{1}{4}\int dy\,\chi(y)\epsilon(y-x)\approx 0. \qquad (8)$$

The canonical Hamiltonian deriving from (7) is found to read $H=\frac{1}{2}\int dx\,\chi^2(x)$.

As in the previous case, the Dirac algorithm does not yield secondary constraints, and since

$$Q(x,y)\equiv[T(x),T(y)]=\tfrac{1}{2}\epsilon(x-y)\neq 0, \qquad (9)$$

one concludes that all constraints are second class.

The extension of Eq. (4) to field theory is trivial $[Q^{-1}(x,y)=\delta'(x-y)]$, and following the procedure indicated in (5) one obtains

$$[\hat{\chi}(x),\hat{\chi}(y)]=i\delta'(x-y), \qquad (10a)$$

$$[\hat{\chi}(x),\hat{\pi}(y)]=\tfrac{1}{2}i\delta(x-y), \qquad (10b)$$

$$[\hat{\pi}(x),\hat{\pi}(y)]=-\tfrac{1}{8}i\epsilon(x-y). \qquad (10c)$$

The Heisenberg equation of motion for $\hat{\chi}(x)$ leads to

$$\dot{\hat{\chi}}(x)=i[\hat{H},\hat{\chi}(x)]=\hat{\chi}'(x), \qquad (11)$$

in complete agreement with the Lagrange equation of motion deriving from (7).

We remark that the equal-time commutation relations (6) and (10) emerge naturally within the theory of constrained systems and need not be defined.

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